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SERIES PARALLEL RESISTOR COMBINATIONS

Up to now we have studied circuits that can be analyzed with one application of KVL (single loop) or KCL (single node-pair).

We have also seen that in some situations it is advantageous to combine resistors to simplify the analysis of a circuit.

Now we examine some more complex circuits where we can simplify the analysis using the technique of combining resistors... ...plus the use of Ohm's law.

**SERIES COMBINATION**

\[ R_S = R_1 + R_2 + \cdots + R_N \]

**PARALLEL COMBINATION**

\[ \frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N} \]

\[ G_p = G_1 + G_2 + \cdots + G_N \]
FIRST WE PRACTICE COMBINING RESISTORS

\[ R_{AB} \rightarrow 2 \, k\Omega \quad 2 \, k\Omega \quad 10 \, k\Omega \]

\[ R_{AB} \rightarrow 4 \, k\Omega \quad 6 \, k\Omega \quad 6 \, k\Omega \]

\[ R_{AB} \rightarrow 6 \, k\Omega \quad 9 \, k\Omega \]

\[ (10 \, k, 2 \, k) \text{ SERIES} \]

\[ 12 \, k\Omega = 10 \, k\Omega + (6 \, k\Omega \text{ in parallel with } 3 \, k\Omega) \]

\[ 6k || 12k = 4k \]

\[ 6k = 2k + (6k \text{ in parallel with } 12k) \]
EXAMPLES COMBINATION SERIES-PARALLEL

If the drawing gets confusing...
Redraw the reduced circuit and start again

Resistors are in series if they carry exactly the same current.
Resistors are in parallel if they are connected exactly between the same two nodes.
**EFFECT OF RESISTOR TOLERANCE**

**NOMINAL RESISTOR VALUE:** 2.7k\(\Omega\)

**RESISTOR TOLERANCE:** 10%

**RANGES FOR CURRENT AND POWER?**

**NOMINAL CURRENT:** \(I = \frac{10}{2.7} = 3.704\,mA\)

**NOMINAL POWER:** \(P = \frac{(10)^2}{2.7} = 37.04\,mW\)

**MINIMUM CURRENT:** \(I_{\text{min}} = \frac{10}{1.1 \times 2.7} = 3.367\,mA\)

**MINIMUM POWER (\(V_{\text{min}}\)):** 33.67 mW

**MAXIMUM CURRENT:** \(I_{\text{max}} = \frac{10}{0.9 \times 2.7} = 4.115\,mA\)

**MAXIMUM POWER:** 41.15 mW
CIRCUIT WITH SERIES-PARALLEL RESISTOR COMBINATIONS

THE COMBINATION OF COMPONENTS CAN REDUCE THE COMPLEXITY OF A CIRCUIT AND RENDER IT SUITABLE FOR ANALYSIS USING THE BASIC TOOLS DEVELOPED SO FAR.

COMBINING RESISTORS IN SERIES ELIMINATES ONE NODE FROM THE CIRCUIT.
COMBINING RESISTORS IN PARALLEL ELIMINATES ONE LOOP FROM THE CIRCUIT.

GENERAL STRATEGY:
• REDUCE COMPLEXITY UNTIL THE CIRCUIT BECOMES SIMPLE ENOUGH TO ANALYZE.
• USE DATA FROM SIMPLIFIED CIRCUIT TO COMPUTE DESIRED VARIABLES IN ORIGINAL CIRCUIT - HENCE ONE MUST KEEP TRACK OF ANY RELATIONSHIP BETWEEN VARIABLES.
We wish to find all the currents and voltages labeled in the ladder network shown.

**FIRST:** Reduce it to a single loop circuit.

**SECOND:** “Backtrack” using KVL, KCL, and Ohm's laws.

**KCL:** \( I_1 - I_2 - I_3 = 0 \)

**Ohm’s:** \( I_2 = \frac{V_a}{6k} \)

**Ohm’s:** \( V_b = 3k \cdot I_3 \)

**KCL:** \( I_5 + I_4 - I_3 = 0 \)

**Ohm’s:** \( V_C = 3k \cdot I_5 \)

**Other Options...**

\( I_4 = \frac{\frac{12}{4+12}}{I_3} \)

\( V_b = 4k \cdot I_4 \)

\( I_1 = \frac{12V}{12k} \)

\( V_a = \frac{3}{3+9} \)
Find $V_o$ in the following network:

$$V_o = \frac{1k}{1k + 2k} (3V) = 1V$$

\[\text{VOLTAGE DIVIDER: } V_o = \frac{1k}{1k + 2k} (3V) = 1V\]

Find $I_o$ in the following circuit:

$$I_o = \frac{1k}{1k + 2k} (3A) = 1A$$

\[\text{CURRENT DIVIDER: } I_o = \frac{1k}{1k + 2k} (3A) = 1A\]
$Y - \Delta$ TRANSFORMATIONS

This circuit has no resistor in series or parallel.

If instead of this we could have this, then the circuit would become like this and be amenable to series-parallel transformations.
\[ R_{ab} = R_2 \parallel (R_1 + R_3) \]

\[ \Delta \rightarrow Y \]

\[ Y \rightarrow \Delta \]

\[ R_{ab} = R_a + R_b \]

\[ R_a + R_b = \frac{R_2(R_1 + R_3)}{R_1 + R_2 + R_3} \]

\[ R_b + R_c = \frac{R_3(R_1 + R_2)}{R_1 + R_2 + R_3} \]

\[ R_c + R_a = \frac{R_1(R_2 + R_3)}{R_1 + R_2 + R_3} \]

\[ R_a = \frac{R_1 R_2}{R_1 + R_2 + R_3} \]

\[ R_b = \frac{R_2 R_3}{R_1 + R_2 + R_3} \]

\[ R_c = \frac{R_3 R_1}{R_1 + R_2 + R_3} \]

\[ \Delta \rightarrow Y \]

SUBTRACT THE FIRST TWO THEN ADD TO THE THIRD TO GET Ra

REPLACE IN THE THIRD AND SOLVE FOR R1
LEARNING EXAMPLE: APPLICATION OF WYE-DELTA TRANSFORMATION

Compute $I_S$

**DELTA CONNECTION**

$$\begin{align*}
R_a &= \frac{R_1 R_2}{R_1 + R_2 + R_3} \\
R_b &= \frac{R_2 R_3}{R_1 + R_2 + R_3} \\
R_c &= \frac{R_3 R_1}{R_1 + R_2 + R_3}
\end{align*}$$

$A \rightarrow Y$

$$R_{EQ} = 6k + (3k + 9k) || (2k + 4k) = 10k$$

$$I_S = \frac{12V}{10k} = 1.2mA$$

One could also use a WYE - DELTA transformation ...