Analysis and Simulation of Phase Noise in Distributed Oscillators

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Abstract—In this paper, the phase noise of a CMOS distributed oscillators is formulated and simulated. A frequency-domain analytical method is employed for the phase noise formulation with the aid of a linear model. The thermal noise and flicker noise (1/f) are considered as the principal noise source contributors. Closed form formulae obtained for predicting distributed oscillator phase noise reveal circuit design issues with regard to phase noise. Harmonic balance (HB) simulation in Agilent advanced design system (ADS) is used to validate the developed formulae. Phase noise plots based on analytical formulae and simulation show a good agreement.

I. INTRODUCTION

A novel oscillator structure, which makes use of the principles different from those have been widely used such as LC and ring oscillators, is the distributed oscillator. The direct ancestor of this device is the distributed amplifier that is suitable for high frequency and microwave applications. Correspondingly, distributed oscillator can operate at frequencies close to f0.

So far, most work on distributed oscillators has been focused on the practical interests like tuning capability, operating frequencies, technology choices, and different applications [1][2][3][4][5][6][7][8]. Although a general method for calculating phase noise was reported in [9], the phase noise prediction was fully based on the circuit behavior and an impulse response (waveform) was used. Therefore, an analytical approach that can establish a connection between the phase noise in distributed oscillator and the circuit parameters would be an useful guideline for designers. The objective of the present work is to reach this goal.

This paper is organized in the following manner. In section II, attention is confined to the analysis of phase noise in distributed oscillators. The result of this analysis is a set of closed form formulae to be used in predicting phase noise performance. In section III, the correctness and accuracy of formulae developed in section II are validated by simulations. Section IV is the conclusion part.

II. PHASE NOISE IN DISTRIBUTED OSCILLATORS

A. Linear Oscillator Model

A small-signal linear model for a generalized oscillator is shown in Fig. 1 where the summing junction at the input is not present in a practical device, but it has been placed in this model to facilitate a simplified theoretical development. Each noise source in this oscillator is modeled as an input signal. At the frequencies close to that of the carrier, \( \omega - \omega_0 + \Delta \omega \), the noise power spectral density is shaped by [10]

\[
\frac{Y_0}{X_i} |j(\omega_0 + \Delta \omega)|^2 = \frac{1}{(\Delta \omega)^2} \left| \frac{dH(j\omega)}{d\omega} \right|^2
\]  

(1)

The open-loop transfer function can be written in the exponential form

\[
H(j\omega) = A(j\omega) e^{j\phi(\omega)}
\]

(2)

The derivative of (2) is found as

\[
\frac{dH(j\omega)}{d\omega} = \left( \frac{dA(j\omega)}{d\omega} + jA(j\omega) \frac{d\phi(\omega)}{d\omega} \right) e^{j\phi(\omega)}
\]

(3)

When \( \omega = \omega_0 \), \( A(j\omega) = 1 \), it can be obtained
Thus, equation (1) takes the form

\[
\frac{|Y|}{X} \left| j(\omega_0 + \Delta \omega) \right|^2 = \frac{1}{(\Delta \omega)^2} \left[ \left( \frac{dA(\omega)}{d\omega} \right)^2 + \left( \frac{d\phi(\omega)}{d\omega} \right)^2 \right]
\]

(5)

B. Phase Noise Analysis in Distributed Oscillators

In this analysis, the principal noise sources considered are thermal noise and \(1/f\) noise. As known, the time domain behavior of an oscillator is nonlinear. However, the linear analysis can be very useful to model the oscillator’s phase noise. Since the additive phase noise characteristics in oscillators are linear, the linear model introduced above is employed to develop the noise prediction formulae as the first step, then the nonlinear effects are taken into account for the final results.

1) Additive noise

Fig. 2 shows a CMOS distributed oscillator. Under condition of \(\beta_{gd} = \beta_{d} = \beta\) and \(Z_m = Z_g = Z_c\), its open-loop transfer function required for the following noise study is given [11]

\[
H(j\omega) = \frac{1}{2} \frac{Z_m}{\alpha} e^{-\alpha_i \omega} - e^{-\alpha_o \omega}
\]

(6)

where \(g_m\) is transistor small-signal transconductance, \(Z_m\) is transmission line characteristic impedance, generally equal to \(50\Omega\), \(N\) is the number of amplifying stages, and \(\alpha_i, \alpha_o, \beta\) are the attenuation and phase constants of gate and drain lines.

