ADDITIONAL ANALYSIS TECHNIQUES

DEVELOP THEVENIN'S AND NORTON'S THEOREMS
These are two very powerful analysis tools that allow us to focus on parts of a circuit and hide away unnecessary complexities

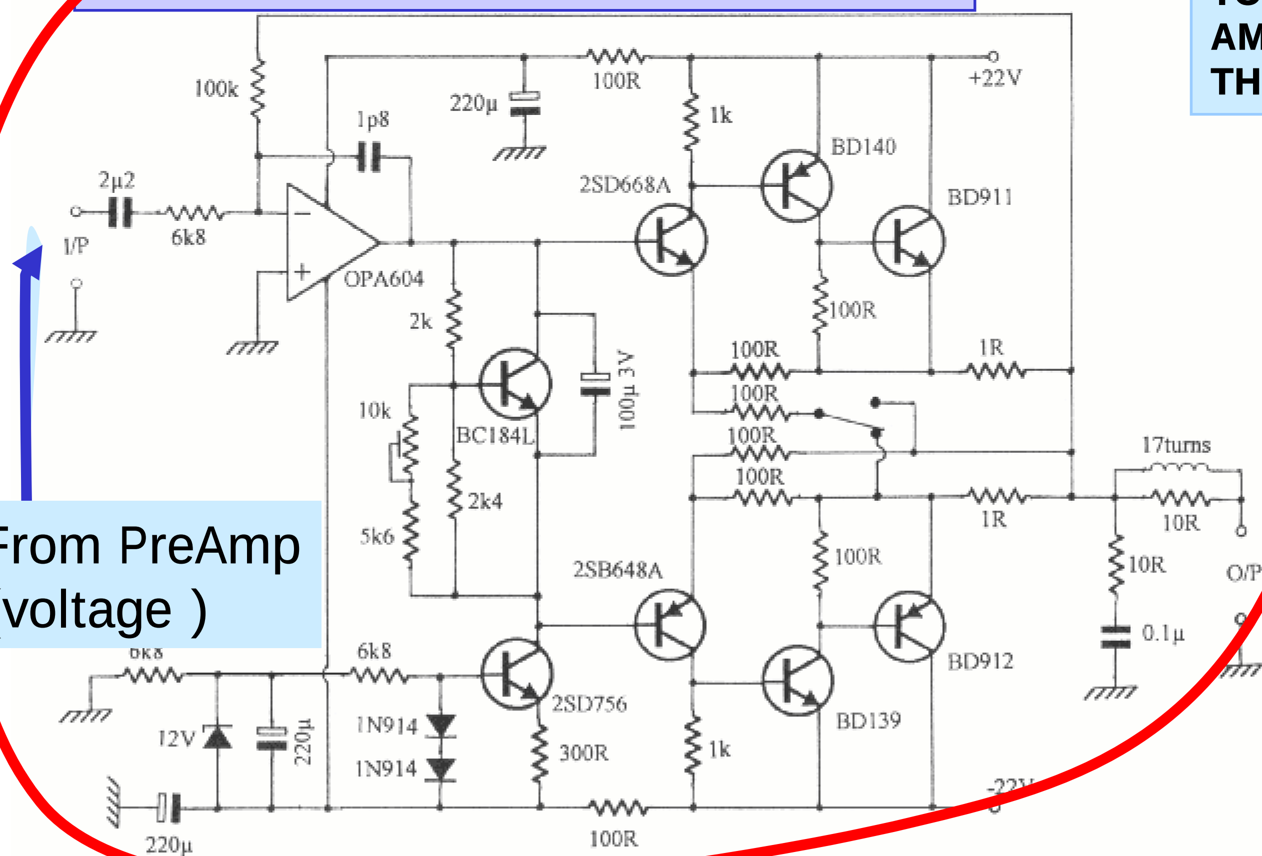
MAXIMUM POWER TRANSFER
This is a very useful application of Thevenin's and Norton's theorems

THEVENIN'S AND NORTON'S THEOREMS

These are some of the most powerful analysis results to be discussed.

They permit to hide information that is not relevant and concentrate in what is important to the analysis

Low distortion audio power amplifier



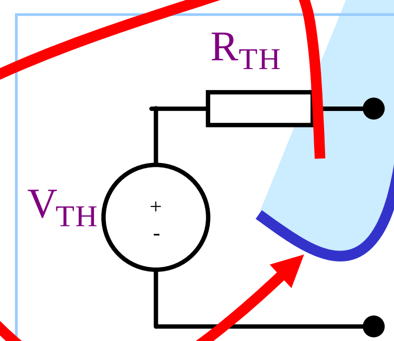
TO MATCH SPEAKERS AND AMPLIFIER ONE SHOULD ANALYZE THIS CIRCUIT

From PreAmp (voltage)

To speakers

Courtesy of M.J. Renardson

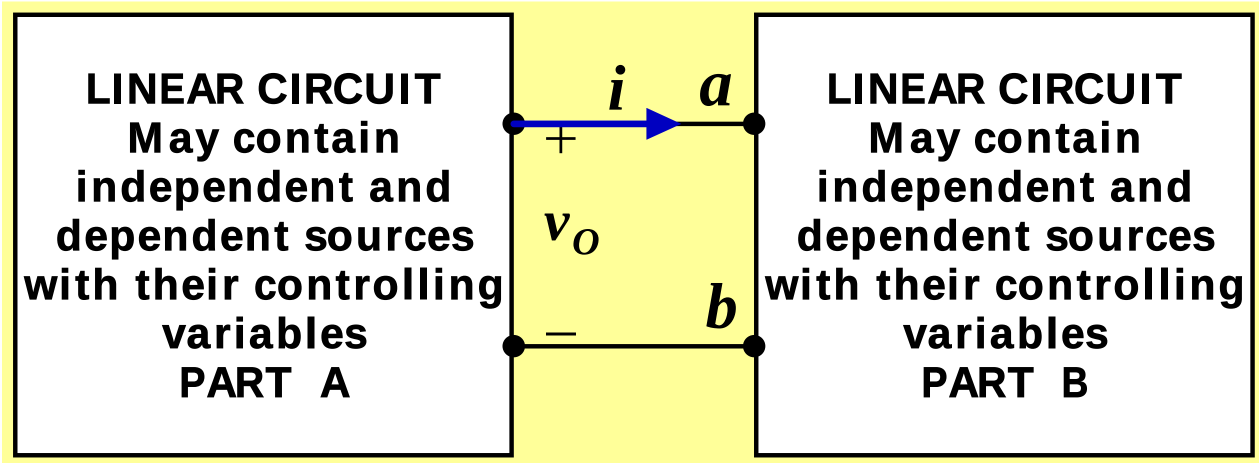
http://angelfire.com/ab3/mjramp/index.html

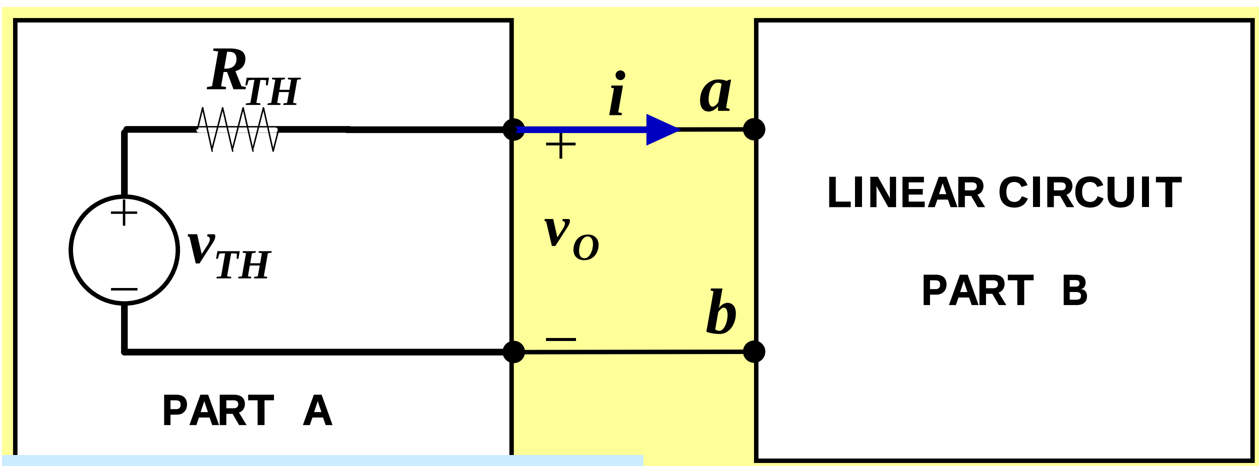


REPLACE AMPLIFIER BY SIMPLER "EQUIVALENT"

TO MATCH SPEAKERS AND AMPLIFIER IT IS MUCH EASIER TO CONSIDER THIS EQUIVALENT CIRCUIT!

