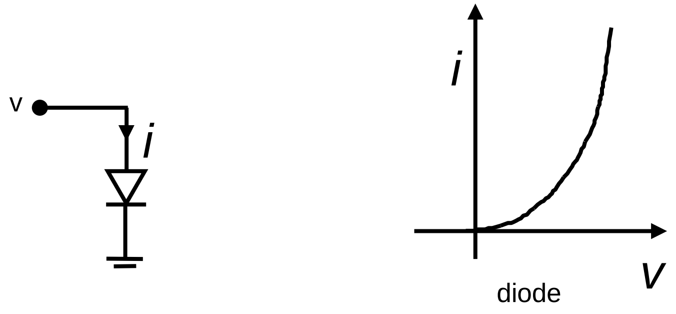
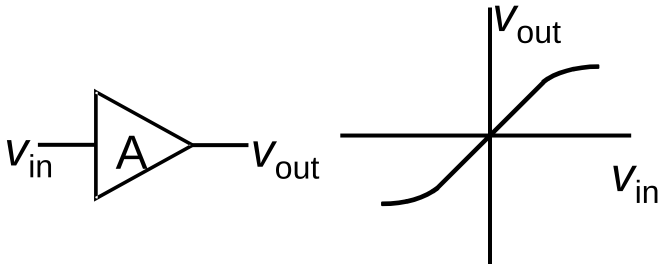
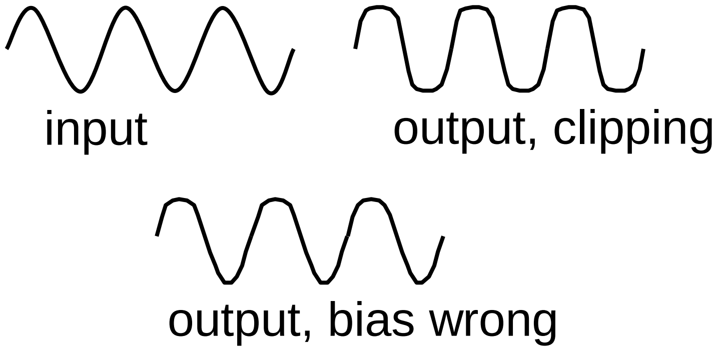
RF Systems Course: RF Concepts II

Linearity and Distortion in RF Circuits

- In an ideal system, output linearly related to input
- real device transfer function usually more complicated
- can be due to active or passive devices, or signal swing being limited by power supply rails.
- Unavoidably, gain curve for any component never straight line
- amplifier saturation-> top and bottom of waveform clipped equally
- If circuit not biased between two clipping levels, then clipping can be non-symmetrical







Power Series Expansion

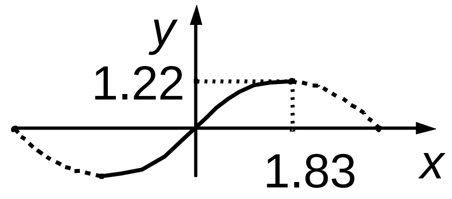
- any nonlinear transfer function can be written as series expansion of power terms unless system has memory

$$v_{\text{out}} = k_0 + k_1 v_{\text{in}} + k_2 v_{\text{in}}{}^2 + k_3 v_{\text{in}}{}^3 + \dots$$

- Ideally infinite number of terms required
- however first 3 terms usually enough

- Symmetrical saturation can be modeled with odd order terms e.g.:

$$y = x - \frac{1}{10}x^3$$



- another example exponential nonlinearity has the form:

$$x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

- contains both even and odd power terms, because it does not have symmetry about the y-axis.
- Real circuits have more complex power series expansions.

Power Series Expansion

- common way of characterizing linearity called the two-tone test.

$$v_{\text{in}} = v_1 \cos\omega_1 t + v_2 \cos\omega_2 t = X_1 + X_2$$

$$v_{\text{out}} = k_0 + k_1 v_{\text{in}} + k_2 v_{\text{in}}^2 + k_3 v_{\text{in}}^3 + \ldots$$

$$v_0 = k_0 + \underbrace{k_1(X_1 + X_2)}_{\text{desired}} + \underbrace{k_2(X_1 + X_2)^2}_{\text{second order}} + \underbrace{k_3(X_1 + X_2)^3}_{\text{third order}}$$

- terms can be broken into various frequency components.
- X_1^2 term has component at dc and at second harmonic of the input:

$$X_1^2 = (v_1 \cos\omega_1 t)^2 = \frac{v_1^2}{2}(1 + \cos 2\omega_1 t)$$

Power Series Expansion

The second-order terms:

$$(X_1 + X_2)^2 = \underbrace{X_1^2}_{\substack{\text{dc}+\\ \text{HD2}}} + \underbrace{2X_1X_2}_{\text{MIX}} + \underbrace{X_2^2}_{\substack{\text{dc}+\\ \text{HD2}}}$$

- comprised of second harmonics HD2, mixing components, sometimes labeled IM2 for second-order intermodulation.
- mixing components will appear at the sum and difference frequencies
- second-order terms cause an additional dc term to appear.

