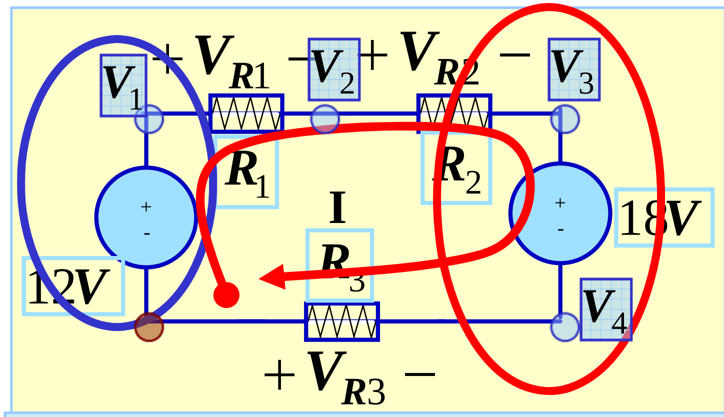
LOOP ANALYSIS

The second systematic technique to determine all currents and voltages in a circuit

IT IS DUAL TO NODE ANALYSIS - IT FIRST DETERMINES ALL CURRENTS IN A CIRCUIT AND THEN IT USES OHM'S LAW TO COMPUTE NECESSARY VOLTAGES

THERE ARE SITUATION WHERE NODE ANALYSIS IS NOT AN EFFICIENT TECHNIQUE AND WHERE THE NUMBER OF EQUATIONS REQUIRED BY THIS NEW METHOD IS SIGNIFICANTLY SMALLER

Apply node analysis to this circuit



There are 4 non reference nodes

There is one super node

There is one node connected to the reference through a voltage source

We need three equations to compute all node voltages

…BUT THERE IS ONLY ONE CURRENT FLOWING THROUGH ALL COMPONENTS AND IF THAT CURRENT IS DETERMINED ALL VOLTAGES CAN BE COMPUTED WITH OHM'S LAW

STRATEGY:
1. Apply KVL
(sum of voltage drops =0)

$$-12[V]+V_{R1}+V_{R2}+18[V]-V_{R3}=0$$

Skip this equation

2. Use Ohm's Law to express voltages in terms of the "loop current."

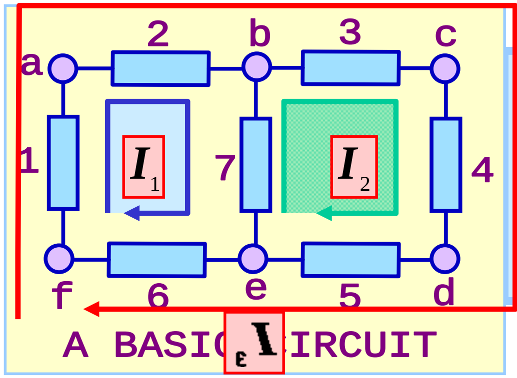
$$-12[V]+R_1I+R_2I+18[V]+R_3I=0$$

Write this one directly

RESULT IS ONE EQUATION IN THE LOOP CURRENT!!!