THEVENIN'S EQUIVALENCE THEOREM





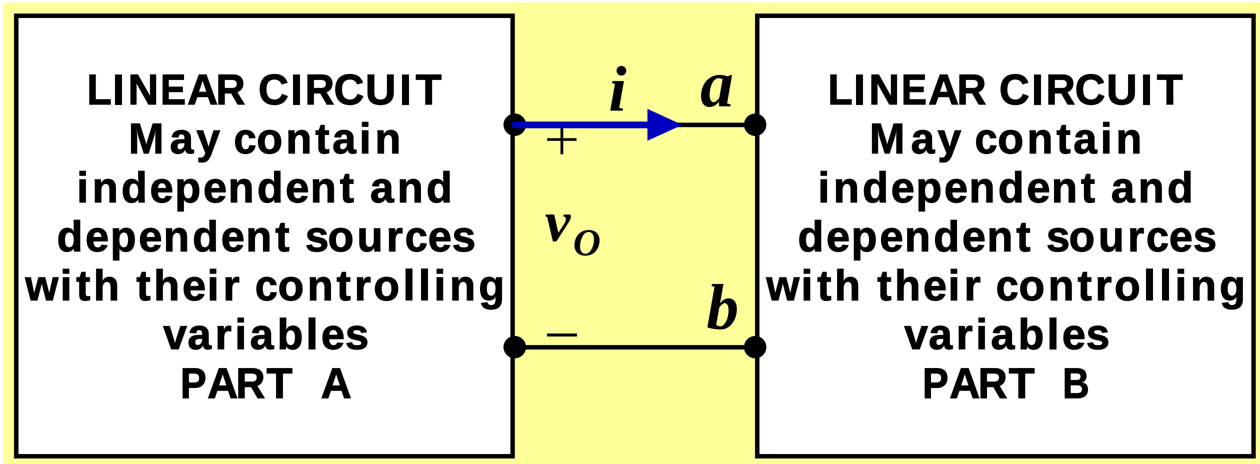
Thevenin Equivalent Circuit

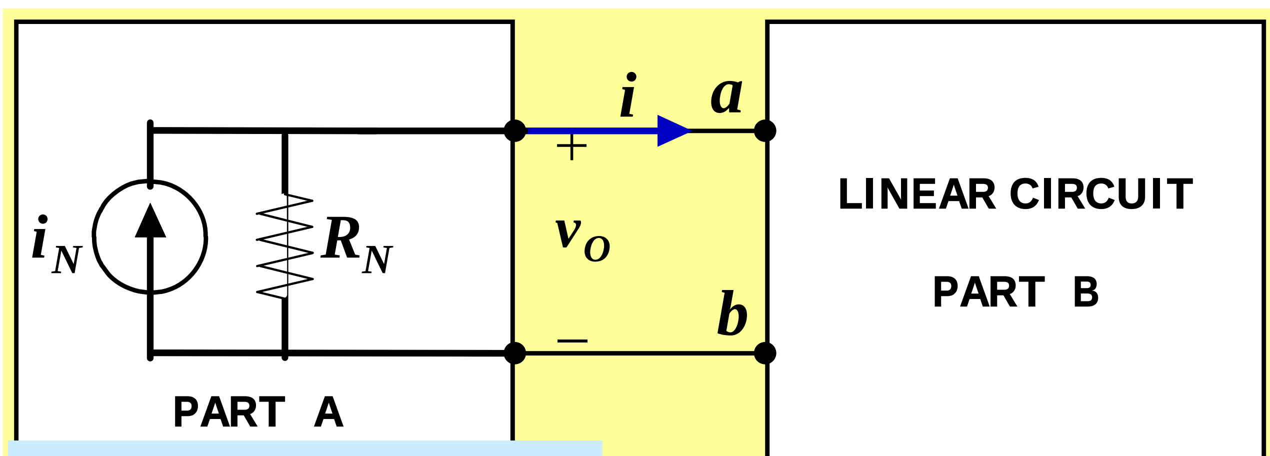
for PART A

v_{TH} Thevenin Equivalent Source

R_{TH} Thevenin Equivalent Resistance

NORTON'S EQUIVALENCE THEOREM



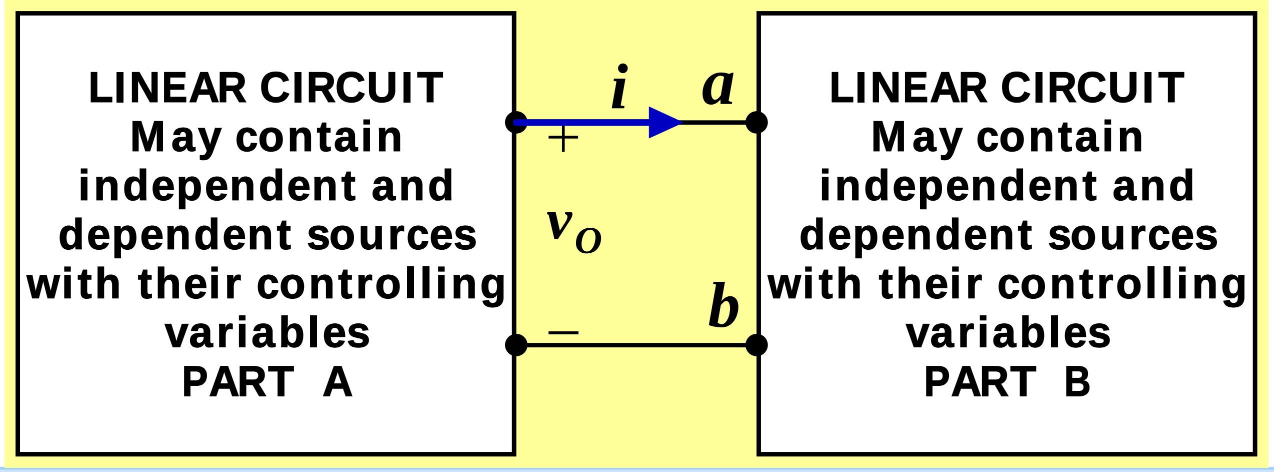


Norton Equivalent Circuit for PART A

i_N Norton Equivalent Source

R_N Norton Equivalent Resistance

OUTLINE OF PROOF



1. Because of the linearity of the models, for any Part B the relationship between *Vo* and the current, *i*, has to be of the form $v_O = m * i + n$