![Figure 2. A CMOS distributed oscillator](image)

![Figure 3. Transistor noise modeled as current source in a distributed oscillator](image)

The amplitude and phase components of (6) are

\[
A(\omega) = \frac{Z_m}{2} \left( e^{-\alpha_i \omega} - e^{-\alpha_o \omega} \right)
\]

\[
\phi(\omega) = -\beta \omega i
\]

(7)

(8)

It is assumed that oscillation amplitude remains small, thus a linear approximation is valid. Additive noise consists of independent contributions occurring along the output path. As shown in Fig. 3, the noise from each transistor in distributed oscillator is modeled as the current sources \(I_{\text{noise}}\), \(I_{\text{noise}2}\), ..., and \(I_{\text{noise}N}\), each being directly connected to the drain line. The modeled noise current sees an impedance of \(Z_d/2 = Z_o/2\). The resulting noise signal appears at node 1 as an equivalent input noise. Let \(I_g = I_d = I\), the output power density contributions from noise sources 1 to \(N\) can be expressed generally as [11]

\[
|V_{\text{noise}}|^2 = \frac{I^2}{2} \left( \frac{Z_m}{2} \right)^2 \left( e^{-\alpha_i (N-\frac{1}{2}) \omega} \right)^2
\]

(9)

where \(I_{\text{noise}}\) is the noise current power spectral density.

If the reasonable assumption that \(I_{\text{noise}1} = I_{\text{noise}2} = \ldots = I_{\text{noise}N} = I_{\text{noise}}\) is made, the total input noise power density at node 1 due to currents \(I_{\text{noise1}}\), \(I_{\text{noise2}}\), ..., and \(I_{\text{noiseN}}\) may be deduced

\[
|V_{\text{noise}}|^2 = \frac{I^2}{2} \left( \frac{Z_m}{2} \right)^2 \sum_{n=1}^{N} \left( e^{-\alpha_i (N-n+\frac{1}{2}) \omega} \right)^2
\]

(10)

The output noise power density can be obtained by substituting (10) to (5). It is expressed as

\[
|V_{\text{noise}}|^2 = \frac{I^2}{2} \left( \frac{Z_m}{2} \right)^2 \left( e^{-\alpha_i (N-\frac{1}{2}) \omega} \right)^2
\]
\[
|V_{\text{out, noise}}|^2 = \frac{j^2_{\text{noise}} \left( \frac{Z_o}{2} \right)^2 \sum_{n=1}^{\infty} e^{-\alpha_0 (N_n-1)} \right)^2}{(\Delta\omega)^2 \left[ \frac{dd(\omega)}{d\omega} + \frac{d\phi(\omega)}{d\omega} \right]^2}
\]

where \(dd(\omega)/d\omega\) and \(d\phi(\omega)/d\omega\) are the derivatives of (7) and (8).

In this paper, the noise source \(I_{\text{noise}}\) includes the thermal noise \(I_{\text{noise}}^2\) and \(1/f\) noise \(I_{\text{noise}}^2\). Therefore, the total output additive noise power is the superposition of the output power spectral density of \(I_{\text{noise}}^2\) and \(I_{\text{noise}}^2\), and it is given by

\[
|V_{\text{out, add-noise}}|^2 = |V_{\text{out, noise}}|^2 + |V_{\text{out, add-noise}}|^2
\]

(12)

\(|V_{\text{out, add-noise}}|^2\) can be obtained by replacing \(j^2_{\text{noise}}\) in (11) to \(I_{\text{noise}}^2\), which is the drain-current-referred thermal noise power spectral density of a MOSFET and can be expressed as [10]

\[I_{\text{noise}}^2 = 4K\beta T^2 \frac{\Delta f}{3\pi}
\]

(13)

For the short-channel devices, the factor \(2/\pi\) in (13) could increase up to 0.873 [10]. \(|V_{\text{out, add-noise}}|^2\) is the contribution of upconverted \(1/f\) noise to the additive phase noise and it will be addressed in the noise mixing section.

2) Noise Mixing

In this section, the nonlinear effects will be discussed. The nonlinearity of the device leads to the mixing of the noise frequency \(\omega_n\), with that of carrier \(\omega_c\) and harmonics whose frequencies are near to the carrier frequency.

For a nonlinear device with assumption of the third-order approximation, if the input includes carrier and noise components \(V_{\text{in}}(t) = A_c \cos \omega_c t + A_n \cos \omega_n t\), then the output may be written as

\[V_{\text{out}}(t) = a_0 + a_1 V_{\text{in}} + a_2 V_{\text{in}}^2 + a_3 V_{\text{in}}^3
\]

(14)

Two considerable terms out of (14) are:

\[V_{\text{out}}(t) \propto a_2 A_c A_n \cos (\omega_c + \omega_n) t\]

and

\[V_{\text{out}}(t) \propto a_2 A_c A_n \cos (2\omega_c - \omega_n) t\].

The important noise components at the output spectrum appear at the frequencies \(\omega_n\), \(\omega_c - \omega_n\), \(\omega_c + \omega_n\), \(2\omega_c - \omega_n\).