SHORTCUT

LOOPS, MESHES AND LOOP CURRENTS



EACH COMPONENT IS CHARACTERIZED BY ITS VOLTAGE ACROSS AND ITS CURRENT THROUGH

A LOOP IS A CLOSED PATH THAT DOES NOT GO TWICE OVER ANY NODE.
THIS CIRCUIT HAS THREE LOOPS

fabef ebcde fabcdef

A MESH IS A LOOP THAT DOES NOT ENCLOSE ANY OTHER LOOP.
fabef, ebcde ARE MESHES

A LOOP CURRENT IS A (FICTICIOUS) CURRENT THAT IS ASSUMED TO FLOW AROUND A LOOP

I_1, I_2, I_3 ARE LOOP CURRENTS

A MESH CURRENT IS A LOOP CURRENT ASSOCIATED TO A MESH. I1, I2 ARE MESH CURRENTS

CLAIM: IN A CIRCUIT, THE CURRENT THROUGH ANY COMPONENT CAN BE EXPRESSED IN TERMS OF THE LOOP CURRENTS

EXAMPLES

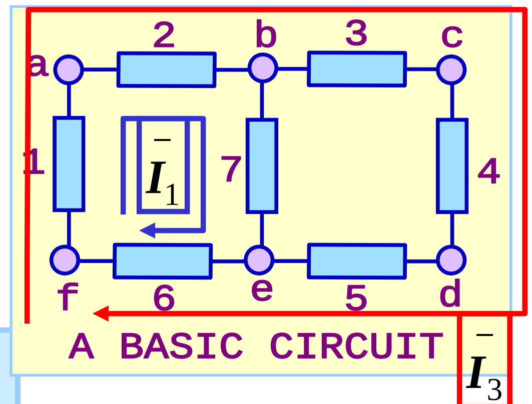
$$I_{af} = -I_1 - I_3$$

$$I_{be} = I_1 - I_2$$

$$I_{bc} = I_2 + I_3$$

THE DIRECTION OF THE LOOP CURRENTS IS SIGNIFICANT

FACT: NOT EVERY LOOP CURRENT IS REQUIRED TO COMPUTE ALL THE CURRENTS THROUGH COMPONENTS



USING TWO LOOP CURRENTS

$$I_{af} = -\bar{I}_1 - \bar{I}_3$$

$$I_{be} = \bar{I}_1$$

$$I_{bc} = \bar{I}_3$$

FOR EVERY CIRCUIT THERE IS A MINIMUM NUMBER OF LOOP CURRENTS THAT ARE NECESSARY TO COMPUTE EVERY CURRENT IN THE CIRCUIT.
SUCH A COLLECTION IS CALLED A MINIMAL SET (OF LOOP CURRENTS).