2. Result must hold for "every valid Part B" that we can imagine

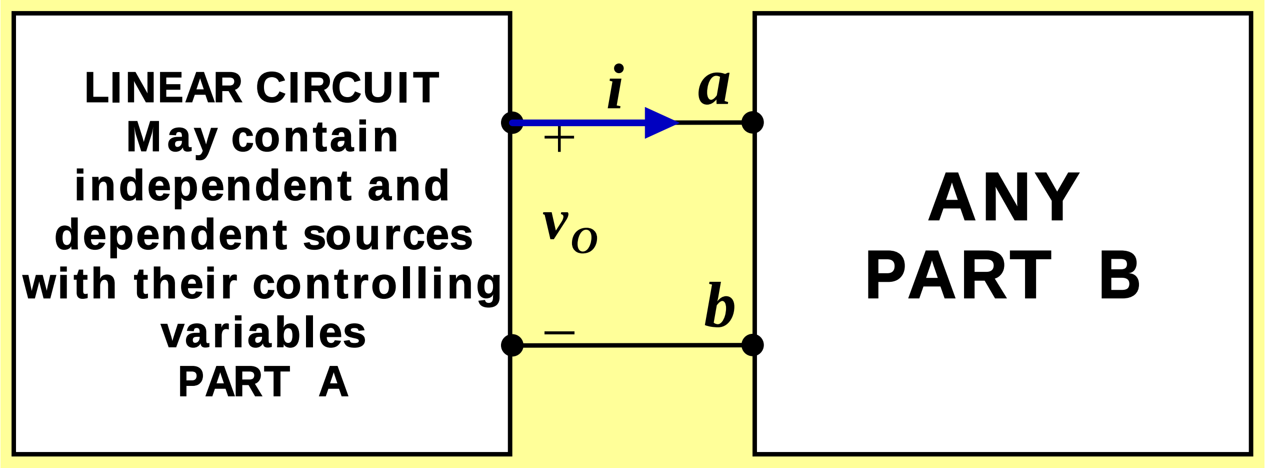
3. If part B is an open circuit then i=0 and… $n = v_{OC}$

4. If Part B is a short circuit then Vo is zero. In this case

$$0 = m * i_{SC} + v_{OC} \Rightarrow m = -\frac{v_{OC}}{i_{SC}} = -R_{TH}$$

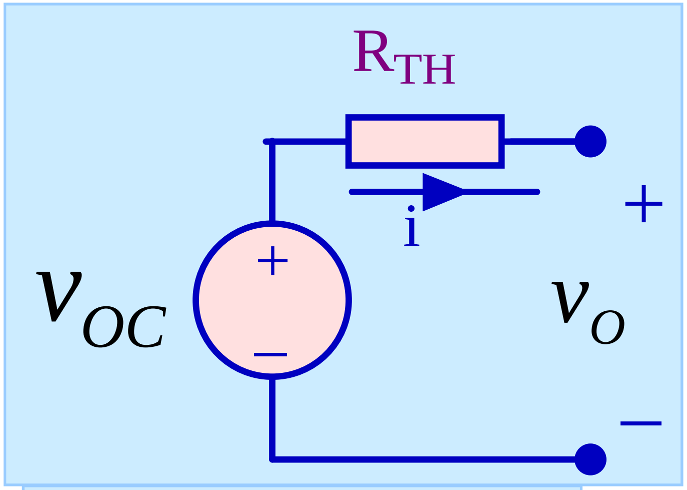
$$v_O = -R_{TH}\, i + v_{OC}$$

THEVENIN APPROACH



$$v_O = -R_{TH} i + v_{OC}$$

For ANY circuit in Part B



PART A MUST BEHAVE LIKE THIS CIRCUIT

This is the Thevenin equivalent circuit for the circuit in Part A

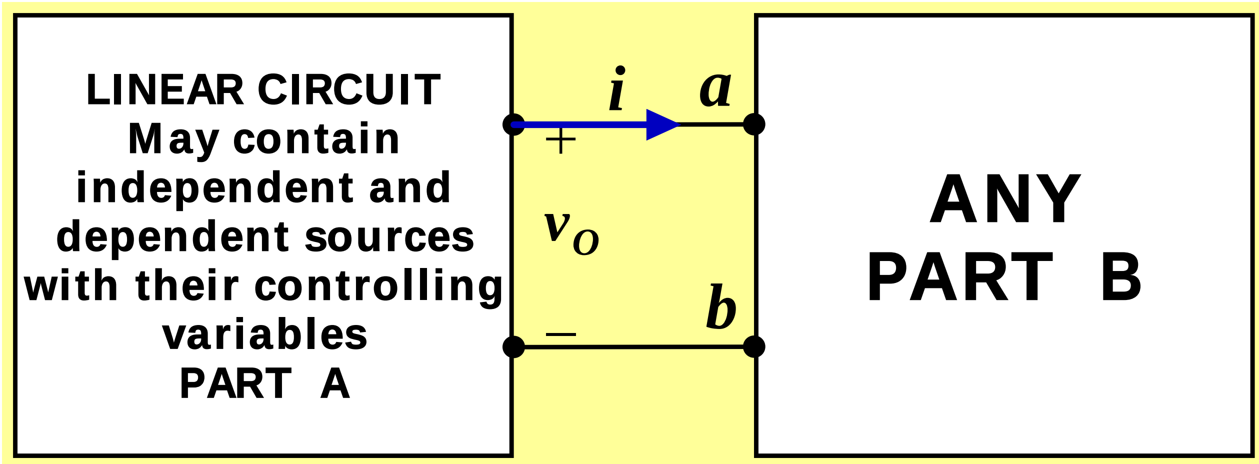
The voltage source is called the THEVENIN EQUIVALENT SOURCE

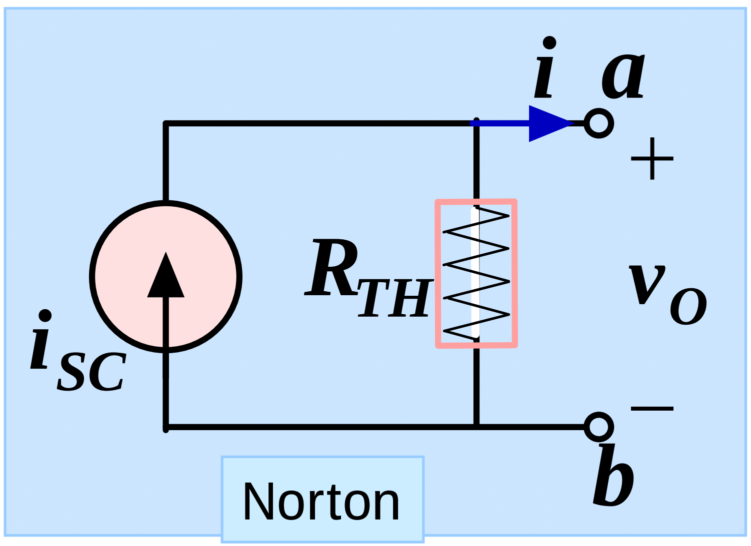
The resistance is called the THEVENIN EQUIVALENT RESISTANCE

Norton Approach

$$v_O = v_{OC} - R_{TH} i \Rightarrow i = \frac{v_{OC}}{R_{TH}} - \frac{v_O}{R_{TH}}$$

$$\frac{v_{OC}}{R_{TH}} = i_{SC}$$

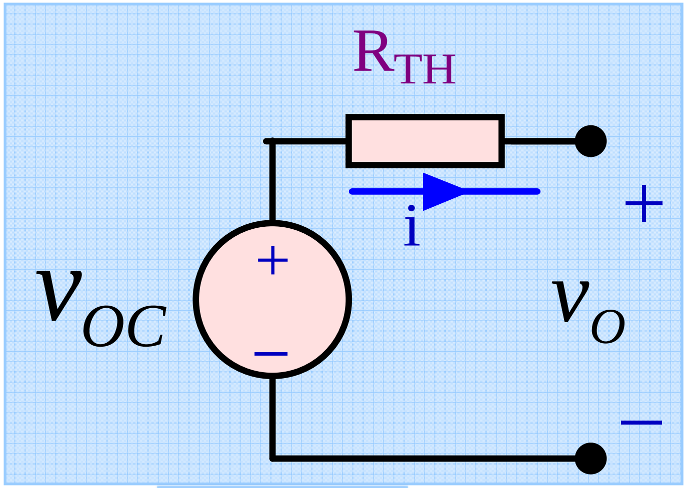




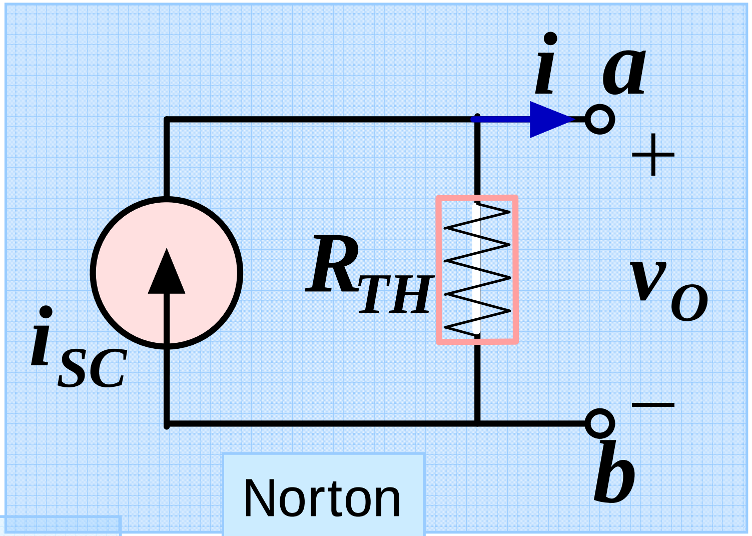
Norton Equivalent Representation for Part A