Power Series Expansion

- The third-order terms can be expanded as follows:

$$(X_1 + X_2)^3 = \underbrace{X_1^3}_{\substack{\text{FUND}\\ +\,\text{HD3}}} + \underbrace{3X_1^2 X_2}_{\substack{\text{IM3}\,+\\ \text{FUND}}} + \underbrace{3X_1 X_2^2}_{\substack{\text{IM3}\,+\\ \text{FUND}}} + \underbrace{X_2^3}_{\substack{\text{FUND}\\ +\text{HD3}}}$$

- Third-order nonlinearity results in third harmonics HD3 and third-order intermodulation IM3.
- Expansion of both the HD3 terms and the IM3 terms show output signals appearing at the input frequencies.
- The effect is that third-order nonlinearity can change the gain, which is seen as gain compression.

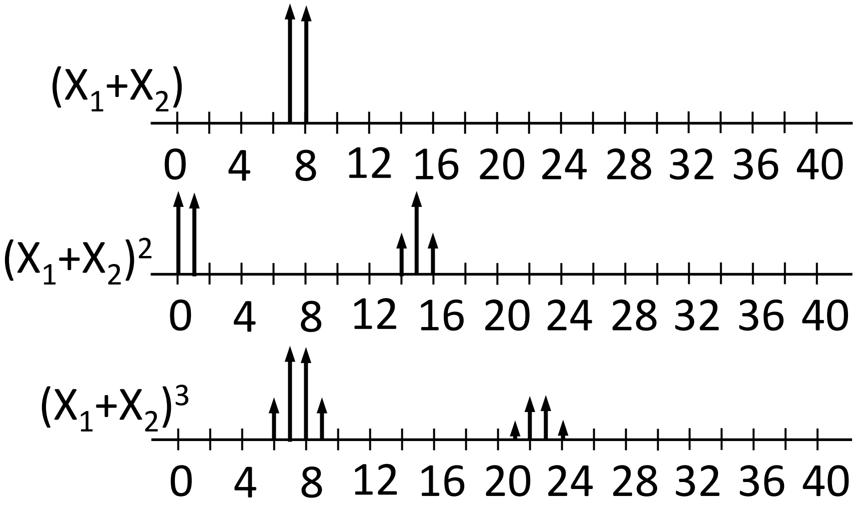
Power Series Expansion

Frequency	Component Amplitude for 1st, 2nd, and 3rd order terms	Component Amplitude for 4th and 5th order terms
DC	$k_o + \frac{k_2}{2}\left(v_1^2 + v_2^2\right)$	$k_4\left(\frac{3v_1^4}{8} + \frac{3v_1^2 v_2^2}{2} + \frac{3v_2^4}{8}\right)$
ω_1	$k_1 v_1 + k_3 v_1\left(\frac{3}{4}v_1^2 + \frac{3}{2}v_2^2\right)$	$k_5\left(\frac{5v_1^5}{8} + \frac{15v_1^3 v_2^2}{4} + \frac{15v_1 v_2^4}{8}\right)$
ω_2	$k_1 v_2 + k_3 v_2\left(\frac{3}{4}v_2^2 + \frac{3}{2}v_1^2\right)$	$k_5\left(\frac{15v_1^4 v_2}{8} + \frac{15v_1^2 v_2^3}{4} + \frac{5v_2^5}{8}\right)$
$2\omega_1$	$\frac{k_2 v_1^2}{2}$	$k_5\left(\frac{v_1^4}{2} + \frac{3v_1^2 v_2^2}{2}\right)$
$2\omega_2$	$\frac{k_2 v_2^2}{2}$	$k_4\left(\frac{3v_1^2 v_2^2}{2} + \frac{v_2^4}{2}\right)$
$\omega_2 \pm \omega_1$	$k_2 v_1 v_2$	$k_4\left(\frac{3v_1^3 v_2}{2} + \frac{3v_1 v_2^3}{2}\right)$
$3\omega_1$	$\frac{k_3 v_1^3}{4}$	$k_5\left(\frac{5v_1^5}{16} + \frac{5v_1^3 v_2^2}{4}\right)$
$3\omega_2$	$\frac{k_3 v_2^3}{4}$	$k_5\left(\frac{5v_1^2 v_2^3}{4} + \frac{5v_2^5}{16}\right)$
$2\omega_1 \pm \omega_2$	$\frac{3}{4}k_3 v_1^2 v_2$	$k_5\left(\frac{5v_1^4 v_2}{4} + \frac{15v_1^2 v_2^3}{8}\right)$
$2\omega_2 \pm \omega_1$	$\frac{3}{4}k_3 v_1 v_2^2$	$k_5\left(\frac{15v_1^3 v_2^2}{8} + \frac{5v_1 v_2^4}{4}\right)$

