Abstract—The conditions for broadside radiation with no open stop-band are derived using a field based metasurface model of a general Leaky-Wave Antenna (LWA). Using the Generalized Sheet Transition Conditions (GSTCs), the unique set of surface susceptibilities describing the metasurface to enable to broadside radiation is obtained and it is found that the LWA must exhibit an orthogonal electric and magnetic field dipole moments forming a Huygens’ source configuration. The susceptibilities must further be lossy and satisfy a unique amplitude and phase relationship between the two.

I. INTRODUCTION

A Leaky-Wave Antenna (LWA) is typically a 2-port traveling wave structure that gradually radiates EM energy to free-space as wave propagates along it [1]. It is characterized by a complex propagation constant \( \gamma(\omega) = \alpha(\omega) + j\beta(\omega) \), where \( \alpha \) is the per unit length leakage along the structure and \( \beta \) is the propagation constant providing the necessary frequency-dependent phase-shifts to scan the radiation beam.

The CRLH-type and general periodic LWAs are capable of providing seamless frequency scanned radiation characteristics from backward to forward region through broadside. Achieving an optimized broadside radiation from LWAs require special care, as they are well-known to exhibit open-stopband at \( \beta = 0 \), which significantly degrades its radiation performance, where the gain abruptly changes when scanning through broadside [2]. A rigorous treatment of this problem is provided in [2], where the necessary conditions of frequency and Q-balancing have been proposed for solving broadside problem, using a universal lattice circuit model representing any arbitrary LWA, which is then further related to various structural symmetries of the unit cells.

While this circuit analysis provides elegant insights into the mechanism of closing the stop-band, it lacks a direct physical mapping of the circuit to the resonant field behavior and the macroscopic polarization response of the structure responsible for broadside radiation. Given that the broadside problem of LWAs is intimately connected to the spatial symmetries of the unit cell, a field analysis of the LWA in terms of required electric and magnetic polarizabilities is essential in forming a complete understanding of the open-stopband closure problem.

Consequently, in this work, broadside radiation from general LWAs is modeled using a zero-thickness electromagnetic metasurface, described in terms of its electric and magnetic surface susceptibilities [3]. It will be shown using the Generalized Sheet transition Conditions (GSTCs), that for complete stop-band closure and thus efficient broadside radiation, the LWA cells must excite both electric and magnetic dipolar moments in the Huygens’ source configuration in a balanced configuration, and the two dipole moments must be lossy.

II. METASURFACE MODEL OF BROADSIDE RADIATION

Consider the configuration of Fig. 1, where the LWA aperture is lying in the \( x-y \) plane. Underneath the aperture is a perfect electrical conductor (PEC) ground plane, which confines the fields inside this effective waveguide. Let there be an input excitation at \( x = 0 \), which launches a \( z \)-polarized wave of frequency \( \omega \), inside the waveguide along \( +x \) with zero back reflection, i.e. perfectly matched. Part of the fields leak out of the aperture, and appear as outgoing plane-wave corresponding to a LWA radiation. The field inside the waveguide decays along the \( x \) direction due to gradual leakage to free space. The LWA aperture can be modeled as a zero-thickness metasurface, which is described in terms of its constitutive parameters expressed as electric and magnetic surface susceptibilities, \( \bar{\chi}_{ee} \) and \( \bar{\chi}_{mn} \). The fields across the metasurface are governed by the surface susceptibilities and the GSTCs. Therefore, any radiation out of this structure while providing zero back reflection inside the waveguide, imposes specific constraints on the surface susceptibilities thereby putting several prescriptions on the LWA properties.

Let us apply this configuration to model broadside radiation with no open stop-band, i.e. zero back-reflection inside the waveguide. The incident E-field inside the guiding structure is given by:
\( \mathbf{E}_0(x, z) = -E_0 e^{-\alpha x} e^{-j\beta_z z} \hat{z} \), with \( \beta_x = 0 \). \hspace{1cm} (1)

Using Maxwell-Faraday’s law, the corresponding incident H-field is given by

\( \mathbf{H}_0(x, z) = -j E_0 \frac{\alpha}{\omega \mu} e^{-\alpha x} \hat{y} \). \hspace{1cm} (2)

The outgoing E-field (linearly polarized along \( x \)) for broadside radiation propagates along \( z \) only (i.e., \( \beta_x = 0 \)), and attenuates along \( x \) due to attenuation inside the structure (following phase matching) and is given by:

\( \mathbf{E}_t(x, z) = E_t e^{-\alpha x} e^{-j\beta_z z} \hat{x} \). \hspace{1cm} (3)

Again using Maxwell-Faraday equation we can get the corresponding outgoing H-field as follows:

\( \mathbf{H}_t(x, z) = E_t \left( \frac{k_z}{\omega \mu} \right) e^{-\alpha x} e^{-j\beta_z z} \hat{y} \). \hspace{1cm} (4)