i_{SC} Norton Equivalent Source

ANOTHER VIEW OF THEVENIN'S AND NORTON'S THEOREMS



Thevenin



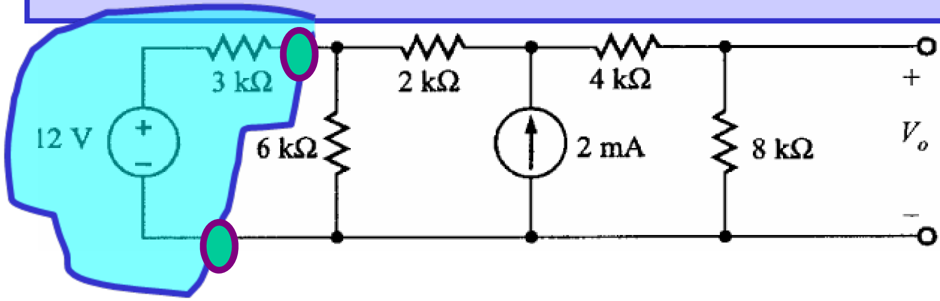
Norton

$$i_{SC} = \frac{v_{OC}}{R_{TH}}$$

This equivalence can be viewed as a source transformation problem
It shows how to convert a voltage source in series with a resistor
into an equivalent current source in parallel with the resistor

SOURCE TRANSFORMATION CAN BE A GOOD TOOL TO REDUCE THE COMPLEXITY OF A CIRCUIT

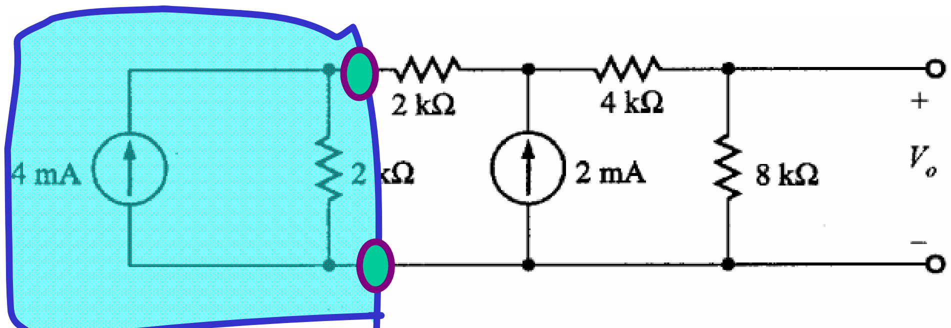
EXAMPLE: SOLVE BY SOURCE TRANSFORMATION



In between the terminals we connect a current source and a resistance in parallel

The equivalent current source will have the value 12V/3k

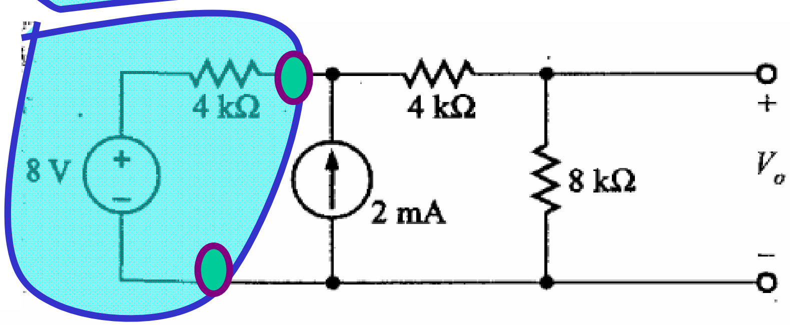
The 3k and the 6k resistors now are in parallel and can be combined



In between the terminals we connect a voltage source in series with the resistor

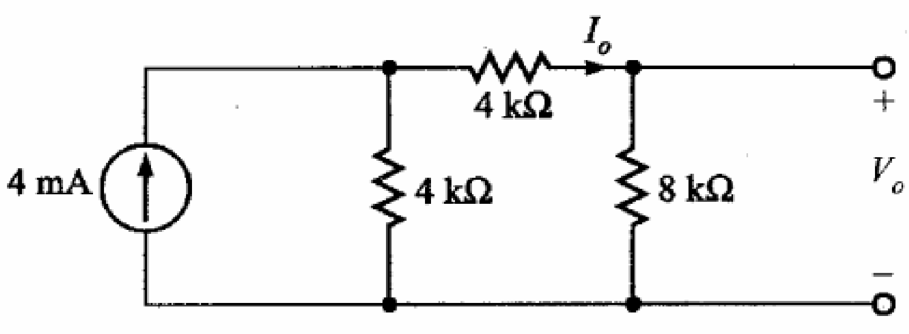
The equivalent source has value 4mA*2k

The 2k and the 2k resistor become connected in series and can be combined



After the transformation the sources can be combined

The equivalent current source has value 8V/4k and the combined current source has value 4mA



Options at this point

1. Do another source transformation and get a single loop circuit

2. Use current divider to compute I_0 and then compute V_0 using Ohm's law

A General Procedure to Determine the Thevenin Equivalent

v_{TH} Open Circuit voltage

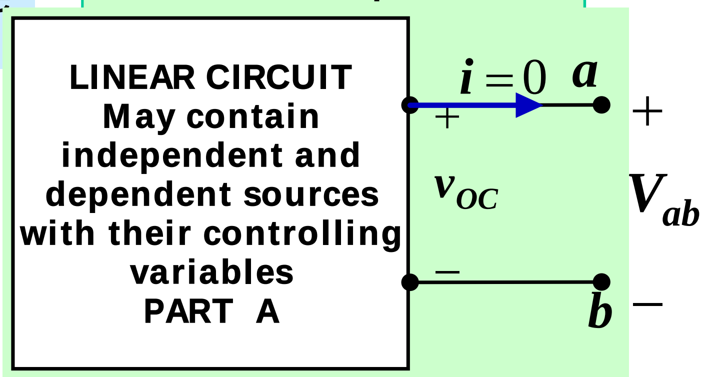
voltage at a-b if Part B is removed

i_{SC} Short Circuit Current

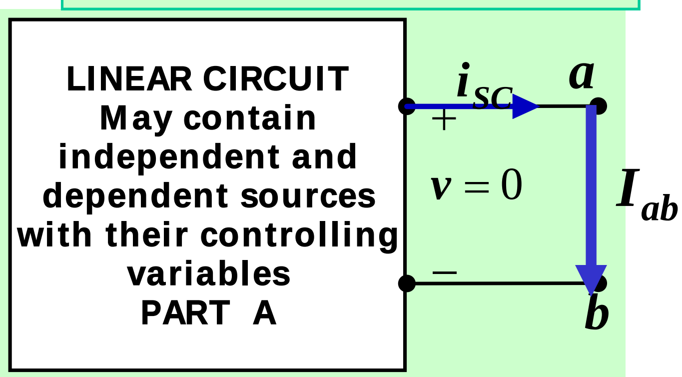
current through a-b if Part B is replaced by a short circuit