Power Series Expansion

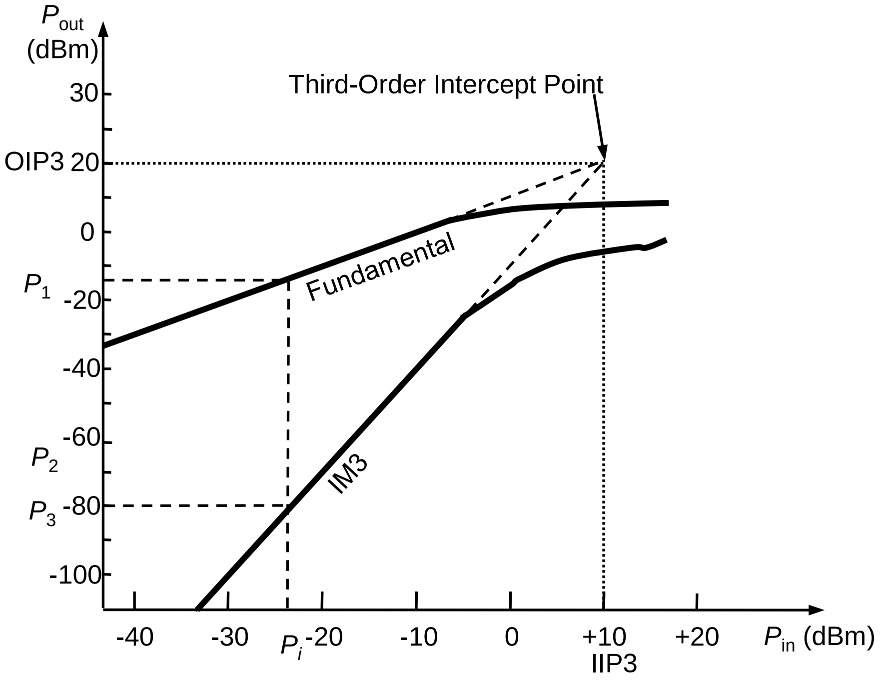
- For amplifier, only terms at input frequency are desired.
- two at frequencies $2\omega_1$-ω_2 and $2\omega_2$-ω_1 are troublesome
- they can fall in the band -> not be easily filtered out.
- called third-order intermodulation terms (IM3 products)

- Consider nonlinear circuit -> 7 MHz and 8 MHz input tones.



	Symbolic Frequency	**example frequency**	**Name**	**Comment**
First Order	f_1, f_2	7, 8	Fundamental	Desired Output
Second Order	0	0	DC	Bias Shifts
	$2f_1, 2f_2$	14, 16	HD2 (Harmonics)	Can filter
	$f_2 \pm f_1$	1, 15	IM2 (Mixing)	Can filter
Third Order	f_1, f_2	7, 8	Fund	Cause Compression
	$3f_1, 3f_2$	21,24	HD3 (Harmonic)	Can filter harmonics
	$2f_1 \pm f_2$, $2f_2 \pm f_1$	6, 22 9, 23	IM3 (Intermod) IM3 (Intermod)	Difference frequencies close to fundamental, dificult to filter

Third-Order Intercept Point



- third-order intercept point is theoretical point where amplitudes of IM3 tones at $2\omega_1 - \omega_2$ *and* $2\omega_2 - \omega_1$ = amplitudes of fundamental tones at ω_1 and ω_2.

- if $v_1 = v_2 = v_i$, then the output voltage at the fundamental frequency:

$$\text{fund} = k_1 v_i + \frac{9}{4} k_3 v_i^{\ 3}$$

- The linear component : $\text{fund} = k_1 v_i$
- can be compared to IM3 term : $\text{IM3} = \frac{3}{4} k_3 v_i^{\ 3}$
- for small v_i, fundamental rises linearly (20 dB/decade) and that the IM3 terms rise as the cube of the input (60 dB/decade)
- theoretical voltage at which these two tones will be equal:

$$\frac{3}{4} k_3 v_{\text{IP3}}^{\ 3} = k_1 v_{\text{IP3}}$$

This can be solved for v_{IP3}: $v_{\text{IP3}} = 2\sqrt{\frac{k_1}{3|k_3|}}$

- for a circuit experiencing compression, k_3 must be negative.
- absolute value of k_3 must be used.
- gives the input voltage at IIP3 point.
- *input third-order intercept point* (IIP3)
- *output third-order intercept point* (OIP3).

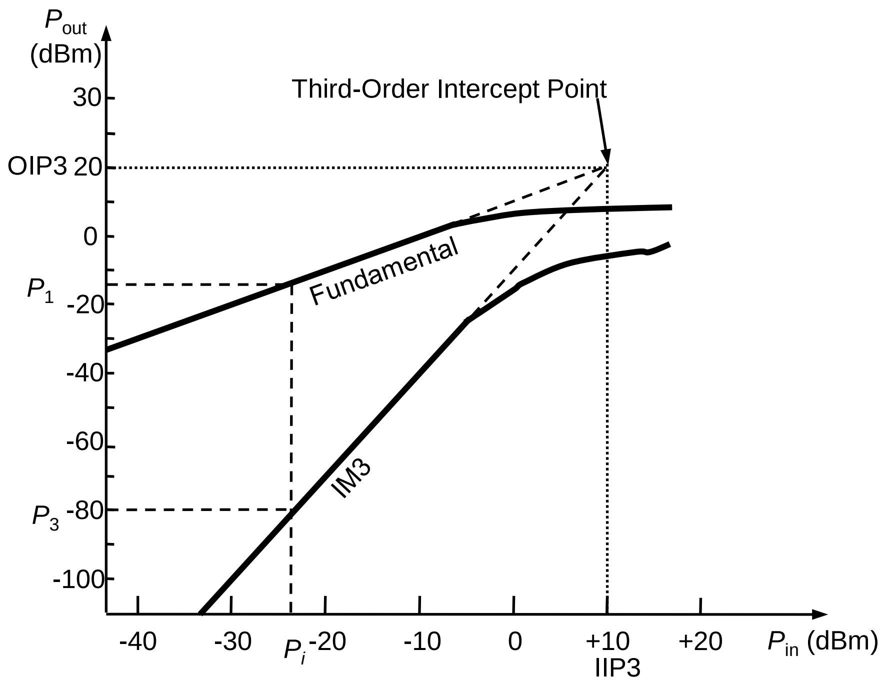
Third-Order Intercept Point

- IIP3 can't be measured, amp would blow up.
- How to find then?
- device with gain G at output power P_1 at fund and power P_3 at IM3 frequency with input power P_i.
- on a log plot P_3 and P_1 versus P_i, IM3 terms have slope of 3, funds have a slope of 1.

