CARLETON UNIVERSITY

FINAL
EXAMINATION Solution
March 2010

DURATION: 3 HOURS

Department Name & Course Number: ELEC 4700

Course Instructor(s): Tom Smy

AUTHORIZED MEMORANDA
CALCULATOR and 1 sheet of notes

Students **MUST** count the number of pages in this examination question paper **before** beginning to write, and report any discrepancy immediately to a proctor. This question paper has 8 pages.

This examination question paper **MAY NOT** be taken from the examination room.

There are six questions on the exam. Do 5 of 7 questions. If you do more I will take your best 5. Note that the questions are equally weighted.

Student Name:

Student Number:

1. (**Marks 25**) Thermal Physics

   (a) What is an intuitive definition for Temperature in a system consisting of a large number of molecules?

   (b) If we model a solid as a 2D rectangular crystalline solid consisting of "balls on springs" then as a function of Temperature (T) determine the following using the $1/2kT$ rule:

      i. The total kinetic energy in the solid.
      ii. The total energy in the solid.
      iii. The heat capacity of the material.

- Temperature is the mean kinetic energy of the system.
- We have two degrees of freedom $1/2kT$ for each. And "springs" in two directions. So:

$$\begin{aligned} E_k &= N_A * (\frac{1}{2}mV_x^2 + \frac{1}{2}mV_y^2) \\ &= 2 * N_A(\frac{1}{2}kT) = N_A * kT \\ E_T &= N_A * (\frac{1}{2}mV_x^2 + \frac{1}{2}MVP_y^2 + \frac{1}{2}km_x x^2 + \frac{1}{2}mK_y y^2) \\ &= 4 * N_A(\frac{1}{2}kT) = 2 * N_A * kT \end{aligned}$$

Heat Capacity is $\frac{dE_T}{dT} = 2 * N_A * k = 2R$

2. (**Marks 25**) Thermal Distributions

   (a) Explain what is the source of noise in a resistor and describe the physics involved.

   (b) What is the mathematical/physical description for the energy distribution of a large number of electrons at thermal equilibrium.

      i. Classically?

      ii. Quantum Mechanically?

      iii. Why do they differ?

      iv. When can we use the classical description as an approximation of the Quantum Mechanical one.

- The primary cause of noise in a resistor is the thermal motion of the electrons. The random in the electron velocities and positions cause charge inhomogeneities which in turn produce potential variations and terminal current and voltage fluctuations.
- The classical description would be the Maxwell-Boltzmann distribution which has the form of:

$$n_E = N * C * e^{(-\frac{E}{kT})} \tag{1}$$

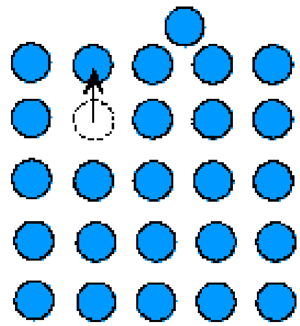
  where C is temperature dependent.
- The quantum distribution is the Fermi-Dirac distribution which is:

$$F_E = \frac{1}{1 + e^{(-\frac{E-E_f}{kT})}} \tag{2}$$

- The differ as the electron is Fermion which must obey the Pauli exclusion principle. Whereas the classical model assumes non-interacting indistinguishable particles.
- The classical expression can be used whenever E is not close to E_f. As in this case there are many unfilled states (electrons or holes) and the Pauli interaction is not important.

3. (**Marks 25**) Thermally Activated processes

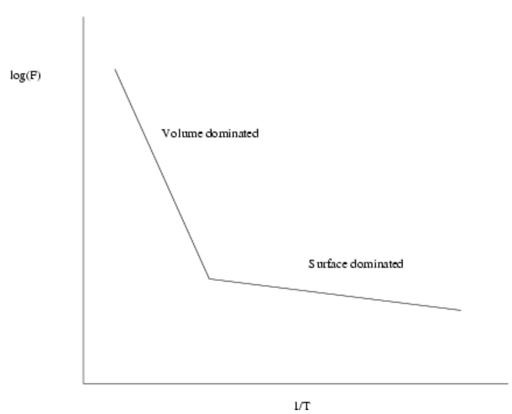
(a) Explain why diffusion of vacancy in a metallic crystal is a thermally activated process. What would be the basic functionality of $D_v(T)$.