$R_{TH} = \frac{v_{TH}}{i_{SC}}$ Thevenin Equivalent Resistance

One circuit problem



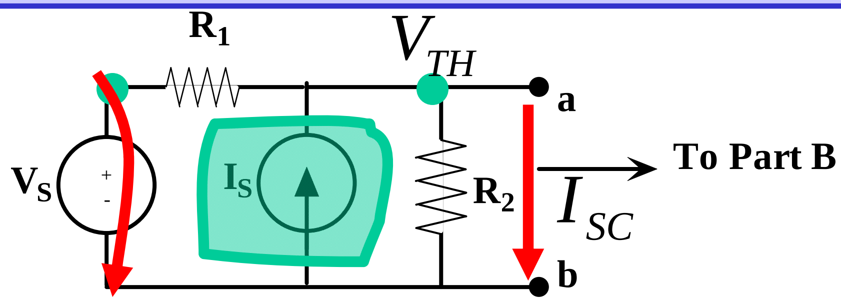
Second circuit problem



1. Determine the Thevenin equivalent source	Remove part B and compute the OPEN CIRCUIT voltage V_{ab}
2. Determine the SHORT CIRCUIT current	Remove part B and compute the SHORT CIRCUIT current I_{ab}

$$v_{TH} = v_{OC}, R_{TH} = \frac{v_{OC}}{i_{SC}}$$

AN EXAMPLE OF DETERMINING THE THEVENIN EQUIVALENT



Part B is irrelevant.
The voltage V_ab will be the value of the Thevenin equivalent source.

What is an efficient technique to compute the open circuit voltage?

NODE ANALYSIS

$$\frac{V_{TH}}{R_2} + \frac{V_{TH} - V_S}{R_1} - I_S = 0$$

$$\left(\frac{1}{R_1} + \frac{1}{R_2}\right)V_{TH} = \frac{V_S}{R_1} + I_S$$

$$V_{TH} = \frac{R_2}{R_1 + R_2}V_S + \frac{R_1 R_2}{R_1 + R_2}I_S$$

$$V_{TH} = \frac{R_1 R_2}{R_1 + R_2}\left(\frac{V_S}{R_1} + I_S\right)$$

Now for the short circuit current
Lets try source superposition

When the current source is open the current through the short circuit is

$$I_{SC}^1 = \frac{V_S}{R_1}$$

When the voltage source is set to zero, the current through the short circuit is

$$I_{SC}^2 = I_S$$

$$I_{SC} = I_S + \frac{V_S}{R_1}$$

$$R_{TH} = \frac{R_1 R_2}{R_1 + R_2}$$

To compute the Thevenin resistance we use

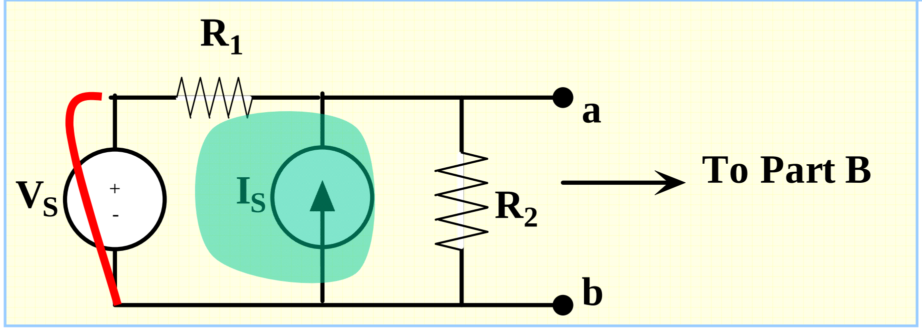
$$R_{TH} = \frac{V_{TH}}{I_{SC}}$$

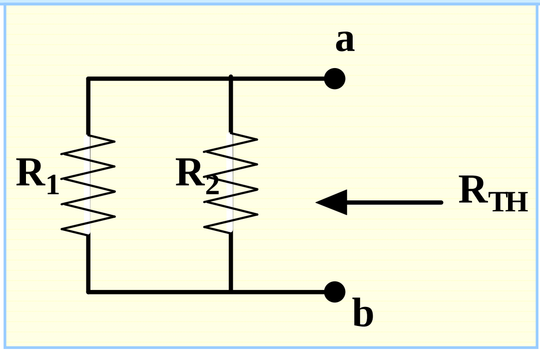
For this case the Thevenin resistance can be computed as the resistance from a - b when all independent sources have been set to zero

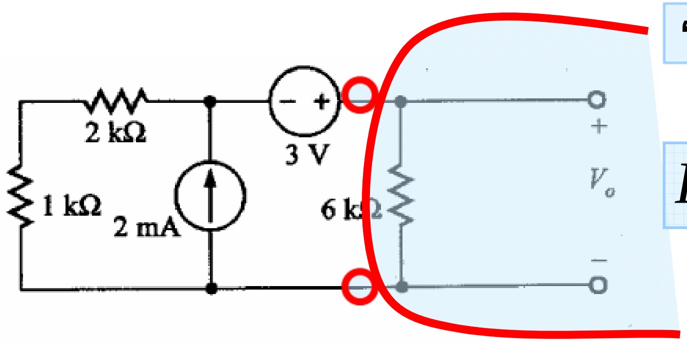
Determining the Thevenin Equivalent in Circuits with Only INDEPENDENT SOURCES

The Thevenin Equivalent Source is computed as the open loop voltage

The Thevenin Equivalent Resistance CAN BE COMPUTED by setting to zero all the sources and then determining the resistance seen from the terminals where the equivalent will be placed



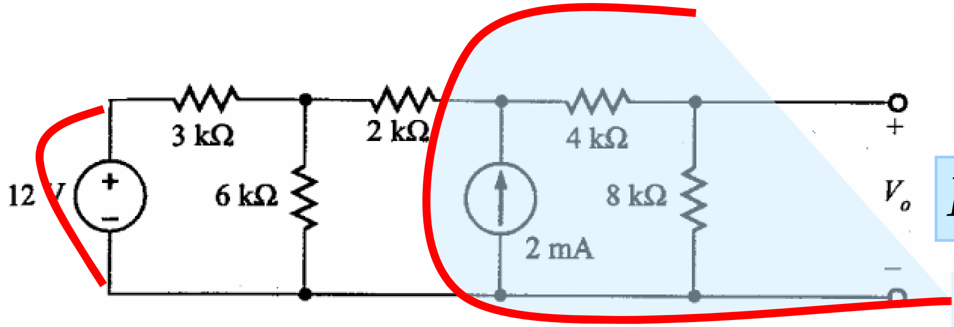




"Part B"

$R_{TH} = 3k\Omega$

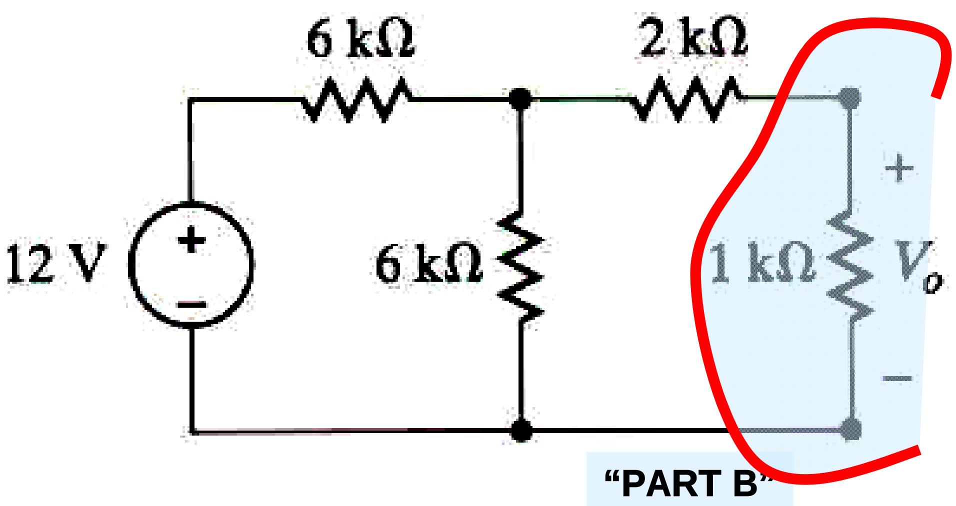
Since the evaluation of the Thevenin equivalent can be very simple, we can add it to our toolkit for the solution of circuits!!

