ADDITIONAL ANALYSIS TECHNIQUES

REVIEW LINEARITY
The property has two equivalent definitions.
We show and application of homogeneity

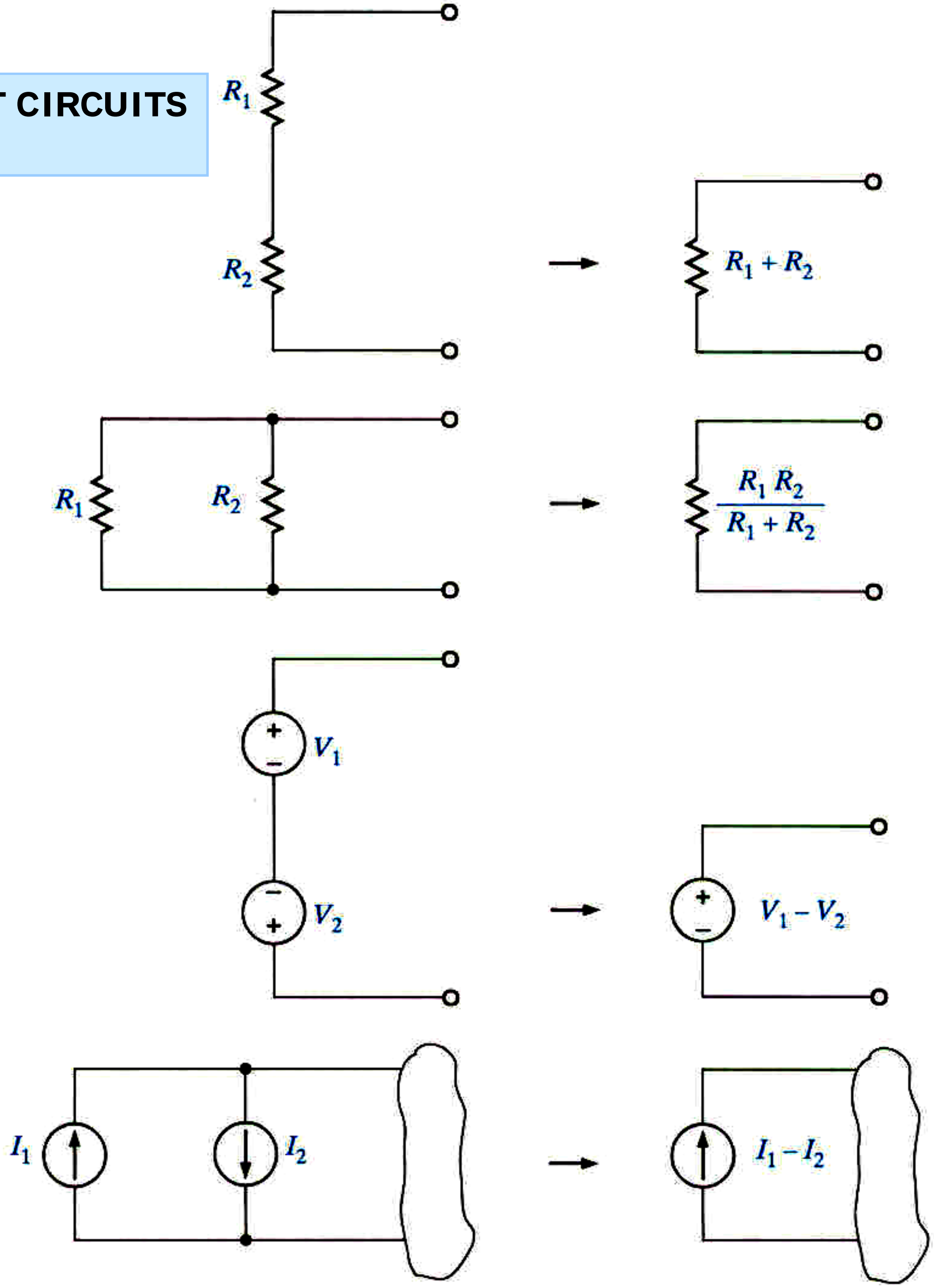
APPLY SUPERPOSITION
We discuss some implications of the superposition property in linear circuits

