97.350 Assignment #1 - Solutions

1. Given the data in the following table for an NMOS transistor with $k' := 20\frac{\mu A}{V^2}$
   Calculate V_{TO}, λ, γ, $2|\phi_F|$, and W/L

	V_{GS} (V)	V_{DS} (V)	V_{SB} (V)	ID (mA)
1	3	5	0	1210
2	5	5	0	4410
3	5	10	0	5292
4	5	5	-2	3265
5	5	5	-5	2381

- In all cases, the device is in saturation, i.e.: $V_{DS} > V_{GS} - V_T$

- From cases 1 and 2: $$\frac{I_{D2}}{I_{D1}} = \frac{\left(V_{GS2} - V_{T0}\right)^2}{\left(V_{GS1} - V_{T0}\right)^2}$$

Therefore: $V_{DS1} = V_{DS2}$ & $V_{BS} = 0$

Hence: $$V_{TO} := \frac{V_{GS2}\cdot\sqrt{\frac{I_{D1}}{I_{D2}}} - V_{GS1}}{\sqrt{\frac{I_{D1}}{I_{D2}}} - 1}$$ $$V_{TO} = 0.8\,V$$

- From cases 2 and 3: $$\frac{I_{D3}}{I_{D2}} = \frac{1 + \lambda\cdot V_{DS3}}{1 + \lambda\cdot V_{DS2}}$$

Therefore: $V_{GS2} = V_{GS3}$

$$\lambda := \frac{\frac{I_{D2}}{I_{D3}} - 1}{V_{DS2} - V_{DS3}\cdot\frac{I_{D2}}{I_{D3}}}$$ $$\lambda = 0.05\,V^{-1}$$

- From $$\frac{W}{L} = \frac{2\cdot I_{D1}}{k'\cdot\left(V_{GS1} - V_{TO}\right)^2\cdot\left(1 + \lambda\cdot V_{DS1}\right)}$$ $$\frac{W}{L} = 20$$

- Solve for V_T: $$V_T = V_{GS} - \sqrt{\frac{2\cdot I_D}{K\cdot\left(1 + \lambda\cdot V_{DS}\right)}}$$ Where $K = k'\cdot\frac{W}{L} = 400\frac{mA}{V^2}$

$$V_{T4} := V_{GS4} - \sqrt{\frac{2\cdot I_{D4}}{K\cdot\left(1 + \lambda\cdot V_{DS4}\right)}}$$ $$V_{T4} = 1.386\,V$$

$$V_{T5} := V_{GS5} - \sqrt{\frac{2\cdot I_{D5}}{K\cdot\left(1 + \lambda\cdot V_{DS5}\right)}}$$ $$V_{T5} = 1.914\,V$$

On the other hand, we can write:

$$V_{T4} = V_{TO} + \gamma\cdot\left(\sqrt{\left|-2\cdot\phi_F + V_{SB4}\right|} - \sqrt{\left|-2\phi_F\right|}\right)$$

$$V_{T5} = V_{TO} + \gamma\cdot\left(\sqrt{\left|-2\cdot\phi_F + V_{SB5}\right|} - \sqrt{\left|-2\phi_F\right|}\right)$$

This can be solved using iteration methods such as Newton's Method, Bisection search or any computer solver using Matlab, Mathcad, etc...:

$\gamma := 1\sqrt{V}$ $\qquad\qquad$ $\phi_F := 1V$

Given

$$V_{T4} = V_{TO} + \gamma\cdot\left(\sqrt{\left|-2\cdot\phi_F + V_{SB4}\right|} - \sqrt{\left|-2\phi_F\right|}\right)$$

$$V_{T5} = V_{TO} + \gamma\cdot\left(\sqrt{\left|-2\cdot\phi_F + V_{SB5}\right|} - \sqrt{\left|-2\phi_F\right|}\right)$$

$$\begin{pmatrix}\gamma \\ \phi_F\end{pmatrix} := \mathrm{Find}(\gamma, \phi_F)$$

$\gamma = 0.7\sqrt{V}$

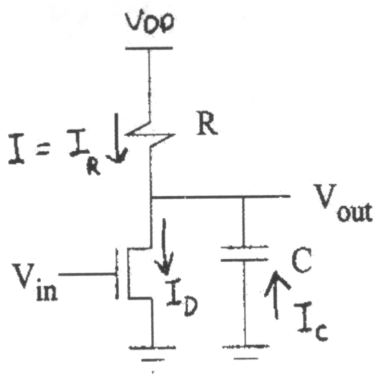
$\phi_F = 0.301\,V$

2. For the following NMOS inverter circuit, assume:

$V_{DD} := 5V \qquad R := 75k\Omega \qquad \frac{W}{L} = \frac{3.6}{1.2} \qquad C := 3pF$

Use the following data for an NMOS transistor:

$V_{T0} := 0.743V \qquad k' := 19.6 \cdot 10^{-6} \frac{A}{V^2} \qquad \lambda := 0.06V^{-1}$



a) Discuss qualitatively why this circuit behaves as an inverter.