The unknown propagation constant \( k_z \) can be obtained using the Helmholtz equation for the E-field, leading to \( k_z = \sqrt{\alpha^2 + k_0^2} \), which is purely real and has the same form as an in-homogenous plane wave (more specifically, an improper complex leaky-wave) [4]. Since \( k_z > k_0 \), this suggests that the propagating wave at broadside is a slow wave. Now, the two fields across the metasurface can be related by the following GSTCs [3], where we identify the specific non-zero vectorial field components of Eqs. (1)-(4):

\[
\begin{pmatrix}
0 \\
-\Delta E_x
\end{pmatrix} = j \omega \mu \begin{pmatrix}
\chi_{xx}^{yy} & \chi_{yy}^{yy} \\
\chi_{xx}^{mm} & \chi_{yy}^{mm}
\end{pmatrix} \begin{pmatrix}
0 \\
H_{y,av}
\end{pmatrix} + j \omega \sqrt{\mu \epsilon} \begin{pmatrix}
\chi_{xx}^{me} & \chi_{yy}^{me} \\
\chi_{xx}^{em} & \chi_{yy}^{em}
\end{pmatrix} \begin{pmatrix}
E_{x,av} \\
0
\end{pmatrix} \hspace{1cm} (5a)
\]

\[
\begin{pmatrix}
0 \\
-\Delta H_y
\end{pmatrix} = j \omega \epsilon \begin{pmatrix}
\chi_{ee}^{xx} & \chi_{ee}^{xy} \\
\chi_{ee}^{yy} & \chi_{ee}^{yy}
\end{pmatrix} \begin{pmatrix}
E_{x,av} \\
0
\end{pmatrix} + j \omega \sqrt{\mu \epsilon} \begin{pmatrix}
\chi_{em}^{xx} & \chi_{em}^{xy} \\
\chi_{em}^{yy} & \chi_{em}^{yy}
\end{pmatrix} \begin{pmatrix}
0 \\
H_{y,av}
\end{pmatrix} \hspace{1cm} (5b)
\]

Eq. 5 thus provide specific constraints on the surface susceptibility tensors to allow such broadside radiation from the metasurface aperture. One of the possible solutions is scalar electric and magnetic surface susceptibilities, forming a Huygens’ configuration:

\[-\Delta E_x = j \omega \mu \chi_{yy}^{yy} H_{y,av}, \quad -\Delta H_y = j \omega \epsilon \chi_{ee}^{xx} E_{x,av} \] \hspace{1cm} (6)

Substituting the expressions for various field components, Eqs. (1)-(4) and assuming unity excitation \( (E_0 = 1) \), we get:

\[
j \frac{\omega \mu}{2} \chi_{yy}^{yy} \left[ -\frac{\alpha}{\omega \mu} + \frac{k_z}{\omega \mu} \right] = -E_t \hspace{1cm} (7a)
\]

\[
j \frac{\omega \epsilon}{2} \chi_{ee}^{xx} E_t = - \left[ E_t \frac{k_z}{\omega \mu} + j \frac{\alpha}{\omega \mu} \right] \hspace{1cm} (7b)
\]

Further, eliminating \( E_t \) from both equations, the electric and the magnetic surface susceptibilities can be related as:

\[
j \frac{\omega}{\mu} \left[ \frac{k_z}{\omega \mu} + j \frac{\omega \epsilon}{4} \chi_{ee}^{xx} \right] = \frac{j}{\omega \mu} \hspace{1cm} (8)
\]

Eqs. 6 and 8 are the central results of this development, from which the following main conclusions can be made to enable broadside radiation with no stop-band characteristics:

1. Following Eq. 6, the metasurface aperture must exhibit orthogonally polarized electric and magnetic surface susceptibilities \( \chi_{ee}^{xx} \) and \( \chi_{yy}^{yy} \), which represents a Huygens’ source configuration. The surface susceptibilities naturally depend on the attenuation factor \( \alpha \) inside the waveguide controlling the leakage factor. The requirement of these two dipolar moments is analogous to requirements of series and shunt resonances in the lattice based circuit model of [2].

2. Following Eq. 7, both electric and magnetic surface susceptibilities are complex, which implies that the metasurface must have ohmic losses. This again is analogous to requirement of finite Q-factors of the series and shunt resonators of [2]. Therefore, an open stop-band of a LWA cannot be closed using a lossless structure.

3. Eq. 8 finally establishes a unique relationship between the strengths of \( \chi_{ee}^{xx} \) and \( \chi_{yy}^{yy} \). This implies that the two dipolar moments must be properly balanced to enable broadside radiation. These moments are further related to equivalent electric and magnetic surface currents, \( P_e \) and \( M_m \), which can be thought of secondary sources of radiation into free-space. It is clear that these two currents do not radiate in phase and feature precise amplitude balance, which may have important consequences in regards to spatial symmetries in practical unit cell designs.

### III. Conclusions

The conditions for broadside radiation with no open stop-band has been derived using a metasurface model of a general LWA. Using the metasurface described using surface susceptibilities and the desired broadside radiation fields, the application of the GSTCs reveal, that the LWA must exhibit an orthogonal electric and magnetic field dipole moments forming a Huygens’ source configuration. The susceptibilities must be lossy and satisfy a unique amplitude and phase relationship between the two. The proposed metasurface model represents a powerful platform for rigorous field analysis of variety of LWAs, leading to specific prescriptions on the LWA unit cells for efficient broadside radiation, akin to balanced series and shunt resonators of the lattice circuit based circuit analysis.

### References