THE METHODS OF NODE AND LOOP ANALYSIS PROVIDE POWERFUL TOOLS TO DETERMINE THE BEHAVIOR OF EVERY COMPONENT IN A CIRCUIT

The techniques developed with combination series/parallel, voltage divider and current divider are special techniques that are more efficient than the general methods, but have a limited applicability. It is to our advantage to keep them in our repertoire and use them when they are more efficient.

In this section we develop additional techniques that simplify the analysis of some circuits. In fact these techniques expand on concepts that we have already introduced: linearity and circuit equivalence

SOME EQUIVALENT CIRCUITS ALREADY USED



LINEARITY

THE MODELS USED ARE ALL LINEAR. MATHEMATICALLY THIS IMPLIES THAT THEY SATISFY THE PRINCIPLE OF SUPERPOSITION

THE MODEL $\boldsymbol{y} = \boldsymbol{T}\boldsymbol{u}$ IS LINEAR IFF

$$\boldsymbol{T}(\alpha_1 \boldsymbol{u}_1 + \alpha_2 \boldsymbol{u}_2) = \alpha_1 \boldsymbol{T}\boldsymbol{u}_1 + \alpha_2 \boldsymbol{T}\boldsymbol{u}_2$$

for all possible input pairs $\boldsymbol{u}_1, \boldsymbol{u}_2$

and all possible scalars α_1, α_2

AN ALTERNATIVE, AND EQUIVALENT, DEFINITION OF LINEARITY SPLITS THE SUPERPOSITION PRINCIPLE IN TWO.

THE MODEL $\boldsymbol{y} = \boldsymbol{T}\boldsymbol{u}$ IS LINEAR IFF

1. $\boldsymbol{T}(\boldsymbol{u}_1 + \boldsymbol{u}_2) = \boldsymbol{T}\boldsymbol{u}_1 + \boldsymbol{T}\boldsymbol{u}_2 \;,\; \forall \boldsymbol{u}_1, \boldsymbol{u}_2$ additivity
2. $\boldsymbol{T}(\alpha \boldsymbol{u}) = \alpha \boldsymbol{T}\boldsymbol{u}, \; \forall \alpha, \forall \boldsymbol{u}$ homogeneity

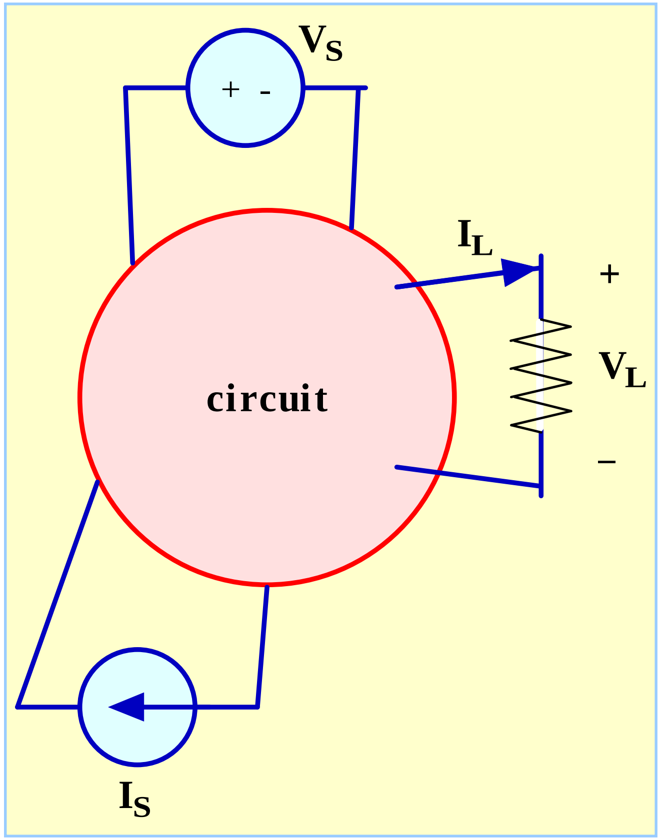
FOR CIRCUIT ANALYSIS WE CAN USE THE LINEARITY ASSUMPTION TO DEVELOP SPECIAL ANALYSIS TECHNIQUES

Source Superposition

This technique is a direct application of linearity.