It Works as an inverter because:

if $(V_{in} < V_T)$ (i.e. low), NMOS is off, $I = 0$, $V_{out} = 5V$ (i.e. high)

if V_{in} is high (i.e. 5 V), NMOS is on, $I \neq 0$, $V_{out} = 5V - I \cdot R$ (i.e. low)

b) Find V_{OH} and V_{OL}

For $V_{in} < V_T$ ---> $V_{OH} = 5V$

For $V_{in} = V_{OH} = 5V$, NMOS is in linear (or triode) mode

$$I_D = \left[\frac{k'}{2} \cdot \left(\frac{W}{L}\right) \cdot \left[(V_{GS} - V_{T0}) \cdot V_{DS} - \frac{V_{DS}^2}{2}\right]\right] = \frac{5V - V_{OL}}{2}$$

Solving the above equation for V_{OL}:

$$I = K_n \cdot V_{OL} \cdot \left(5 - V_T - \frac{V_{OL}}{2}\right) = \frac{5V - V_{OL}}{R} = I_R = I_D \qquad V_{OL} := 0.26V$$

c) Calculate t_{plh}, t_{phl}, and t_p

$t_{plh} := 0.69 \cdot R \cdot C \qquad t_{plh} = 155.25\,ns$

For t_{phl} we should calculate I_{ave}

$I_C = I_D - I_R$

$V_{out} := 5V$ Hence $V_{DS} := V_{out} \qquad V_{GS} := 5V$

$$I_{Dsat} := \frac{k'}{2} \cdot \left(\frac{W}{L}\right) \cdot (V_{GS} - V_{T0})^2 \cdot (1 + \lambda \cdot V_{DS}) \qquad I_{Dsat} = 692.625\,\mu A$$

$I_R = 0A$

Therefore: $I_{C1} := I_{Dsat} \qquad I_{C1} = 692.625\,\mu A$

$V_{out} := 2.5V$ Hence $V_{DS} := V_{out}$ $V_{GS} := 5V$

$$I_{Dlin} := k' \cdot \left(\frac{W}{L}\right) \cdot \left[(V_{GS} - V_{T0}) \cdot V_{DS} - \frac{V_{DS}^2}{2} \right] \qquad I_{Dlin} = 442.029\,\mu A$$

$$I_R := \frac{V_{out}}{R} \qquad I_R = 33.333\,\mu A$$

Therefore: $I_{C2} := I_{Dlin} - I_R$ $I_{C2} = 408.696\,\mu A$

$$I_{ave} := \frac{I_{C1} + I_{C2}}{2} \qquad I_{ave} = 550.66\,\mu A$$

Since $C = 3\,pF$ is large, we ignore parasitics.

$$t_{phl} := \frac{2.5V \cdot C}{I_{ave}} \qquad t_{phl} = 13.62\,ns$$

$$t_p := \frac{t_{phl} + t_{plh}}{2} \qquad t_p = 84.435\,ns$$

d) Are the rising and falling delays equal? Why?

$t_{plh} >> t_{phl}$ because R = 75 kΩ is much larger then the effective on-resistance of the NMOS.

e) Calculate the static power dissipation for: (i) Vin = 0 V and (ii) Vin = 5 V.

if $V_{in} = V_{OL}$ then $V_{out} = V_{OH} = 5V$ and $I_{Vdd} = 0$ $P_s = 0W$

$V_{DS} := 0.26V$

$$I_D := k' \cdot \left(\frac{W}{L}\right) \cdot \left[(V_{GS} - V_{T0}) \cdot V_{DS} - \frac{V_{DS}^2}{2} \right] \qquad I_D = 63.094\,\mu A$$

if $V_{in} = V_{OH}$ then $V_{out} = V_{OL}$

and $I_{Vdd} = 63.1\mu A$ $P_s := I_D \cdot V_{DD}$ $P_s = 0.315\,mW$

f) Calculate the dynamic power dissipation assuming that the gate is clocked as fast as possible.

$$f_{max} := \frac{1}{2 \cdot t_p} \qquad f_{max} = 5.922\,MHz \qquad V_{swing} := V_{DD} - V_{OL}$$

$$V_{swing} = 4.74\,V$$

$$P_d := C \cdot V_{swing} \cdot V_{DD} \cdot f_{max} \qquad P_d = 0.421\,mW$$