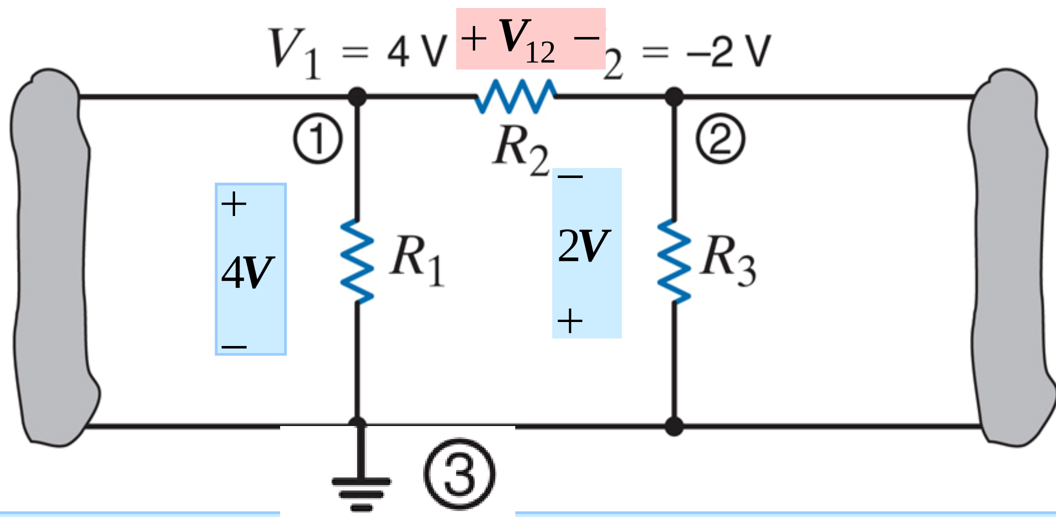
RESISTIVE CIRCUITS

- MULTI NODE/LOOP CIRCUIT ANALYSIS

DEFINING THE REFERENCE NODE IS VITAL



THE STATEMENT $V_1 = 4V$ IS MEANINGLES

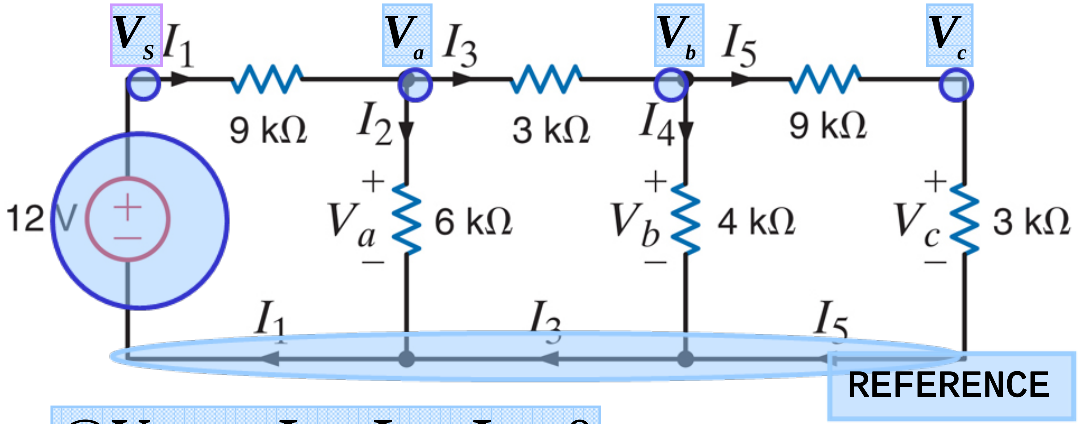
UNTIL THE REFERENCE POINT IS DEFINED

BY CONVENTION THE GROUND SYMBOL SPECIFIES THE REFERENCE POINT.

ALL NODE VOLTAGES ARE MEASURED WITH RESPECT TO THAT REFERENCE POINT

$\mathbf{V}_{12} =$ ______?

THE STRATEGY FOR NODE ANALYSIS



1. IDENTIFY ALL NODES AND SELECT A REFERENCE NODE
2. IDENTIFY KNOWN NODE VOLTAGES
3. AT EACH NODE WITH UNKNOWN VOLTAGE WRITE A KCL EQUATION (e.g.,SUM OF CURRENT LEAVING =0)
4. REPLACE CURRENTS IN TERMS OF NODE VOLTAGES

AND GET ALGEBRAIC EQUATIONS IN THE NODE VOLTAGES ...