$$\frac{\mathrm{OIP3}-P_1}{\mathrm{IIP3}-P_i}=1 \qquad \frac{\mathrm{OIP3}-P_3}{\mathrm{IIP3}-P_i}=3$$

$$\mathrm{IIP3}=P_i+\frac{1}{2}\left[P_1-P_3\right] \qquad \mathrm{IIP3}=P_1+\frac{1}{2}\left[P_1-P_3\right]-G$$

$$G=\mathrm{OIP3}-\mathrm{IIP3}=P_1-P_i$$



Second-Order Intercept Point

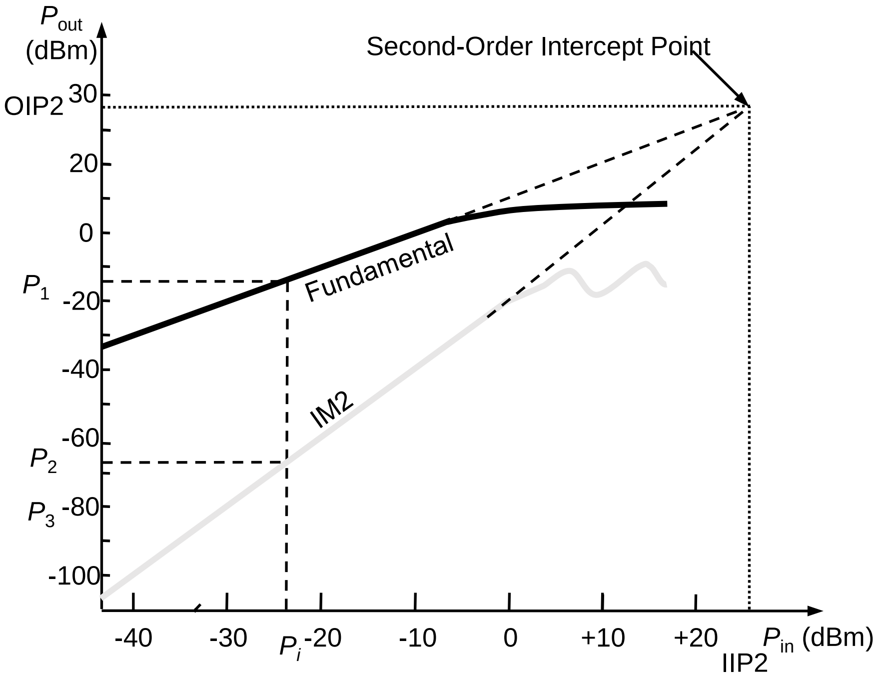
- *second-order intercept point* (IP2) similar to IIP3.
- Which to use? Talk about that later -> depends on system!
- E.g. 2nd order distortion important in direct down-conversion receivers.
- If two tones are present at the input:

$$v_{\mathrm{IM2}} = k_2 v_i^{\,2}$$

- IM2 terms rise at 40 dB/dec rather than at 60 dB/dec
- voltage which IM2 term = fund term:

$$k_2 v_{\mathrm{IP2}}^{\,2} = k_1 v_{\mathrm{IP2}}$$

$$v_{\mathrm{IP2}} = \frac{k_1}{k_2}$$



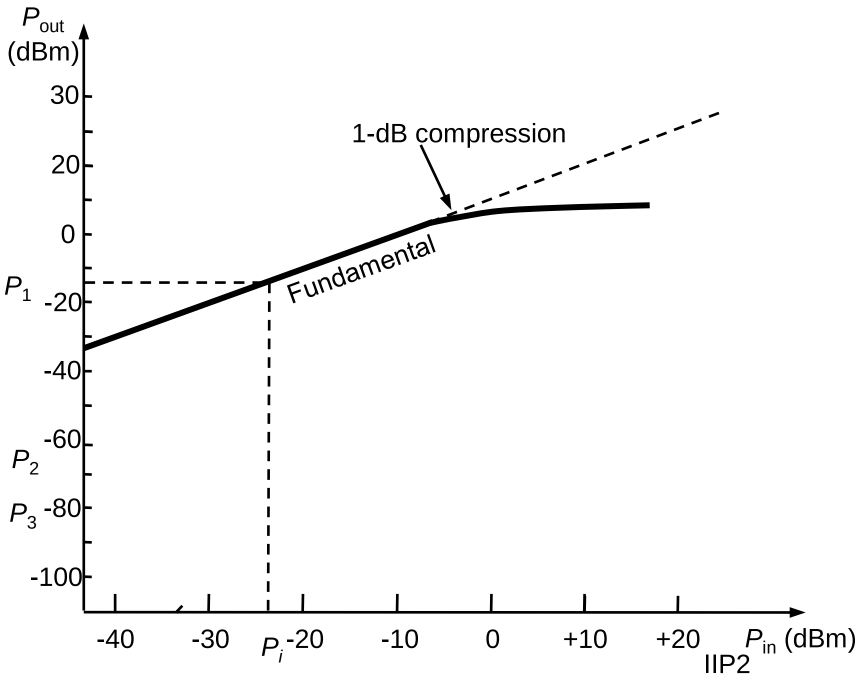
- device with G, output power P_1 at fund, power of P_2 at IM2, input power of P_i
- on a log IM2 terms have a slope of 2 fund have slope of 1