It is normally useful when the circuit has only a few sources.

FOR CLARITY WE SHOW A CIRCUIT WITH ONLY TWO SOURCES



Due to Linearity

$$V_L = a_1 V_S + a_2 I_S$$

CONTRIBUTION BY V_S: V_L^1

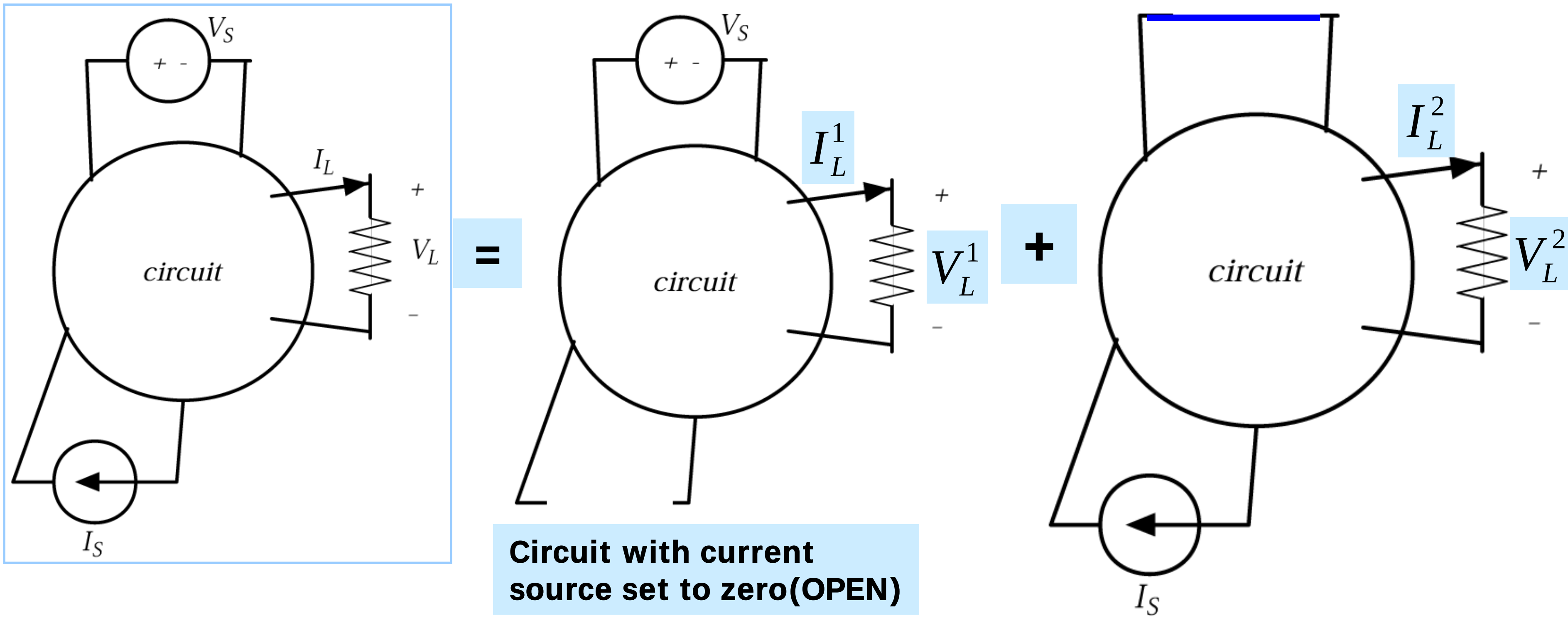
CONTRIBUTION BY I_S: V_L^2

$V^1{}_L$ — **Can be computed by setting the current source to zero and solving the circuit**