$$@V_a: \quad -I_1 + I_2 + I_3 = 0$$

$$\frac{V_a - V_s}{9k} + \frac{V_a}{6k} + \frac{V_a - V_b}{3k} = 0$$

$$@V_b: \quad -I_3 + I_4 + I_5 = 0$$

$$\frac{V_b - V_a}{3k} + \frac{V_b}{4k} + \frac{V_b - V_c}{9k} = 0$$

$$@V_c: \quad -I_5 + I_6 = 0$$

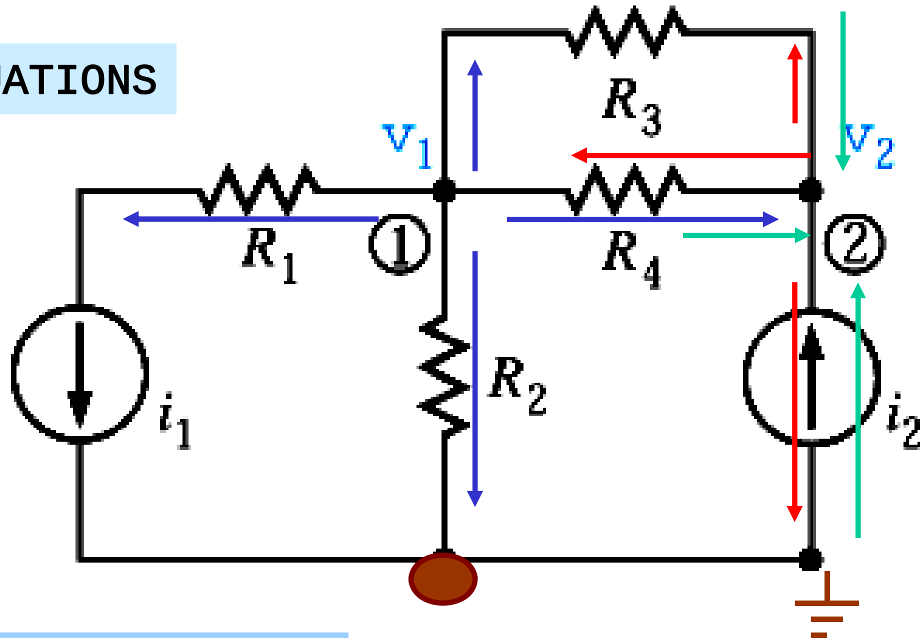
$$\frac{V_c - V_b}{9k} + \frac{V_c}{3k} = 0$$

SHORTCUT: SKIP WRITING THESE EQUATIONS...

AND PRACTICE WRITING THESE DIRECTLY

EXAMPLE

WRITE THE KCL EQUATIONS



@ NODE 1 WE VISUALIZE THE CURRENTS LEAVING AND WRITE THE KCL EQUATION

$$i_1 + \frac{v_1}{R_2} + \frac{v_1 - v_2}{R_3} + \frac{v_1 - v_2}{R_4} = 0$$

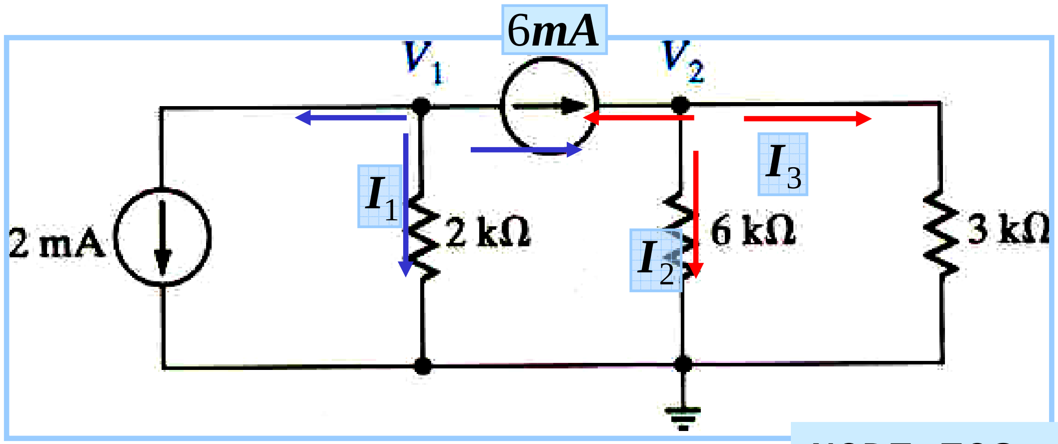
REPEAT THE PROCESS AT NODE 2

$$-i_2 + \frac{v_2 - v_1}{R_4} + \frac{v_2 - v_1}{R_3} = 0$$

OR VISUALIZE CURRENTS GOING INTO NODE

$$i_2 + \frac{v_1 - v_2}{R_3} + \frac{v_1 - v_2}{R_4} = 0$$

Find all the branch currents



Node analysis

$$@V_1: \frac{V_1}{2k} + 2mA + 6mA = 0 \quad \Rightarrow V_1 = -16V$$

$$@V_2: \quad -6mA + \frac{V_2}{6k} + \frac{V_2}{3k} = 0 \Rightarrow \quad V_2 = 12V$$

NODE EQS. BY INSPECTION

$$\frac{1}{2k}V_1 + (0)V_2 = -(2+6)mA$$

$$(0)V_1 + \left(\frac{1}{6k} + \frac{1}{3k}\right)V_2 = 6mA$$

IN MOST CASES THERE ARE SEVERAL DIFFERENT WAYS OF SOLVING A PROBLEM

Once node voltages are known

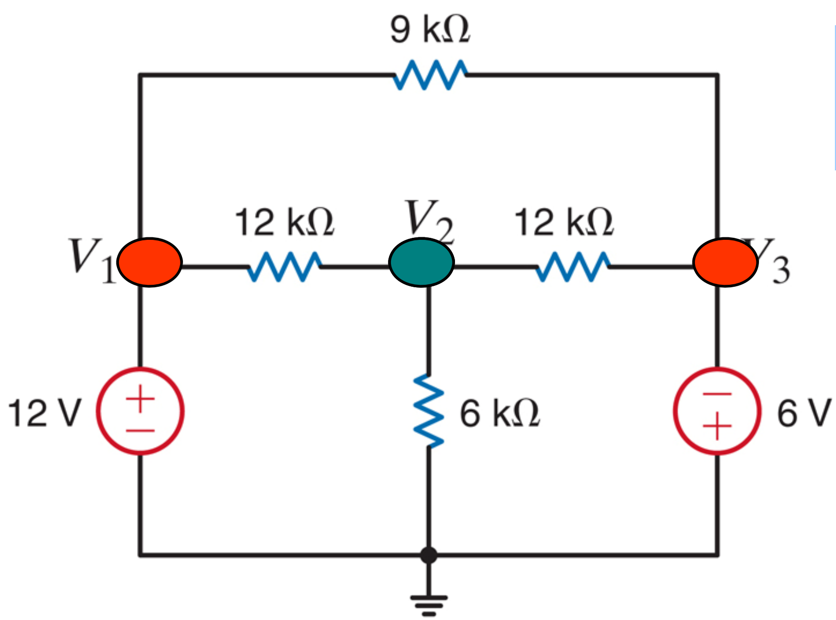
$$I_1 = \frac{V_1}{2k} \qquad I_2 = \frac{V_2}{6k} \qquad I_3 = \frac{V_2}{3k}$$

$$I_1 = -8mA$$

$$I_2 = \frac{3k}{3k+6k}(6mA) = 2mA$$

$$I_3 = \frac{6k}{3k+6k}(6mA) = 4mA$$

CURRENTS COULD BE COMPUTED DIRECTLY USING KCL AND CURRENT DIVIDER!!