$$\frac{\mathrm{OIP2} - P_1}{\mathrm{IIP2} - P_i} = 1 \qquad \frac{\mathrm{OIP2} - P_2}{\mathrm{IIP2} - P_i} = 2$$

$$\mathrm{IIP2} = P_i + [P_1 - P_2] = P_1 + [P_1 - P_2] - G$$

1dB Compression Point

- directly measurable unlike IP3
- requires only one tone rather than two (although any number of tones can be used).
- P_{1dB} is power level where the output power is 1 dB less than it would have been in an ideally linear device.
- At P_{1dB} ratio of actual output voltage v_o to ideal output voltage v_{oi} is:



$$20\log_{10}\left(\frac{v_o}{v_{oi}}\right) = -1\,\text{dB} \qquad \frac{v_o}{v_{oi}} = 0.89125$$

$$v_o = k_1 v_i + \frac{3}{4}k_3 v_i^3 \qquad v_{oi} = k_1 v_i$$

$$\frac{k_1 v_{1dB} + \frac{3}{4}k_3 v_{1dB}^3}{k_1 v_{1dB}} = 0.89125$$

for nonlinearity that causes compression, rather than one that causes expansion, k_3 has to be negative.

$$v_{1dB} = 0.38\sqrt{\frac{k_1}{|k_3|}}$$

- If more than one tone is applied, P_{1dB} occurs at lower voltage.
- For two equal tones applied to system:

$$v_o = k_1 v_i + \frac{9}{4}k_3 v_i^3 \qquad \frac{k_1 v_{1dB} + \frac{9}{4}k_3 v_{1dB}^3}{k_1 v_{1dB}} = 0.89125$$

$$v_{1dB} = 0.22\sqrt{\frac{k_1}{|k_3|}}$$

Relationship between 1dB Compression Point and IP3 Points

- can find relationship between these two points.

$$\frac{v_{\mathrm{IP3}}}{v_{\mathrm{1dB}}} = \frac{2\sqrt{\dfrac{k_1}{3|k_3|}}}{0.38\sqrt{\dfrac{k_1}{|k_3|}}} = 3.04$$

- these voltages related by a factor of 3.04 (9.66 dB)
- independent of particulars of nonlinearity
- For P_{1dB} with two tones applied, ratio is larger.

$$\frac{v_{\mathrm{IP3}}}{v_{\mathrm{1dB}}} = \frac{2\sqrt{\dfrac{k_1}{3|k_3|}}}{0.22\sqrt{\dfrac{k_1}{|k_3|}}} = 5.25$$

- related by factor of 5.25 (14.4 dB)

Example: Determining IIP3 and 1-dB Compression Point from Measurement Data

- amp Freq= 2 GHz, G= 10 dB
- two equal signals applied -> 2.0 GHz and 2.01 GHz
- At output, four tones 1.99, 2.00, 2.01 and 2.02 GHz.
- power levels are –70, -20, -20, and –70 dBm
- IIP3 and P_{1dB}?

The tones at 1.99 and 2.02 GHz are the IP3 tones.

$$\mathrm{IIP3} = P_1 + \frac{1}{2}\left[P_1 - P_3\right] - G = -20 + \frac{1}{2}\left[-20 + 70\right] - 10 = -5\mathrm{dBm}$$

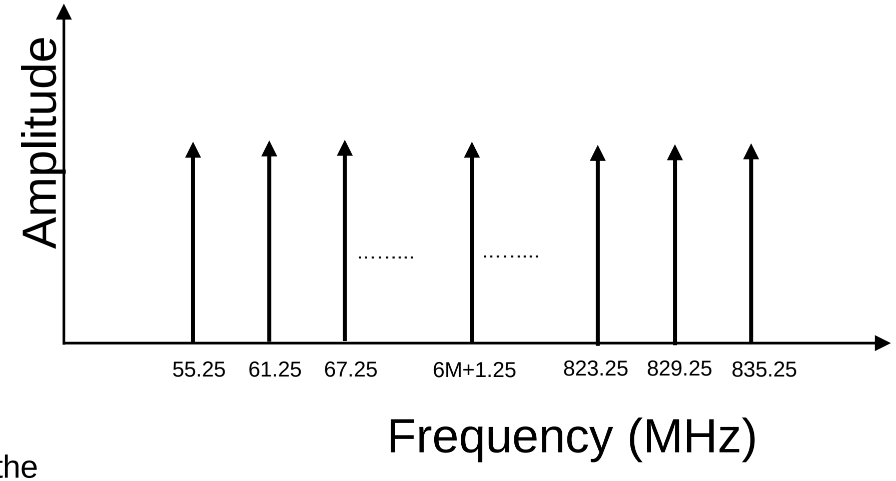
P_{1dB} for a single tone is 9.66 dB lower, about –14.7 dBm at input