- Breaking of bonds – requires energy
- Process is characterized by activation energy E_v
- $D_v = D_0 e^{-E_v/KT}$

(b) For a long metallic Aluminum rod (single crystal) with a source of Na atoms at the left end what would be the process of mass transport for the Na. Consider both the surface and the volume. If we were to plot the log of the flow of Na in steady-state as a function of $1/T$ what would it look like? Would this change if the rods cross-sectional area was increased dramatically?

- Surface Diffusion – $F_s \approx P_S D_s e^{-E_s/KT} dN_a/dx$
- Volume Diffusion – $F_v \approx A_S D_v e^{-E_v/KT} dN_a/dx$
- Total Diffusion – $F_t = F_s + F_v$



4. (**Marks 25**) Electron transport due to drift:

Electron transport in a metal due to drift is described by the following equation $J = q\mu nE$.

(a) Physically what is the explanation for resistance in a bulk material? What are the basic sources of this resistance? How do you include all these sources to determine a net resistivity for the material.

(b) For a thick metal film discuss the physical basis for this equation. How is the mobility expressed as a function of physical parameters?

(c) What changes must be made to this equation for a Quantum Mechanical model? Why?

(d) If the solid has two other materials in solid solution (at very low concentrations) how would you express the mobility μ as a function of the concentrations. Justify your expression physically.

(e) At high frequencies what effects would need to be included. Describe the effect.

(f) For a very thin metal film how would the physics change and what would happen to the mobility?

- Resistance is caused by the scattering of the electrons by various sources. These include phonons (heat), impurities, and defects. Each of the scattering processes reduces the mean free path of the electrons causing a resistivity the total resistivity is the sum of these.
- The effect of the field is to cause a small perturbation of the thermal velocities. This produces a net drift of the electrons which is characterized by drift velocity, a mobility and resistance. The mobility is characterised by a life time τ or mfp and an effective mass.
- Additional Quantum corrections.
  - The presence of a band structure
  - Use of the Fermi-Dirac function
  - Discrete electron states that “fill-up” – Pauli exclusion principle
  - The concept of “holes”
  - Effective mass
- Skin effect would increase the resistance by decreasing the effective cross-sectional area of the wire.
- In small line where the dimensions of the line are less then the mean free path of the electrons surface scattering will dominate the resistance of the material.

5. (**Marks 25**) Hole/electron transport The current of holes in a semiconductor is given by

$$J_p = qp\mu_p E_x + qD_p dp/dx \quad (3)$$

and the 1D continuity equation by,

$$\frac{\partial p}{\partial t} = -\frac{1}{q}\frac{\partial J_p}{\partial x} - \frac{p - p_0}{\tau_p} + G_{ph} \quad (4)$$

(a) Given that E_x and p are functions of x derive a single equation that could be solved for p.

(b) What physical equation needs to be solved in conjunction with this one to find p and therefore J_p?

(c) If temperature was a function of position within the device would your expression above change? What would need to be incorporated?

(d) If $E_x = 0$ simplify the equation and what physical effects are represented?

(e) If $p \approx p_0$ and p_0 is a constant what is the expression and what physical effects are modeled?

(f) At steady-state with $p \approx p_0$ and p_0 varying with x explain the flow of holes in the device in terms current flows produced from different physical effects.

- Take $\frac{\partial}{\partial x}$ of J_p

$$\frac{\partial J_p}{\partial x} = qp\mu_p \frac{\partial E_x}{\partial x} + qD_p \frac{\partial^2 p}{\partial x^2} \quad (5)$$

sub into 4

$$\frac{\partial p}{\partial t} = -\frac{1}{q}\left(qp\mu_p \frac{\partial E_x}{\partial x} + qD_p \frac{\partial^2 p}{\partial x^2}\right) + \frac{p - p_0}{\tau_p} + G_{ph} \quad (6)$$

$$\frac{\partial p}{\partial t} = -p\mu_p \frac{\partial E_x}{\partial x} - D_p \frac{\partial^2 p}{\partial x^2} + \frac{p - p_0}{\tau_p} + G_{ph} \quad (7)$$

- We would need to know E_x which would require solving for the potential within the device using Poisson's equation.
- As μ_p and D_p are functions of T we would need to account for $\frac{\partial \mu_p}{\partial T}\frac{\partial T}{\partial x}$ and $\frac{\partial D_p}{\partial T}\frac{\partial T}{\partial x}$ terms
- $E_x = 0$ gives,

$$\frac{\partial p}{\partial t} = -D_p \frac{\partial^2 p}{\partial x^2} + \frac{p - p_0}{\tau_p} + G_{ph} \quad (8)$$

Diffusion, recombination and photo generation.