$V^2{}_L$ — **Can be computed by setting the voltage source to zero and solving the circuit**

SOURCE SUPERPOSITION

Circuit with voltage source set to zero (SHORT CIRCUITED)



Due to the linearity of the models we must have

$$I_L = I_L^1 + I_L^2 \qquad V_L = V_L^1 + V_L^2$$

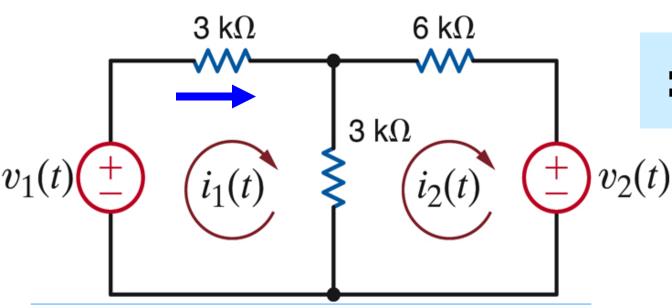
Principle of Source Superposition

The approach will be useful if solving the two circuits is simpler, or more convenient, than solving a circuit with two sources

We can have any combination of sources. And we can partition any way we find convenient

EXAMPLE

WE WISH TO COMPUTE THE CURRENT i1



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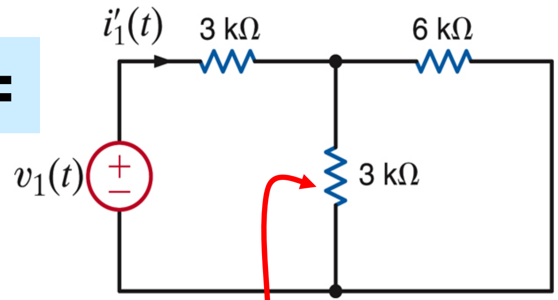

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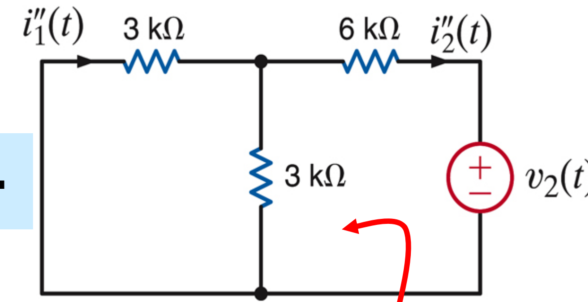


$$6ki_1(t) - 3ki_2(t) = v_1(t)$$

$$-3ki_1(t) + 9ki_2(t) = -v_2(t)$$

Loop equations

$$R_{eq} = 3 + 3 \| 6 \; [k]$$

$$i_1'(t) = \frac{v_1(t)}{3k + \frac{(3k)(6k)}{3k + 6k}} = \frac{v_1(t)}{5k}$$

Contribution of v1

$$R_{eq} = 6 + (3 \| 3) \; [k]$$

$$i_2'' = \frac{v_2}{R_{eq}}$$

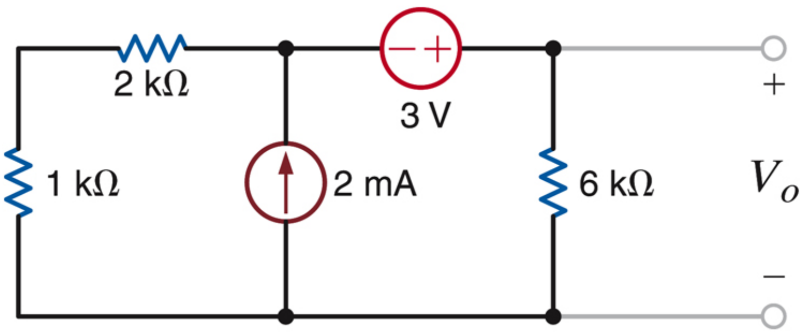
$$i_1''(t) = \frac{-2v_2(t)}{15k}\left(\frac{3k}{3k + 3k}\right) = \frac{-v_2(t)}{15k}$$