3 nodes plus the reference. In principle one needs 3 equations...

…but two nodes are connected to the reference through voltage sources. Hence those node voltages are known!!!

…Only one KCL is necessary

$$\frac{V_2}{6k} + \frac{V_2 - V_3}{12k} + \frac{V_2 - V_1}{12k} = 0$$

$$V_1 = 12[V]$$

$$V_3 = -6[V]$$

THESE ARE THE REMAINING TWO NODE EQUATIONS

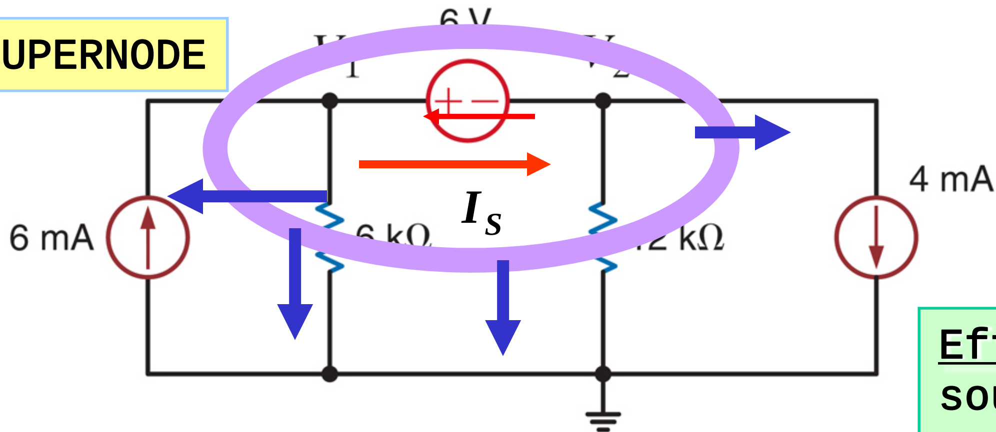
Hint: Each voltage source connected to the reference node saves one node equation

SOLVING THE EQUATIONS

$$2V_2 + (V_2 - V_3) + (V_2 - V_1) = 0$$

$$4V_2 = 6[V] \Rightarrow V_2 = 1.5[V]$$

THE SUPERNODE TECHNIQUE



Conventional analysis requires all currents at a node

@V_1

$$-6mA+\frac{V_1}{6k}+I_S=0$$

@V_2

$$-I_S+4mA+\frac{V_2}{12k}=0$$

2 eqs, 3 unknowns...Panic!!
The current through the source is not related to the voltage of the source

Math solution: add one equation

$$V_1-V_2=6[V]$$

Efficient solution: enclose the source, and all elements in parallel, inside a surface.

Apply KCL to the surface!!!

$$-6mA+\frac{V_1}{6k}+\frac{V_2}{12k}+4mA=0$$

The source current is interior to the surface and is not required

We STILL need one more equation

$$V_1-V_2=6[V]$$

Only 2 eqs in two unknowns!!!

ALGEBRAIC DETAILS

The Equations

(1) $\dfrac{V_1}{6k} + \dfrac{V_2}{12k} - 6mA + 4mA = 0$ *\12k

(2) $V_1 - V_2 = 6[V]$

Solution

1. Eliminate denominators in Eq(1). Multiply by …

$$2V_1 + V_2 = 24[V]$$

$$V_1 - V_2 = 6[V]$$

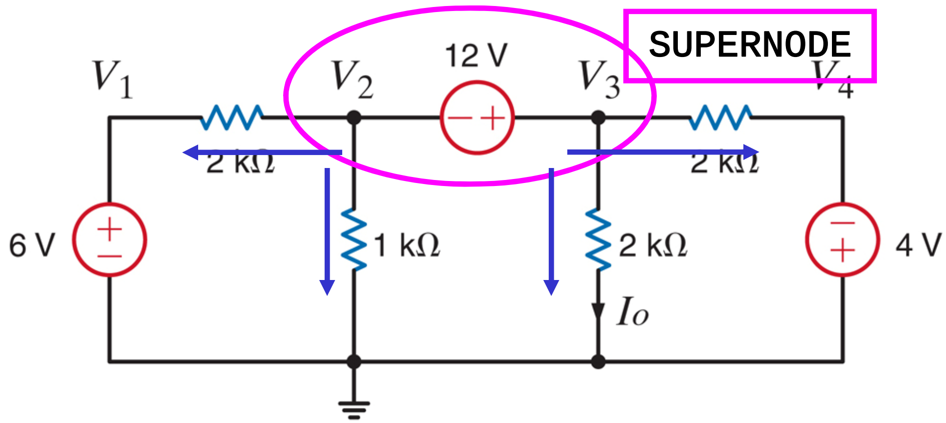
2. Add equations to eliminate V_2

$$3V_1 = 30[V] \Rightarrow V_1 = 10[V]$$

3. Use Eq(2) to compute V_2

$$V_2 = V_1 - 6[V] = 4[V]$$

Find Value of I_O



$V_1 = 6V$

$V_4 = -4V$

SOURCES CONNECTED TO THE REFERENCE

CONSTRAINT EQUATION $V_3 - V_2 = 12V$

KCL @ SUPERNODE

$$\frac{V_2 - 6}{2k} + \frac{V_2}{1k} + \frac{V_3}{2k} + \frac{V_3 - (-4)}{2k} = 0 \quad * / 2k$$