Broad Band Measures of Linearity

- IIP3 and P_{1dB} not only way to measure linearity.
- measures of linearity common in wide band systems handling many signals simultaneously are called *composite triple-order beat* (CTB) and *composite second-order beat* (CSO).
- In these tests of linearity, N signals of voltage v_i are applied to the circuit equally spaced in frequency.
- E.g. tones spaced 6 MHz apart (this is the spacing for a cable television system).
- tones never placed at frequency that is an exact multiple of spacing (in this case 6 MHz).
- so that IM3 and IM2 fall at different freqs
- If we take three signals, 3rd order nonlinearity:

$$(x_1+x_2+x_3)^3 = \underbrace{x_1^3+x_2^3+x_3^3}_{\text{HM3}} + \underbrace{3x_1^2x_2+3x_1^2x_3+3x_2^2x_1+3x_3^2x_1+3x_2^2x_3+3x_3^2x_2}_{\text{IM3}} + \underbrace{6x_1x_2x_3}_{\text{TB}}$$

- last term causes CTB,
- It creates terms at frequencies $\omega_1 \pm \omega_2 \pm \omega_3$ of magnitude $1.5k_3v_i$



- TB term twice IM3
- except when all three add (ω_1 + ω_2 + ω_3), tones can fall into the channels being used, many will fall into the same channel.
- E.g. 67.25-73.25+79.25=73.25 MHz, or 49.25-55.25+79.25=73.25 MHz
- many more *triple beat* (TB) products than IM3 products.
- these become more important in a wide band system.

Broad Band Measures of Linearity: CTB

- Max number of terms will fall on the tone at the middle of the band.
- With N tones, the number of tones falling there will be

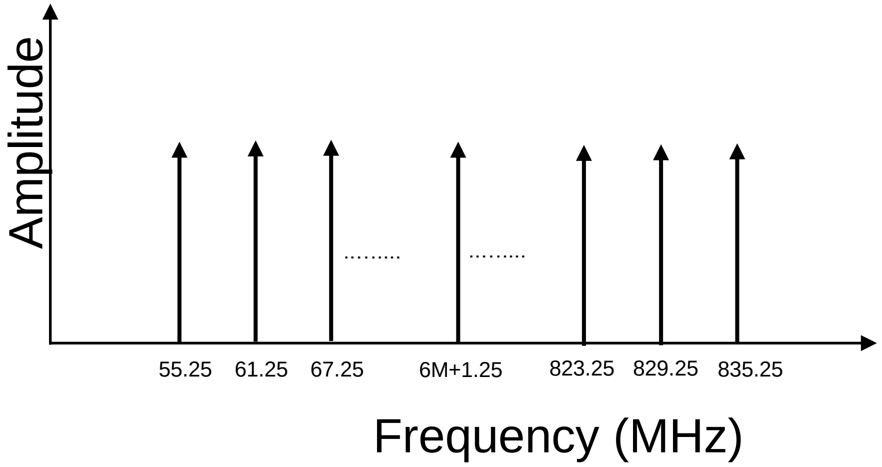
$$\text{Tones} = \frac{3}{8}N^2$$

- if signal power is backed off from the IP3 power by some amount, then power in IM3 tones will be backed off three times as much
- if each fundamental tone is at a power level of P_s (in dBm), then power of TB:

$$\text{TB(dBm)} = P_{\text{IP3}} - 3(P_{\text{IP3}} - P_s) + 6$$

- Therefore

$$\text{CTB(dB)} = P_s - \left[P_{\text{IP3}} - 3(P_{\text{IP3}} - P_s) + 6 + 10\log(\frac{3}{8}N^2) \right]$$



Broad Band Measures of Linearity: CSO

- have N signals at same power level, 2nd order distortion products of each pair of signals fall at $\omega_1 \pm \omega_2$.
- signals fall at freqs above or below the carriers not on top of them provided carriers not some even multiple of the channel spacing
- E.g. 49.25+55.25=104.5 MHz, 1.25 MHz above closest carrier at 103.25 MHz.
- the sum terms will fall 1.25 MHz above the closest carrier,
- the difference terms such as 763.25 – 841.25 = 78 MHz will fall 1.25 MHz below the closest carrier at 79.25 MHz.
- 2nd order, 3rd order terms can be measured separately.
- number of terms that fall next to any given carrier will vary.
- Some of $\omega_1 + \omega_2$ terms fall out of band and max number in band will fall next to the highest frequency carrier.
- number of 2nd order beats above any given carrier:

$$N_B = (N-1)\frac{f - 2f_L + d}{2\left(f_H - f_L\right)}$$

- For difference freq 2nd order beats, there are more of these at lower freqs, max number will be next to the lowest frequency carrier.
- the number of 2nd order products next to any carrier:

$$N_B = (N-1)\left(1 - \frac{f - d}{f_H - f_L}\right)$$

Each 2nd order beat is an IP2 tone.
If each fund tone is at P_s, then power of *second-order beat* (SO) tones:

$$\mathrm{SO(dBm)} = P_{\mathrm{IP2}} - 2(P_{\mathrm{IP2}} - P_s)$$

$$\mathrm{CSO(dB)} = P_s - \left[P_{\mathrm{IP2}} - 2(P_{\mathrm{IP2}} - P_s) + 10\log(N_B)\right]$$

RF Building Blocks

- In this course transistor level design will not be considered
- complete RF circuits will be basic building blocks for all designs.
- It is a great asset to a systems designer to have some knowledge of the design of all these components as knowing the level of effort required to achieve a given performance target may lead to making a more optimal performance tradeoff.