For the thermal noise with a broadband spectrum, only the noise component \(\omega_n\) close to the carrier frequency \(\omega_c\) and its corresponding component \(2\omega_c - \omega_n\) are of importance. The magnitudes at \(\omega_n\) and \(2\omega_c - \omega_n\) depend on the noise shaping properties of the feedback oscillator system. These two components have approximately equal magnitude. This effect is accounted for by doubling the output noise power density predicted by (11) [10], [11]. It can be expressed as

\[
|V_{\text{out, noise}}|^2 = \frac{2I_{\text{noise}}^2 \left( \frac{Z_o}{2} \right)^2 \sum_{n=1}^{\infty} e^{-\alpha_0 (N_n-1)} \right)^2}{(\Delta\omega)^2 \left[ \frac{dd(\omega)}{d\omega} + \frac{d\phi(\omega)}{d\omega} \right]^2}
\]

(15)

As for the \(1/f\) noise, it is upconverted to the band of interest and the resulting noise components are located at frequencies \(\omega_0 - \omega_n\) and \(\omega_0 + \omega_n\) with the amplitudes of \(a_2 A_c A_n\). The value of the coefficient \(a_2\) can be approximately determined by measuring the magnitude of the second harmonic of oscillator's output spectrum through simulation. The contribution of \(1/f\) noise to the phase noise can be considered as an additive noise. According to (11), we have

\[
|V_{\text{out, add-noise}}|^2 = \frac{2a_2^2 A_c^2 I_{\text{noise}}^2 \left( \frac{Z_o}{2} \right)^2 \sum_{n=1}^{\infty} e^{-\alpha_0 (N_n-1)} \right)^2}{(\Delta\omega)^2 \left[ \frac{dd(\omega)}{d\omega} + \frac{d\phi(\omega)}{d\omega} \right]^2}
\]

(16)

where the constant 2 reflects the effects from the two frequency components \(\omega_0 - \omega_n\) and \(\omega_0 + \omega_n\). \(I_{\text{noise}}^2\) is the current power spectral density of \(1/f\) noise.

An empirical relation of \(1/f\) noise is given by [12]

\[I_{\text{noise}}^2 = \frac{M g_m^2}{C_m \omega L f^2}
\]

(17)

where \(M\) is an empirical parameter, \(W\) and \(L\) are the width and length of the transistor, \(C_m\) is the oxide capacitance per unit area, and \(\beta\) is typically close to 1. A more complex expression for \(I_{\text{noise}}^2\), following [13], could also be used later in the design example.

Therefore, the total output noise power density after adjustment due to the intermodulation effects becomes
\[ |\varphi_{out, noise, total}|^2 = \frac{\left(2I_{noi}^2 + 2a_I^2A_I^2\right)^2\left(2Z_{o}\sum_{n=1}^N e^{-\frac{a_I^2(N-n)^2}{2}}\right)}{2} \]

\[ = 10\log \frac{\left(\varphi_{out, noise, total}\right)^2}{A^2/2} \]

where \(A_0\) is the carrier amplitude.

By observing (19), one can find that \(\frac{dA(\omega)\omega}{d\omega}\) is negligible compared to \(\frac{d\phi(\omega)\omega}{d\omega}\) at frequencies close to \(\omega_0\).

Also \(I_{noise}^2 \propto g_m\), \(I_{noise}^2 \propto g_m^2/C_w\), \(\sum_{n=1}^N e^{-\frac{a_I^2(N-n)^2}{2}} \propto R \omega_0 N\), and \(\frac{d\phi(\omega)\omega}{d\omega} \propto N^2(C + C_w)^2\). Upon these proportional relations, equation (19) yields a direct design parameters’ dependence

\[ L(\Delta\omega) \propto 10\log \left[2(g_m + g_m^2)R\omega_0 N^2(C + C_w)^2\right] \]

According to (20), designer could examine the trade-offs between the circuit parameters and phase noise at the design stage.

### III. Simulation Validation

The accuracy of the phase noise analytical prediction is validated by HB simulation in ADS. The HB simulation setup follows that found in the ADS manual. TSMC CMOS 0.13\mu m technology is employed and the transistor model is BSIM3v3. In calculation, equation (19) is used to produce the phase noise. The equations for \(I_{noise}^2\) and \(I_{noise}^2\) are those adopted in SPICE2 thermal noise model and BSIM3v3 flicker noise model, respectively [13].

A 4-stage distributed oscillator operating at 17.4GHz is used for examination. Fig. 4 depicts a comparison of phase noise spectrum plots between calculation and simulation for the case mentioned above. As can be seen in Fig. 4, a good agreement is shown between calculation and simulation.

### IV. Conclusion

This paper presents a theoretical analysis on distributed oscillator’s phase noise. The closed form formulae for predicting the phase noise have been obtained. Based on the derived formulae, the phase noise dependence on circuit parameters is also given to provide better understanding. The correctness and accuracy of the theoretical analysis on phase noise is verified by HB simulation in ADS.

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**References**