V_2 IS NOT NEEDED FOR I_o

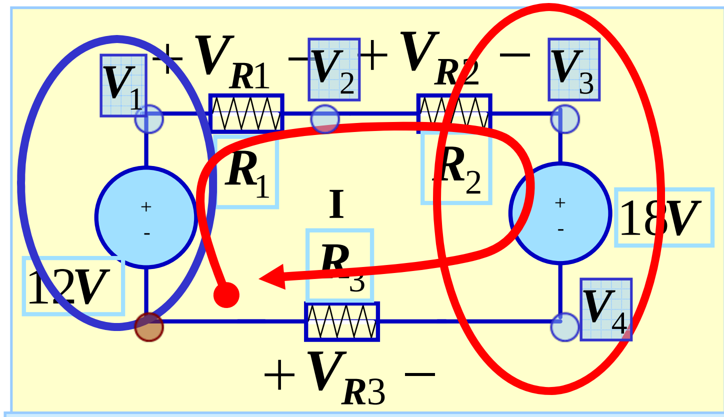
$$3V_2 + 2V_3 = 2V$$

$$-V_2 + V_3 = 12V \quad * / 3 \text{ and add}$$

$$5V_3 = 38V$$

OHM'S LAW $I_o = \frac{V_3}{2k} = 3.8mA$

Apply node analysis to this circuit



There are 4 non reference nodes

There is one super node

There is one node connected to the reference through a voltage source

We need three equations to compute all node voltages

…BUT THERE IS ONLY ONE CURRENT FLOWING THROUGH ALL COMPONENTS AND IF THAT CURRENT IS DETERMINED ALL VOLTAGES CAN BE COMPUTED WITH OHM'S LAW

STRATEGY:
1. Apply KVL
(sum of voltage drops =0)

$$-12[V]+V_{R1}+V_{R2}+18[V]-V_{R3}=0$$

Skip this equation

2. Use Ohm's Law to express voltages in terms of the "loop current."

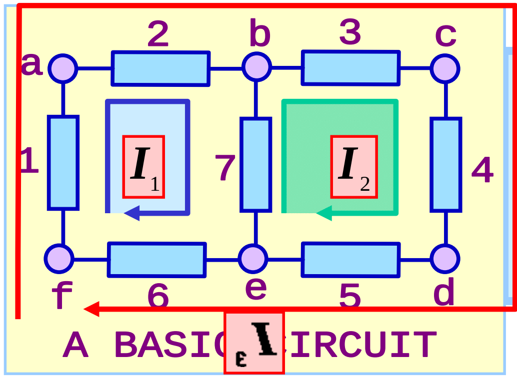
$$-12[V]+R_1I+R_2I+18[V]+R_3I=0$$

Write this one directly

RESULT IS ONE EQUATION IN THE LOOP CURRENT!!!

SHORTCUT

LOOPS, MESHES AND LOOP CURRENTS



EACH COMPONENT IS CHARACTERIZED BY ITS VOLTAGE ACROSS AND ITS CURRENT THROUGH

A LOOP IS A CLOSED PATH THAT DOES NOT GO TWICE OVER ANY NODE.
THIS CIRCUIT HAS THREE LOOPS

fabef ebcde fabcdef

A MESH IS A LOOP THAT DOES NOT ENCLOSE ANY OTHER LOOP.
fabef, ebcde ARE MESHES

A LOOP CURRENT IS A (FICTICIOUS) CURRENT THAT IS ASSUMED TO FLOW AROUND A LOOP

I_1, I_2, I_3 ARE LOOP CURRENTS

A MESH CURRENT IS A LOOP CURRENT ASSOCIATED TO A MESH. I1, I2 ARE MESH CURRENTS

CLAIM: IN A CIRCUIT, THE CURRENT THROUGH ANY COMPONENT CAN BE EXPRESSED IN TERMS OF THE LOOP CURRENTS

EXAMPLES

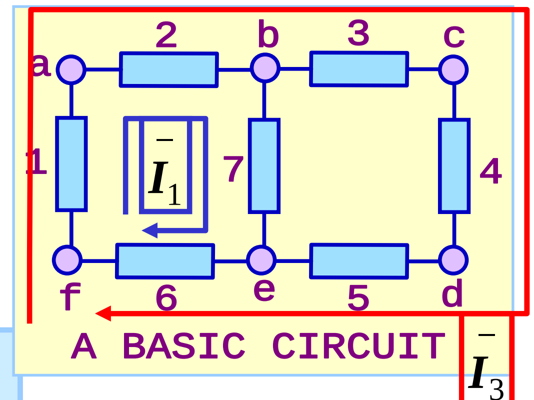
$$I_{af} = -I_1 - I_3$$

$$I_{be} = I_1 - I_2$$

$$I_{bc} = I_2 + I_3$$

THE DIRECTION OF THE LOOP CURRENTS IS SIGNIFICANT

FACT: NOT EVERY LOOP CURRENT IS REQUIRED TO COMPUTE ALL THE CURRENTS THROUGH COMPONENTS



USING TWO LOOP CURRENTS

$$I_{af} = -\bar{I}_1 - \bar{I}_3$$

$$I_{be} = \bar{I}_1$$

$$I_{bc} = \bar{I}_3$$

FOR EVERY CIRCUIT THERE IS A MINIMUM NUMBER OF LOOP CURRENTS THAT ARE NECESSARY TO COMPUTE EVERY CURRENT IN THE CIRCUIT.
SUCH A COLLECTION IS CALLED A MINIMAL SET (OF LOOP CURRENTS).