Low Noise Amplifiers (LNAs)

- LNAs typically placed at front of radio -> amplify weak signals while adding as little noise as possible.
- Typical specifications gain, noise figure, IP3, IP2, power consumption, input impedance and 1dB compression point.
- there is a compromise between performance parameters.
- max possible frequency of operation is related first to the choice of technology.
- operating at higher freq and achieving lower noise, higher linearity, and higher gain can be achieved at the expense of higher power dissipation.
- if high operating freq is not critical, may be possible to operate at a lower current density -> lower power dissipation.
- typical design might have few mA as bias current, noise figure of about 2 dB, IIP3 of about -15 dBm.
- Linearity limited by the available swing at the output -> made worse by having higher gain.
- Typical IIP3 range from about -20 dBm or even less for a low power designs to about 0 dBm with additional bias current.

RF Building Blocks

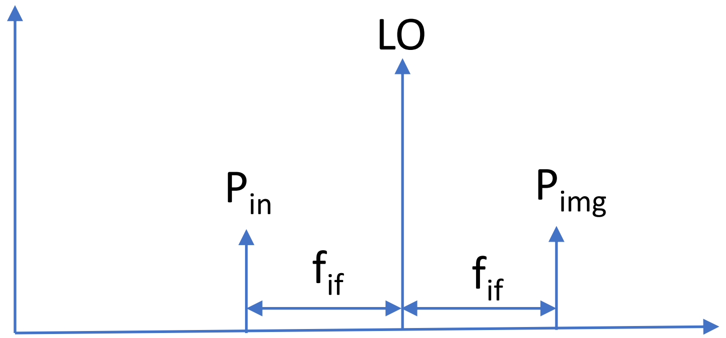
Mixers

- Mixers are freq translation devices.
- have two inputs, one output.
- in radio one input driven with sign wave or square wave ref signal (local oscillator (LO))
- mixer multiplies this with data input -> translates input data to sum/difference freq.
- desired output frequency selected by filtering.
- Gain in mixer -> amplitude of output signal at desired freq divided by the amp of input signal:

$$Gain = \frac{Sig_{out}\left(\omega_{\text{desired}}\right)}{Sig_{in}\left(\omega_{\text{in}}\right)}$$

- Most mixers operate by switching mechanism that is equivalent to multiplying the input signal by plus or minus 1.
- This action has a max gain of $2/\pi$ or about – 4dB.
- gain is achieved only if switches are large and are driven with a large enough LO.
- Active mixers can have different gain.
- higher power dissipation is required to achieve higher gain.

RF Building Blocks



- Mixer NF complicated to define -> freq translation involved.
- mixers have modified definition of NF:

$$F = \frac{N_{otot}(\omega_{desired})}{N_{o(source)}(\omega_{desired})}$$

- source generates noise at all freq, many of these freq will produce noise at output frequency due to mixing
- Usually two dominant freqs are input freq and image freq (freq same distance on other side of LO).
- SSB NF and DSB NF
- difference is value of denominator
- DSB NF -> all noise due to source at output freq. considered (noise of source at input and image frequencies)
- SSB NF -> only noise at output freq due to source that originated at RF frequency considered
- DSB NF even ideal noiseless mixer has NF = 3dB.
- noise of source doubled at output due to mixing RF and image frequency noise

$$NF_{DSB} = NF_{SSB} - 3\text{ dB}$$

- In practice input filter also affect output noise
- Which to use?
- depends type of radio architecture
- mixers much noisier than LNAs.
- finite linearity just like an amplifier.
- In passive mixer, min possible noise is loss.
- Thus min NF ~ 4 dB.
- active mixers more noise from gain stage
- Typical specifications: gain, NF (either SSB or DSB), IP3, IP2, power consumption, P_{1dB}

RF Building Blocks

Filters

- Filters are freq selective -> pass certain freqs, attenuate others.
- two major types: bandpass and low pass.
- BPF passes (usually narrow) band of freq, attempts to reject signals at all other freqs
- LPF passes freq up to some cutoff
- can be made of either active or passive components.
- Typical specifications are bandwidth of the pass band, pass band attenuation, and roll off rate in the stop band.

RF Building Blocks

Voltage-Controlled Oscillators and Frequency Synthesizers

- VCO is signal source whose freq can be controlled by DC voltage
- VCOs (frequency synthesizer) often provide any needed local oscillator (LO) signals
- complete freq synthesizer specify max sideband spur levels, tuning range, tuning resolution or step size, integrated phase noise
- many different types of Oscillators
- Noise in oscillators goes up roughly in proportion to the carrier freq, inversely proportional to the offset freq from carrier
- Generally, higher power dissipation achieves higher output signal levels and lower PN
- Higher tuning range in a single band typically results in higher noise
- For wide tuning range it is important to have this in multiple bands.