Contribution of v2

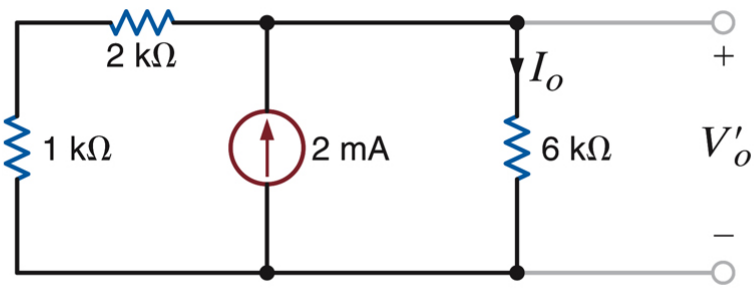
Once we know the "partial circuits" we need to be able to solve them in an efficient manner

EXAMPLE

Compute V_0 using source superposition



We set to zero the voltage source



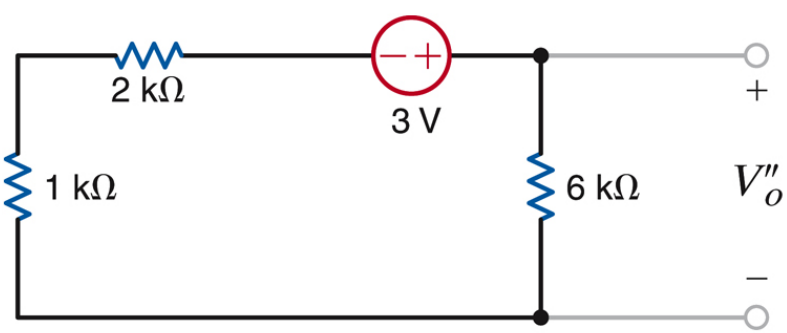
$$I_o = (2 \times 10^{-3})\left(\frac{1k + 2k}{1k + 2k + 6k}\right)$$