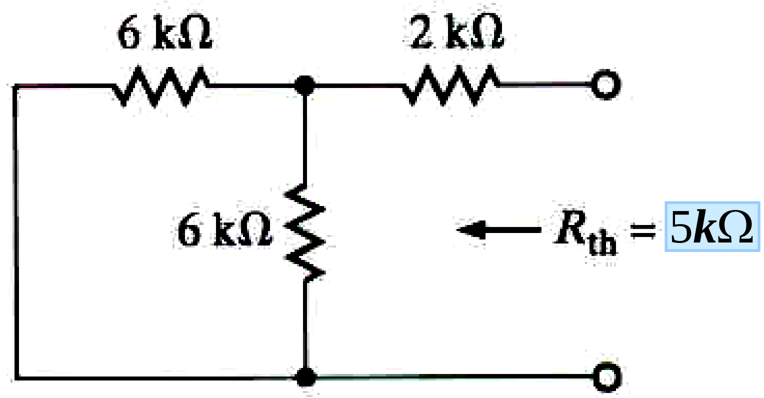


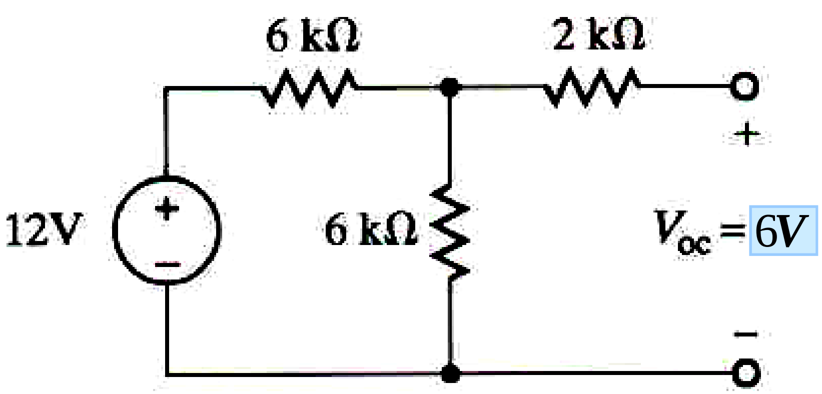
$R_{TH} = 4k\Omega$

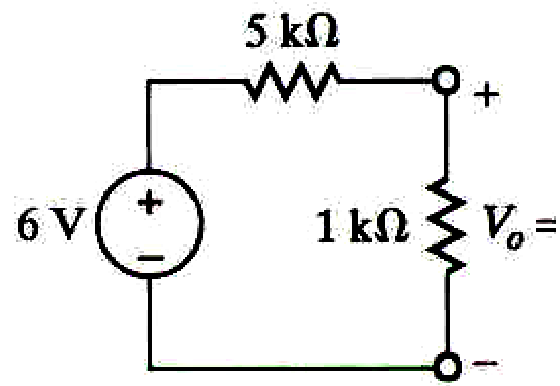
"Part B"

Find V_o in the following network using Thévenin's theorem.





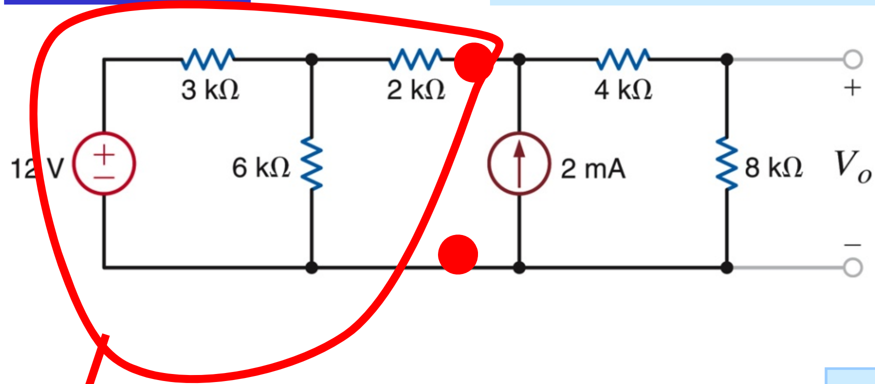




$$V_O = \frac{1k}{1k + 5k}(6V) = 1[V]$$

EXAMPLE

COMPUTE Vo USING THEVENIN



In the region shown, one could use source transformation twice and reduce that part to a single source with a resistor.

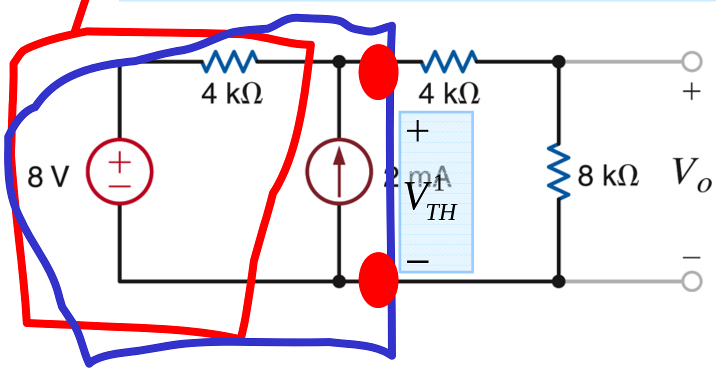
... Or we can apply Thevenin Equivalence to that part (viewed as "Part A")

$$R_{TH} = 4k\Omega$$

$$V_{TH} = \frac{6}{3+6} 12[V] = 8[V]$$

For the open loop voltage the part outside the region is eliminated

The original circuit becomes...



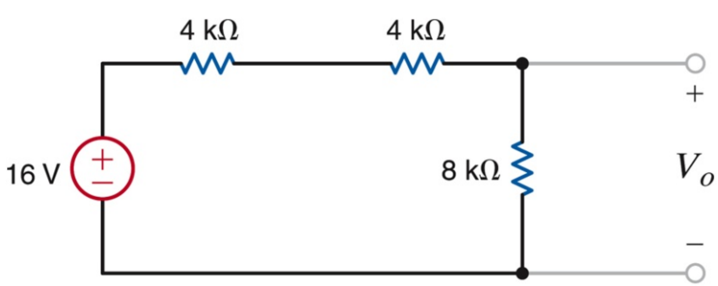
And one can apply Thevenin one more time!

For open loop voltage use KVL

$$R^1{}_{TH} = 4k\Omega$$

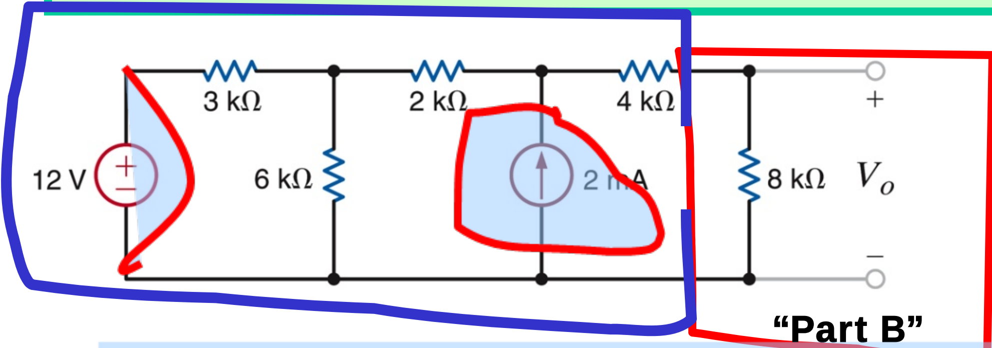
$$V^1_{TH} = 4k * 2mA + 8V = 16V$$

...and we have a simple voltage divider!!