RF Building Blocks

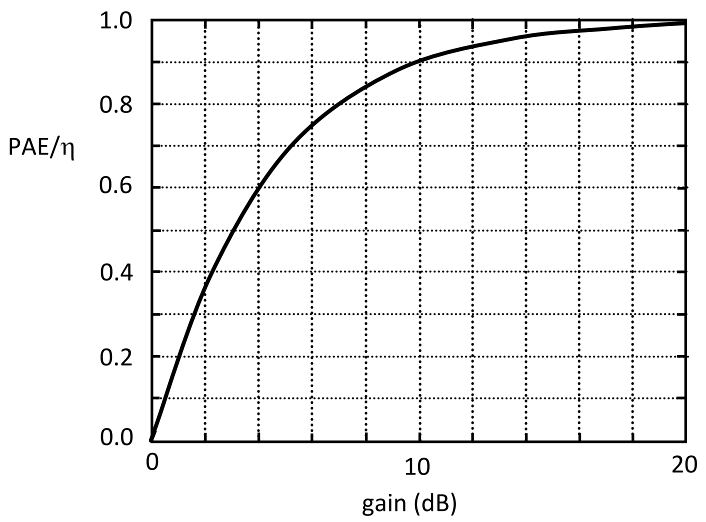
Variable Gain Amplifiers

- Some amps need to have gain levels that are programmable.
- have gain levels and gain steps (the distance between two gain levels) must be specified.
- requirement for adjustable gain results in compromise with other performance parameters, e.g. noise or linearity.
- noise will likely be higher and thus first stage of an LNA would usually not be designed to have variable gain.
- linearity may suffer and so variable gain is typically not used for the final stage of a power amplifier.

RF Building Blocks

- PAs responsible for delivering power to antenna
- consume large amount of DC power so efficiency (η) very important.
- η also called *dc-to-RF efficiency*:

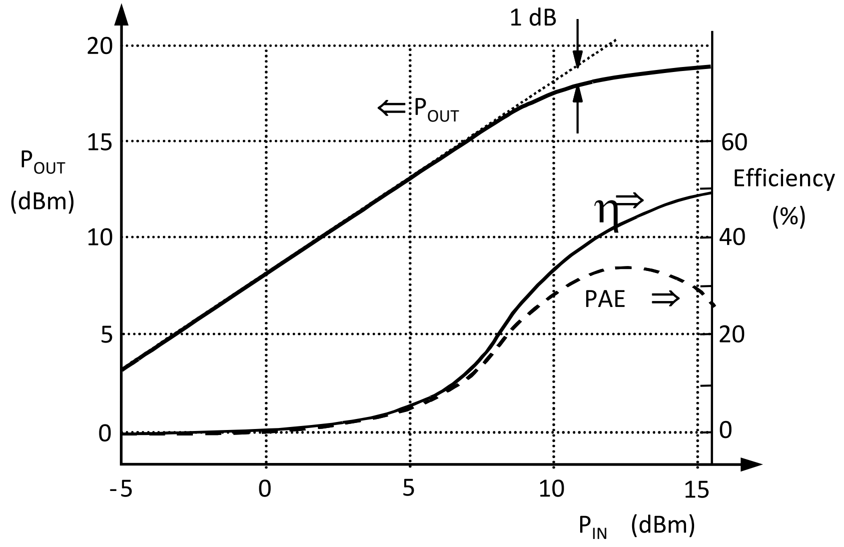
$$\eta = \frac{P_{\text{out}}}{P_{\text{dc}}}$$



Power-added efficiency (PAE) is same as efficiency, however takes gain of amp into account:

$$\text{PAE} = \frac{P_{\text{out}} - P_{\text{in}}}{P_{\text{dc}}} = \frac{P_{\text{out}} - P_{\text{out}}/G}{P_{\text{dc}}} = \eta\left(1 - \frac{1}{G}\right)$$

- for high gain, PAE is same as η
- but if gain is 3 dB, the PAE is only half of the dc-to-RF efficiency
- while η keeps increasing for higher input power, as amp compresses and gain decreases, PAE decreases
- there is optimal value of PAE and it occurs few dB beyond the P_{1dB}
- When specifying a PA, often most important parameters are η , PAE, output power, Output P_{1dB}, OIP3, gain, output impedance.
- Efficiency is typically traded off with linearity.



RF Building Blocks

Phase Shifters

- Often phase shifters used to implement phase shift in an LO tone.
- common requirement is to have two copies of input signal at 90°
- commonly called I and Q for in phase and quadrature phase
- Common specifications for phase shifter: power consumption, phase shift, expected phase error or mismatch.
- Phase shifters may be designed based on circuit components such as resistors, capacitors and inductors or as delay lines.
- In either case additional buffers may be required to compensate for signal attenuation, thus additional power dissipation is expected.
- There may also be errors due to process variations.
- All of these devices also have limited BW.
- In a filtering approach, one way to reduce the error and to increase BW is to use more stages, that is, to use cascaded filter sections.

RF Building Blocks

Analog-to-Digital (A/D) and Digital-to-Analog (D/A) Converters

- A/D and D/A converters are used as the interface between the analog radio and the digital base band.
- Depending on the nature of a project they may be implemented with the radio or with the DSP core.
- specifications for these parts are power consumption, number of bits, sampling frequency, and the clock jitter of the reference used to drive them.
- Operating them at higher speed usually requires higher power dissipation and typically results in an effective lower number of bits.
- When specifying D/A converters linearity power dissipation, and max signal size may be additional concerns.

RF Building Blocks

RF Switch

- A switch is often needed to select between a transmit path and a receive path in a radio.
- Such a switch will need to be low loss and have good isolation between the three terminals.
- If the switch is made from active components linearity may be an issue as well.

Antenna

- antenna is what changes RF energy into EM radiation that can be transmitted through the air and vice versa.
- Antennas have ability to tradeoff directivity (how much energy is transmitted in a given direction) with gain.
- Thus a highly directive antenna can have a lot more gain than one that transmits in almost every direction.