Current division

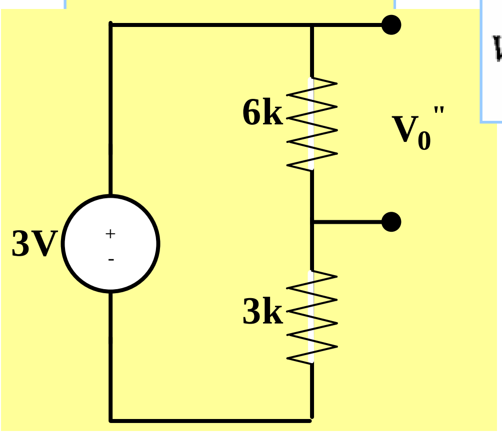
$$V'_o = I_o(6k) = 4\ V$$

Ohm's law

Now we set to zero the current source



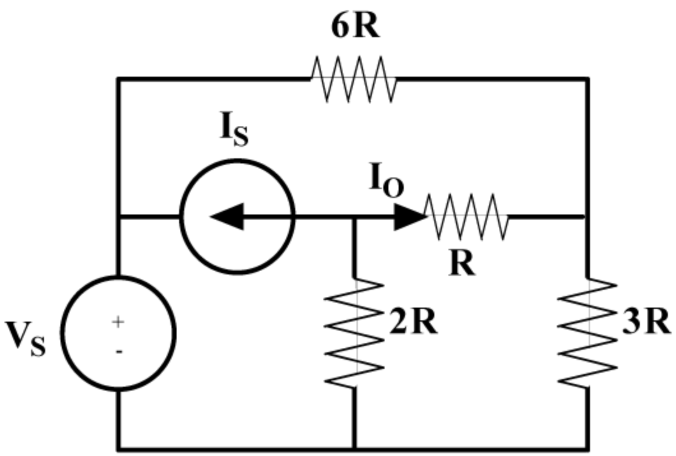
Voltage Divider



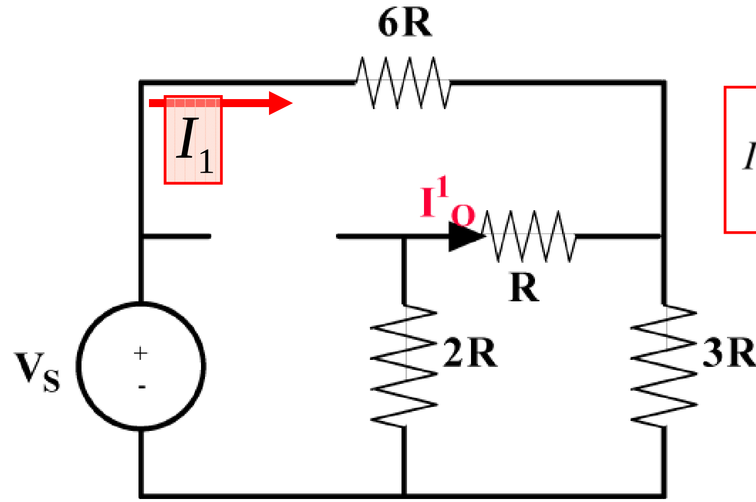
$$V''_o = 3\left(\frac{6k}{1k + 2k + 6k}\right) = 2[V]$$

$$V_0 = V_0' + V_0'' = 6[V]$$

USE SOURCE SUPERPOSITION TO COMPUTE I_O



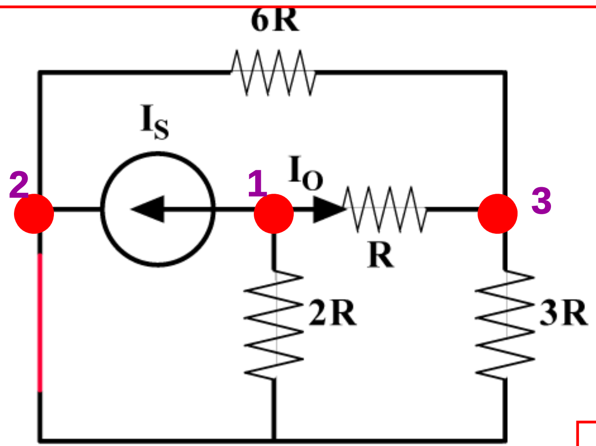
open current source



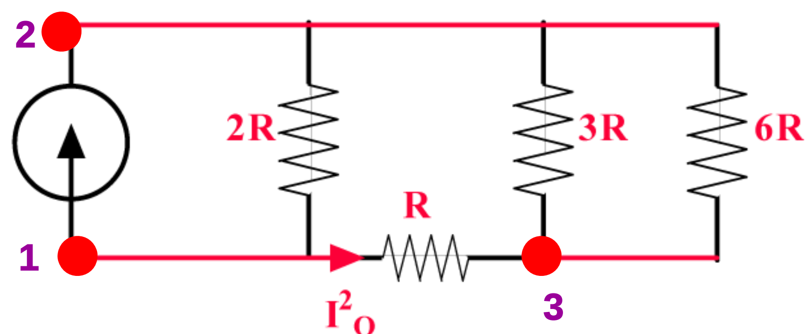
$$I_1 = \frac{V_S}{6R + \| 3R, 3R \|}$$

$$I_O^1 = -\frac{I_1}{2}$$

short circuit voltage source



in case of doubt: REDRAW CIRCUIT!



NOW USE CURRENT DIVIDER

$$I^2{}_O = -\frac{2R}{2R + R + \| 3R, 6R \|} I_S$$

$$I^2{}_O = -\frac{2}{5} I_S$$

$$I_O = I^1{}_O + I^2{}_O$$

$$I_O = -\frac{V_S}{15R} - \frac{2}{5} I_S$$