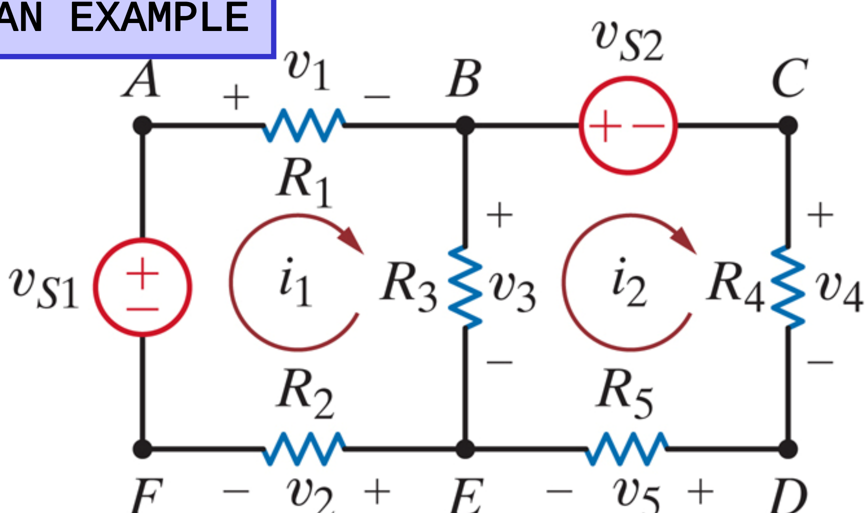
FOR A GIVEN CIRCUIT LET
B NUMBER OF BRANCHES
N NUMBER OF NODES

THE MINIMUM REQUIRED NUMBER OF LOOP CURRENTS IS

$$L = B - (N - 1)$$

MESH CURRENTS ARE ALWAYS INDEPENDENT

AN EXAMPLE



$B = 7$

$N = 6$

$L = 7 - (6 - 1) = 2$

TWO LOOP CURRENTS ARE REQUIRED.
THE CURRENTS SHOWN ARE MESH CURRENTS. HENCE THEY ARE INDEPENDENT AND FORM A MINIMAL SET

DETERMINATION OF LOOP CURRENTS

KVL ON LEFT MESH

$$v_1 + v_3 + v_2 - v_{S1} = 0$$

KVL ON RIGHT MESH

$$v_{S2} + v_4 + v_5 - v_3 = 0$$

USING OHM'S LAW

$$v_1 = i_1 R_1, v_2 = i_1 R_2, v_3 = (i_1 - i_2) R_3$$

$$v_4 = i_2 R_4, v_5 = i_2 R_5$$

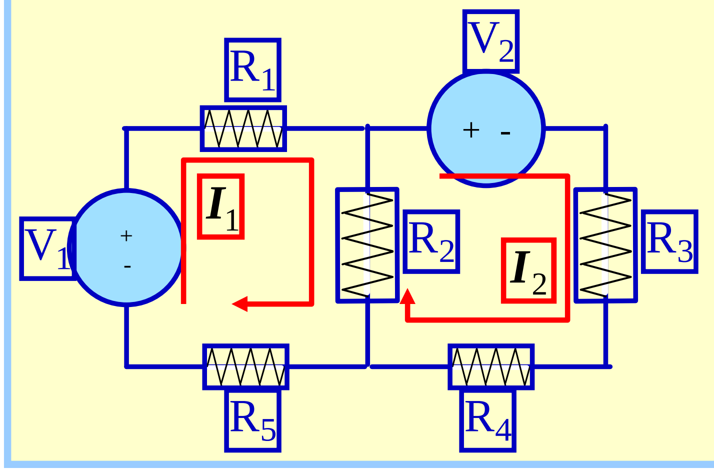
REPLACING AND REARRANGING

$$i_1(R_1 + R_2 + R_3) - i_2(R_3) = v_{S1}$$

$$-i_1(R_3) + i_2(R_3 + R_4 + R_5) = -v_{S2}$$

DEVELOPING A SHORTCUT

WRITE THE MESH EQUATIONS



WHENEVER AN ELEMENT HAS MORE THAN ONE LOOP CURRENT FLOWING THROUGH IT WE COMPUTE NET CURRENT IN THE DIRECTION OF TRAVEL

DRAW THE MESH CURRENTS. ORIENTATION CAN BE ARBITRARY. BUT BY CONVENTION THEY ARE DEFINED CLOCKWISE

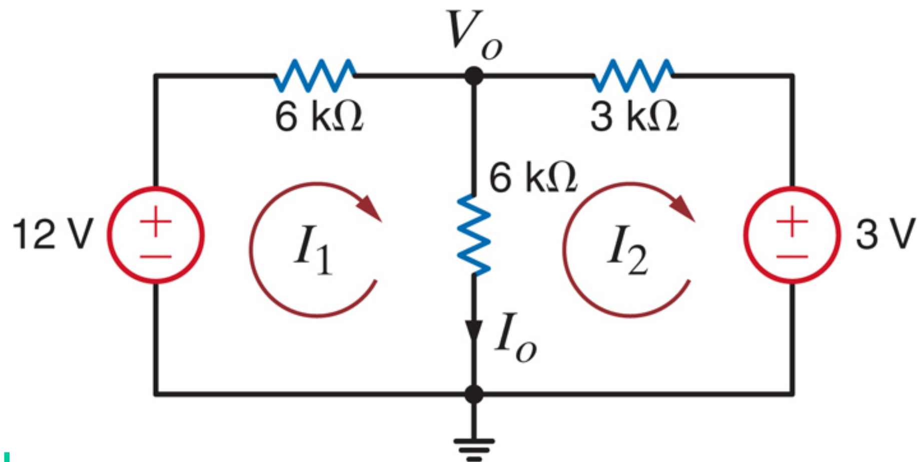
NOW WRITE KVL FOR EACH MESH AND APPLY OHM'S LAW TO EVERY RESISTOR.