- $p \approx p_0$ and p_0 is a constant gives,

$$0 = -p_0\mu_p \frac{\partial E_x}{\partial x} + G_{ph} \quad (9)$$

Drift and photogeneration.

- At steady-state with $p \approx p_0$ and p_0 varying with x

$$0 = -p_0\mu_p \frac{\partial E_x}{\partial x} - D_p \frac{\partial^2 p_0}{\partial x^2} + G_{ph} \quad (10)$$

Drift and diffusion, built-in field driven by photogeneration.

6. (**Marks 25**) Schottky Diode:

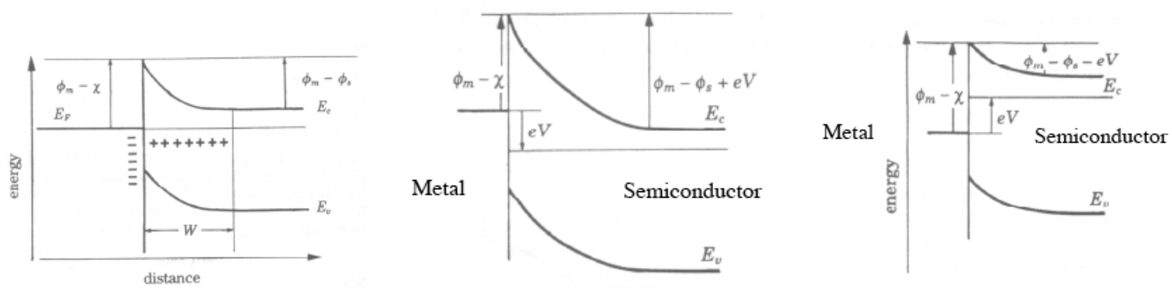
(a) Draw the band structure of a metal-N type Schottky with no bias. You can place the work functions as you wish (with in reason). Label all relevant energy levels.

(b) For both forward and reverse bias draw the band structure of the device. Carefully note the applied bias in the drawing.

(c) Explain the operation of the device in terms of physical currents from the two materials

(d) What is the expected I-V characteristic? Sketch it. Explain why the device produces little current in reverse bias and a large current in forward bias. In other words why it is non-linear.

- Band diagram



equ rev fwd

- To compute the electron flow from semiconductor to metal we use thermionic emission theory. The current that flows from the metal into the semiconductor is given by

$$J_{ms} = \frac{q4\pi m^* K^2 T^2}{h^3} e^{-q(\phi_m - \chi)/KT}. \tag{11}$$

Where $C = q4\pi m^* K^2/h^3$ is defined as Richardson's Constant. So we have the current as

$$J_{ms} = CT^2 e^{-q(\phi_m - \chi)/KT}. \tag{12}$$

The current flowing from the semiconductor into the metal is

$$J_{sm} = CT^2 e^{-q(\phi_m - \phi_s - V)/KT}. \tag{13}$$

- The net current is given by

$$J_{net} = J_s(e^{qV/KT} - 1) \quad , \quad J_s = CT^2 e^{-q(\phi_m - \phi_s)/KT} \tag{14}$$

Ideal diode equation – non-linear as operation is majority carrier excitation over a barrier.

7. (**Marks 25**) Optical effects and Lasers:

(a) What are the three basic light/material interactions that occur involving electrons in a semiconductor.

(b) Why is GaAs a good optical semiconductor and Si not?

(c) Describe the basic operation of a laser. Sketch the physical structure.

(d) For a laser what factors determines the output spectrum of the light? For a gas laser is the temperature of the gas important? If so why?

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  - Spontaneous emission
  - Absorption
  - Stimulated emission
- GaAs - direct semiconductor – no Δp needed for optical transition.
- Laser structure:
  - 2 Mirrors to form a resonant light cavity
  - Gain material to provide amplification – two or three level energy system.
  - Pumping mechanism
- Laser operation – stimulate emission amplifies the light “bouncing” between the two mirrors. Pumping maintains a population inversion.
- Output spectrum determined by basic energy transition and longitudinal mode selection of the allowable wavelengths. Temperature produces Doppler broadening of the gain spectrum allowing a range of wavelengths to lase.