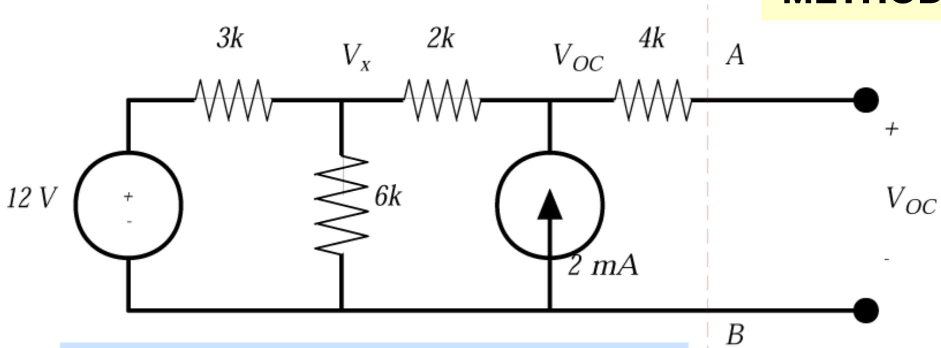
$$V_0 = \frac{8}{8+8} 16[V] = 8V$$

Or we can use Thevenin only once to get a voltage divider

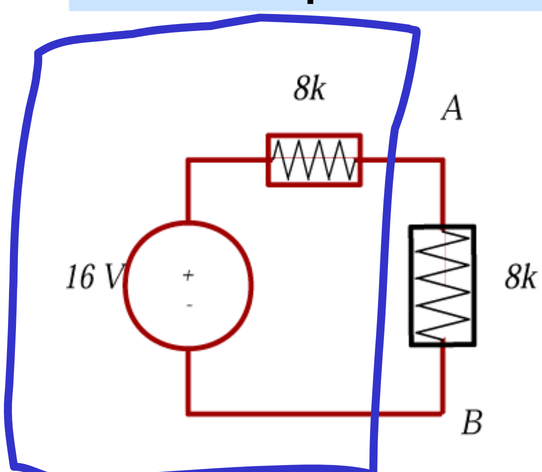


For the Thevenin voltage we have to analyze the following circuit

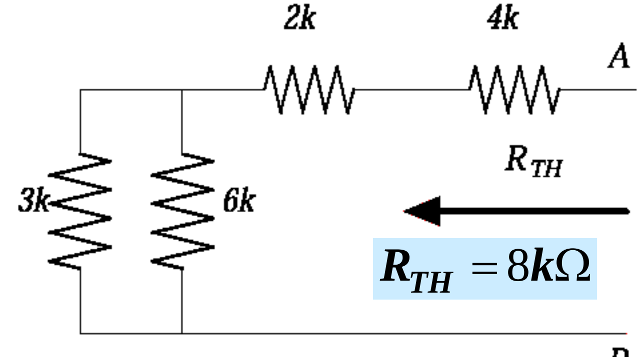
METHOD??



Thevenin Equivalent of "Part A"