AT EACH LOOP FOLLOW THE PASSIVE SIGN CONVENTION USING <u>LOOP CURRENT REFERENCE DIRECTION</u>

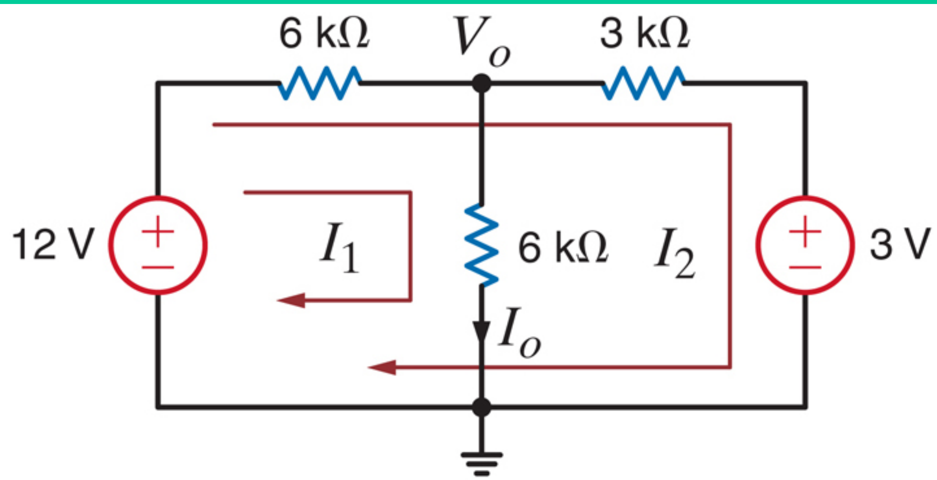
$$-V_1 + I_1R_1 + (I_1 - I_2)R_2 + I_1R_5 = 0$$

$$V_2 + I_2R_3 + I_2R_4 + (I_2 - I_1)R_2 = 0$$

EXAMPLE: FIND Io



AN ALTERNATIVE SELECTION OF LOOP CURRENTS



SHORTCUT: POLARITIES ARE NOT NEEDED. APPLY OHM'S LAW TO EACH ELEMENT AS KVL IS BEING WRITTEN

KVL @ I_1 $-12 + 6kI_1 + 6k(I_1 - I_2) = 0$

KVL @ I_2 $6k(I_2 - I_1) + 3kI_2 + 3 = 0$

REARRANGE

$12kI_1 - 6kI_2 = 12$

$-6kI_1 + 9kI_2 = -3$ $*/2$ and add

$12kI_2 = 6 \Rightarrow I_2 = 0.5mA$

$12kI_1 = 12 + 6kI_2 \Rightarrow I_1 = \frac{5}{4}mA$

EXPRESS VARIABLE OF INTEREST AS FUNCTION OF LOOP CURRENTS $I_O = I_1 - I_2$

KVL @ I_1 $-12 + 6k(I_1 + I_2) + 6kI_1 = 0$

KVL @ I_2 $-12 + 6k(I_1 + I_2) + 3kI_2 + 3 = 0$

NOW $I_O = I_1$

THIS SELECTION IS MORE EFFICIENT

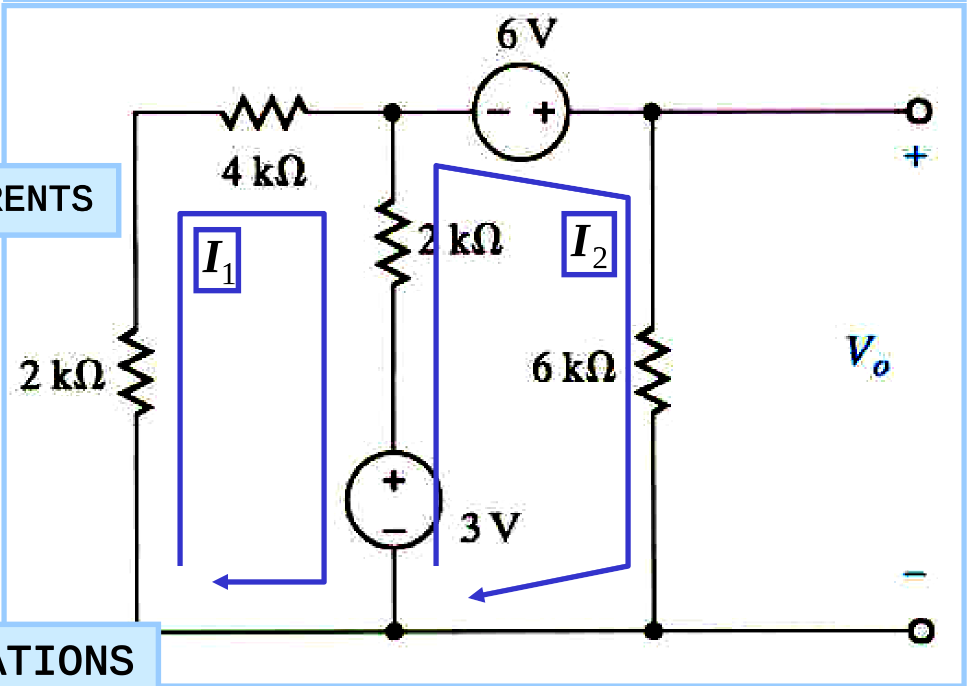
REARRANGE

$12kI_1 + 6kI_2 = 12$ $*/3$

$6kI_1 + 9kI_2 = 9$ $*/2$ and substract

$24kI_1 = 18 \Rightarrow I_1 = \frac{3}{4}mA$

Use mesh equations to find V_o



1. DRAW THE MESH CURRENTS

2. WRITE MESH EQUATIONS

MESH 1 $(2k + 4k + 2k)I_1 - 2kI_2 = -3[V]$

MESH 2 $-2kI_1 + (2k + 6k)I_2 = (6V + 3V)$

DIVIDE BY 1k. GET NUMBERS FOR COEFFICIENTS ON THE LEFT AND mA ON THE RHS

3. SOLVE EQUATIONS

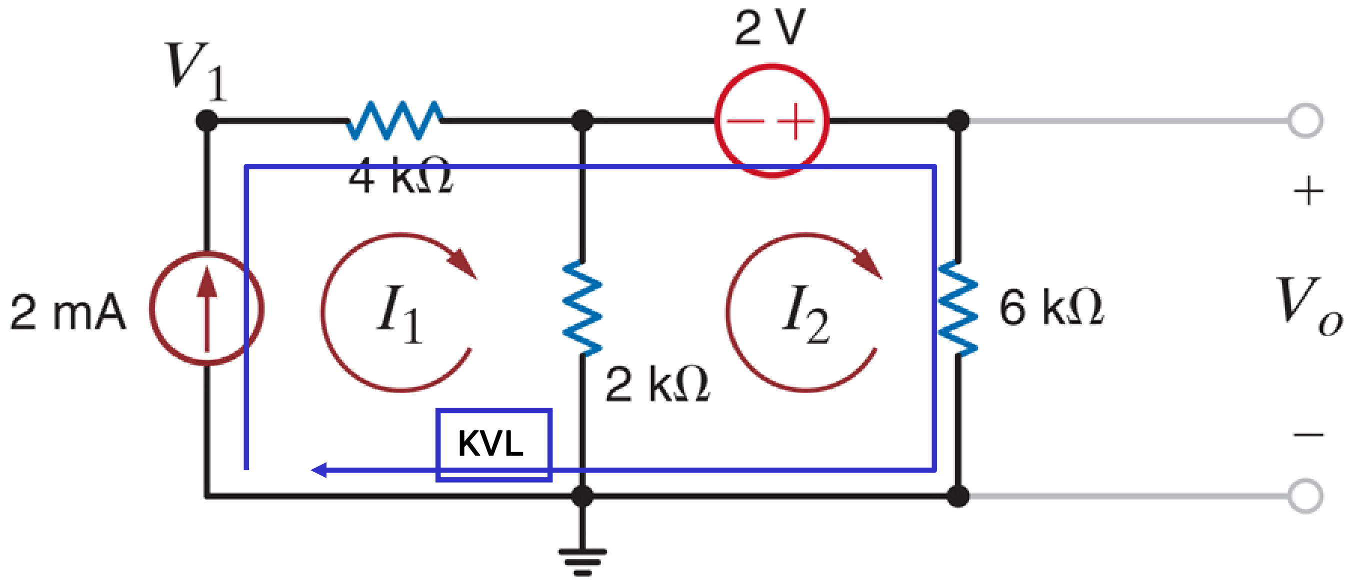
$$8I_1 - 2I_2 = -3[mA]$$

$$-2I_1 + 8I_2 = 9[mA]$$ $*/4$ and add

$$30I_2 = 33[mA]$$

$$V_O = 6kI_2 = \frac{33}{5}[V]$$

find both V_o and V_1 in the circuit



THERE IS NO RELATIONSHIP BETWEEN V1 AND THE SOURCE CURRENT! HOWEVER ...

MESH 1 CURRENT IS CONSTRAINED

MESH 1 EQUATION $I_1 = 2mA$

MESH 2 $2k(I_2 - I_1) - 2 + 6kI_2 = 0$

"BY INSPECTION" $-2kI_1 + 8kI_2 = 2V$

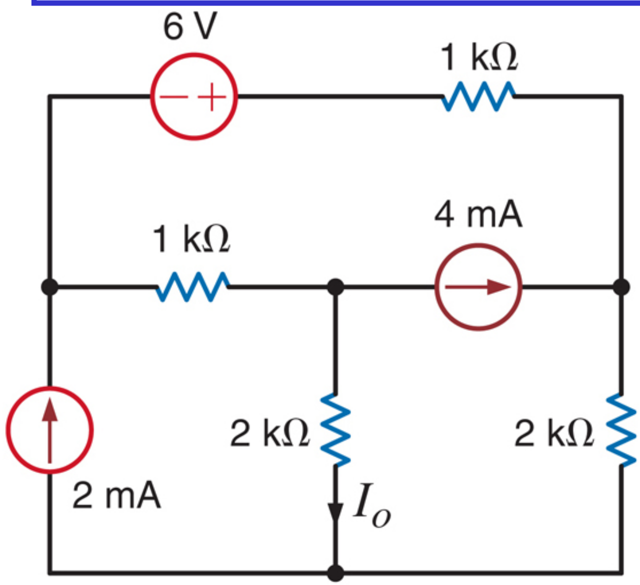
$$I_2 = \frac{2k \times (2mA) + 2V}{8k} = \frac{3}{4}mA \Rightarrow V_O = 6kI_2 = \frac{9}{2}[V]$$

CURRENT SOURCES THAT ARE NOT SHARED BY OTHER MESHES (OR LOOPS) SERVE TO DEFINE A MESH (LOOP) CURRENT AND REDUCE THE NUMBER OF REQUIRED EQUATIONS