For the Thevenin resistance



Source superposition, for example

Contribution of the voltage source

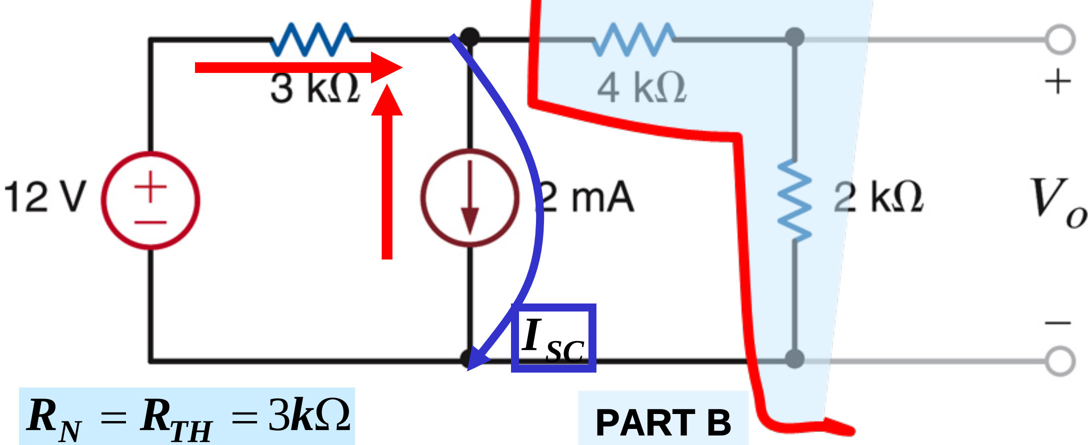
$$V_{OC}^{1} = \frac{6}{3+6} 12V = 8V$$

Contribution of the current source

$$V_{OC}^{2} = (2k + 2k) * (2mA) = 8V$$

Simple Voltage Divider

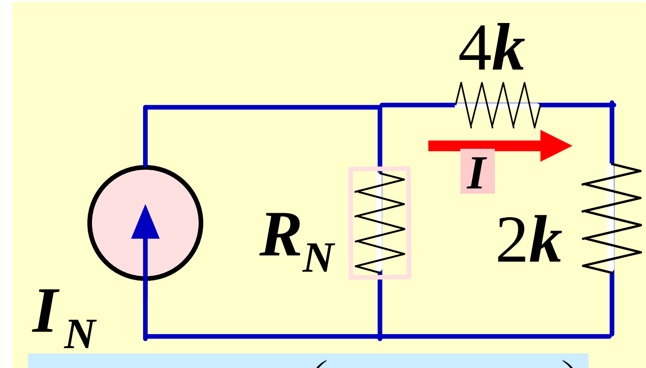
COMPUTE Vo USING NORTON



PART B

$$R_N = R_{TH} = 3k\Omega$$

$$I_{SC} = I_N = \frac{12V}{3k} - 2mA = 2mA$$

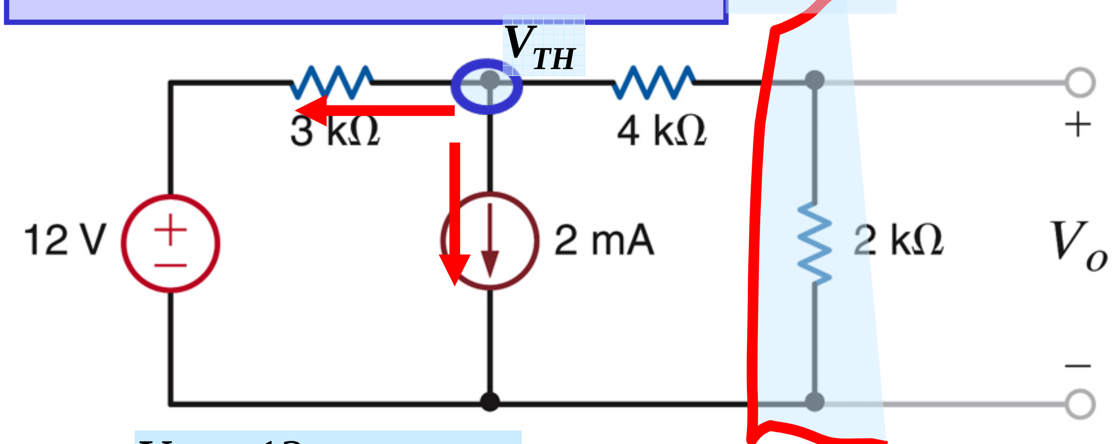


$$V_O = 2kI = 2k\left(\frac{R_N}{R_N + 6k} I_N\right)$$

$$V_O = 2\frac{3}{9}(2) = \frac{4}{3}[V]$$

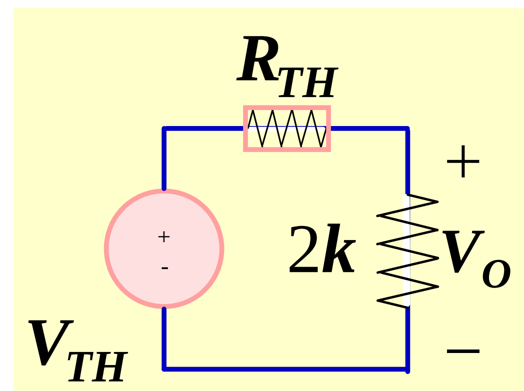
COMPUTE Vo USING THEVENIN

PART B



$$\frac{V_{TH} - 12}{3k} + 2mA = 0$$

$$R_{TH} = 3k + 4k$$

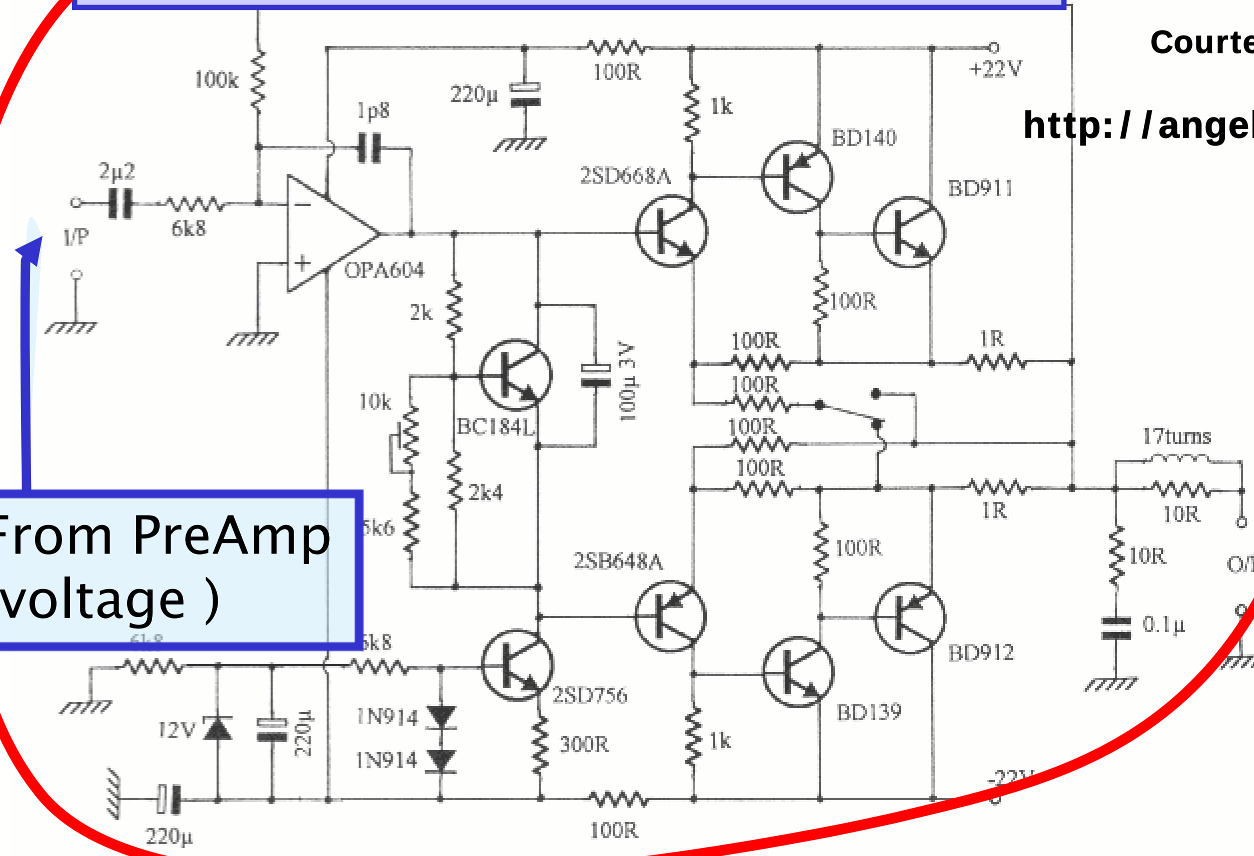


$$V_O = \frac{2}{2+7}(6V) = \frac{4}{3}[V]$$

MAXIMUM POWER TRANSFER

Courtesy of M.J. Renardson

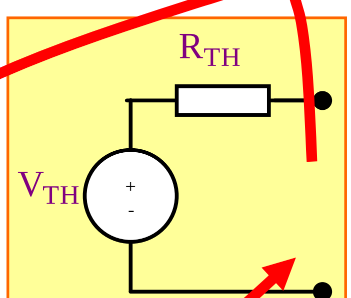
http://angelfire.com/ab3/mjramp/index.html

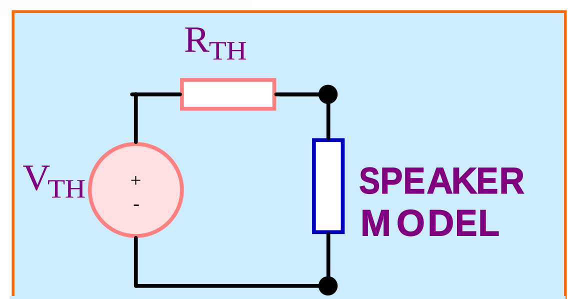


From PreAmp (voltage)

To speakers

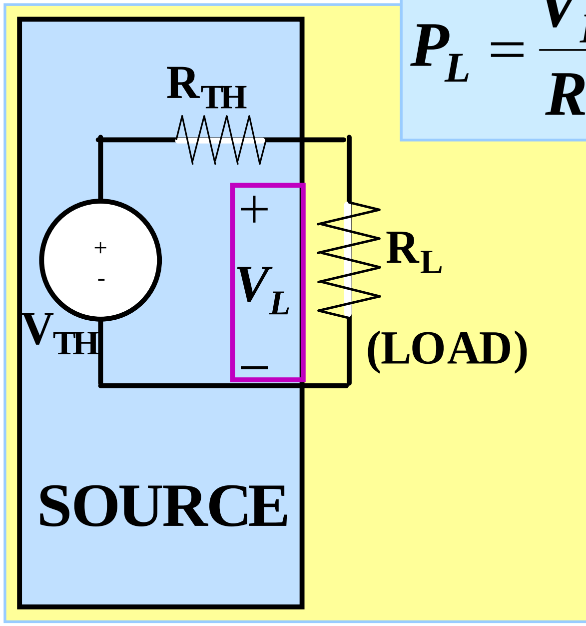
The simplest model for a speaker is a resistance...





BASIC MODEL FOR THE ANALYSIS OF POWER TRANSFER

MAXIMUM POWER TRANSFER



$$P_L = \frac{V_L^2}{R_L}; V_L = \frac{R_L}{R_{TH} + R_L} V_{TH}$$

$$P_L = \frac{R_L}{(R_{TH} + R_L)^2} V_{TH}^2$$

For every choice of R_L we have a different power. How do we find the maximum value?

Consider P_L as a function of R_L and find the maximum of such function

$$\frac{dP_L}{dR_L} = V_{TH}^2 \left(\frac{(R_{TH} + R_L)^2 - 2R_L (R_{TH} + R_L)}{(R_{TH} + R_L)^4} \right) \quad 3$$

Set the derivative to zero to find extreme points. For this case we need to set to zero the numerator

$$R_{TH} + R_L - 2R_L = 0 \Rightarrow R_L^* = R_{TH}$$

The maximum power transfer theorem

The load that maximizes the power transfer for a circuit is equal to the Thevenin equivalent resistance of the circuit.

The value of the maximum power that can be transferred is

$$P_L(\max) = \frac{V_{TH}^2}{4R_{TH}}$$

ONLY IN THIS CASE WE NEED TO COMPUTE THE THEVENIN VOLTAGE