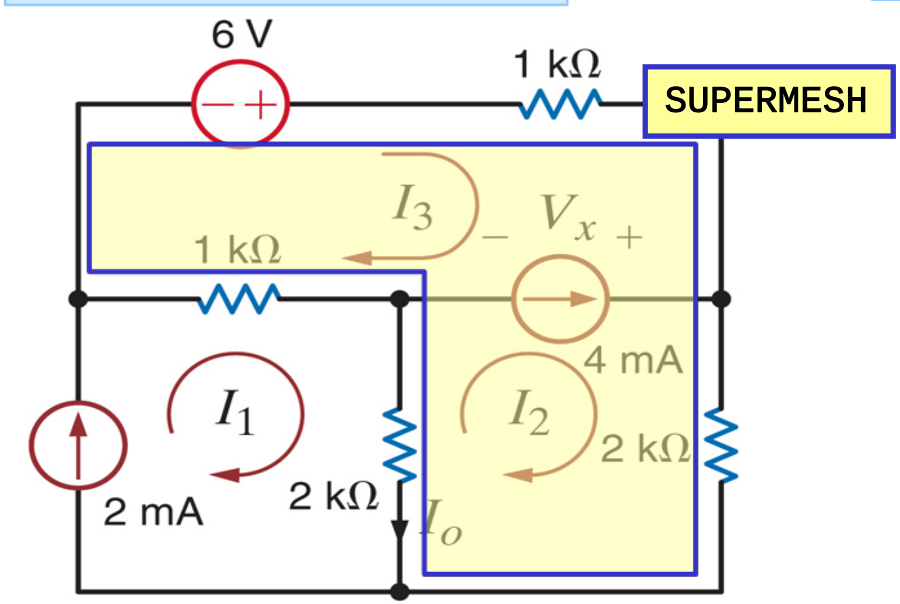
TO OBTAIN V1 APPLY KVL TO ANY CLOSED PATH THAT INCLUDES V1

$$-V_1 + 4kI_1 - 2 + 6kI_2 = 0$$

CURRENT SOURCES SHARED BY LOOPS - THE SUPERMESH APPROACH



1. SELECT MESH CURRENTS



2. WRITE CONSTRAINT EQUATION DUE TO MESH CURRENTS SHARING CURRENT SOURCES

$$I_2 - I_3 = 4mA$$

3. WRITE EQUATIONS FOR THE OTHER MESHES

$$I_1 = 2mA$$

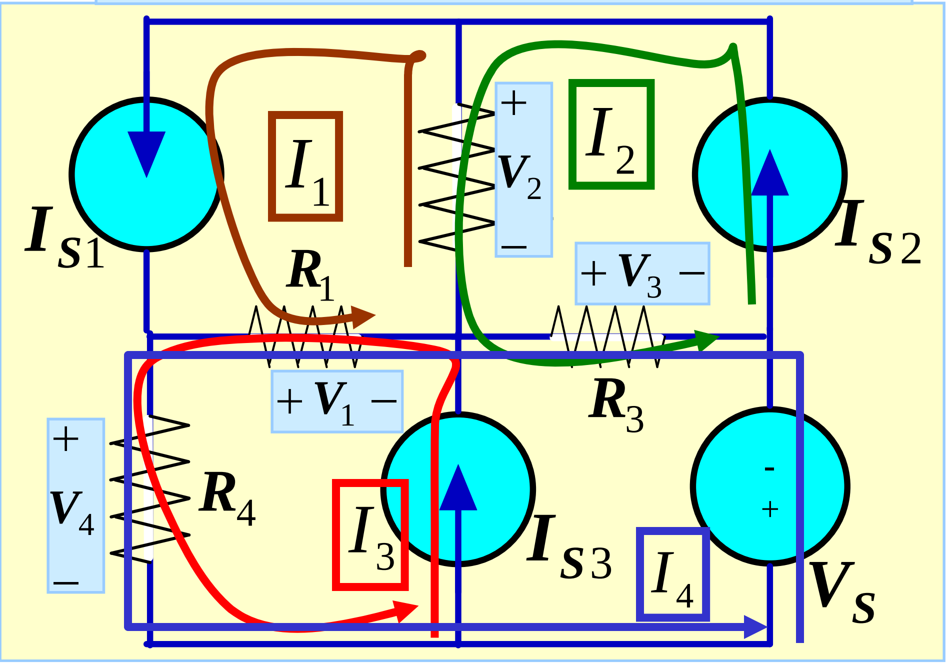
4. DEFINE A SUPERMESH BY (MENTALLY) REMOVING THE SHARED CURRENT SOURCE

5. WRITE KVL FOR THE SUPERMESH

$$-6 + 1kI_3 + 2kI_2 + 2k(I_2 - I_1) + 1k(I_3 - I_1) = 0$$

NOW WE HAVE THREE EQUATIONS IN THREE UNKNOWNS. THE MODEL IS COMPLETE

FIND VOLTAGES ACROSS RESISTORS



Now we need a loop current that does not go over any current source and passes through all unused components.

HINT: IF ALL CURRENT SOURCES ARE REMOVED THERE IS ONLY ONE LOOP LEFT

MESH EQUATIONS FOR LOOPS WITH CURRENT SOURCES

$$I_1 = I_{s1}$$

$$I_2 = I_{S2}$$

$$I_3 = I_{S3}$$

KVL OF REMAINING LOOP

$$V_S + R_3(I_4 - I_2) + R_1(I_4 + I_3 - I_1) + R_4(I_4 + I_3) = 0$$

SOLVE FOR THE CURRENT I4.
USE OHM'S LAW TO COMPUTE REQUIRED VOLTAGES

$$V_1 = R_1(I_1 - I_3 - I_4)$$

$$V_2 = R_2(I_2 - I_1)$$

$$V_3 = R_3(I_2 - I_4)$$

For loop analysis we notice...

Three independent current sources.
Four meshes.
One current source shared by two meshes.

Careful choice of loop currents should make only one loop equation necessary. Three loop currents can be chosen using meshes and not sharing any source.