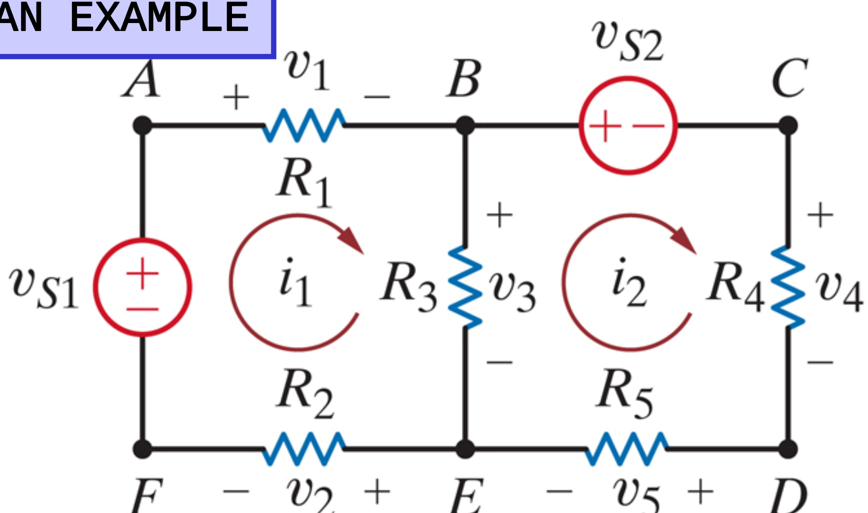
FOR A GIVEN CIRCUIT LET
B NUMBER OF BRANCHES
N NUMBER OF NODES

THE MINIMUM REQUIRED NUMBER OF LOOP CURRENTS IS

$$L = B - (N - 1)$$

MESH CURRENTS ARE ALWAYS INDEPENDENT

AN EXAMPLE



$B = 7$

$N = 6$

$L = 7 - (6 - 1) = 2$

TWO LOOP CURRENTS ARE REQUIRED.
THE CURRENTS SHOWN ARE MESH CURRENTS. HENCE THEY ARE INDEPENDENT AND FORM A MINIMAL SET

DETERMINATION OF LOOP CURRENTS

KVL ON LEFT MESH

$$-v_1 + v_3 + v_2 - v_{S1} = 0$$

KVL ON RIGHT MESH

$$v_{S2} + v_4 + v_5 - v_3 = 0$$

USING OHM'S LAW

$$v_1 = i_1 R_1, v_2 = i_1 R_2, v_3 = (i_1 - i_2) R_3$$

$$v_4 = i_2 R_4, v_5 = i_2 R_5$$

REPLACING AND REARRANGING

$$i_1(R_1 + R_2 + R_3) - i_2(R_3) = v_{S1}$$

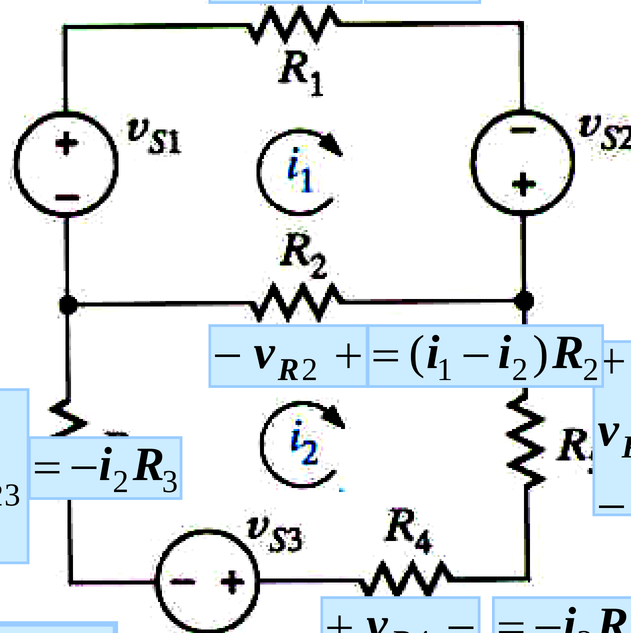
$$-i_1(R_3) + i_2(R_3 + R_4 + R_5) = -v_{S2}$$

WRITE THE MESH EQUATIONS

BOOKKEEPING
BRANCHES = 8
NODES = 7

LOOP CURRENTS NEEDED = 2

AND WE ARE TOLD TO USE MESH CURRENTS! THIS DEFINES THE LOOP CURRENTS TO BE USED



$+\ v_{R1}\ - = i_1 R_1$

$-\ v_{R2}\ + = (i_1 - i_2) R_2$

$+\ v_{R3}\ - = -i_2 R_3$

$+\ v_{R5}\ - = i_2 R_5$

$+\ v_{R4}\ - = -i_2 R_4$

IDENTIFY ALL VOLTAGE DROPS

WRITE KVL ON EACH MESH

TOP MESH: $-v_{S1} + v_{R1} - v_{S2} + v_{R2} = 0$

BOTTOM: $-v_{R2} + v_{R5} - v_{R4} + v_{S3} - v_{R3} = 0$

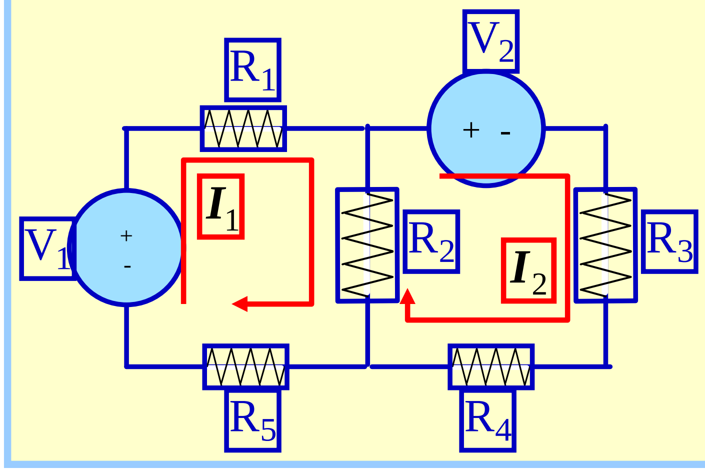
USE OHM'S LAW

$$-v_{S1} + i_1 R_1 - v_{S2} + (i_1 - i_2) R_2 = 0$$

$$i_2 R_3 + (i_2 - i_1) R_2 + i_2 R_5 + i_2 R_4 + V_{S3} = 0$$

DEVELOPING A SHORTCUT

WRITE THE MESH EQUATIONS



WHENEVER AN ELEMENT HAS MORE THAN ONE LOOP CURRENT FLOWING THROUGH IT WE COMPUTE NET CURRENT IN THE DIRECTION OF TRAVEL

DRAW THE MESH CURRENTS. ORIENTATION CAN BE ARBITRARY. BUT BY CONVENTION THEY ARE DEFINED CLOCKWISE

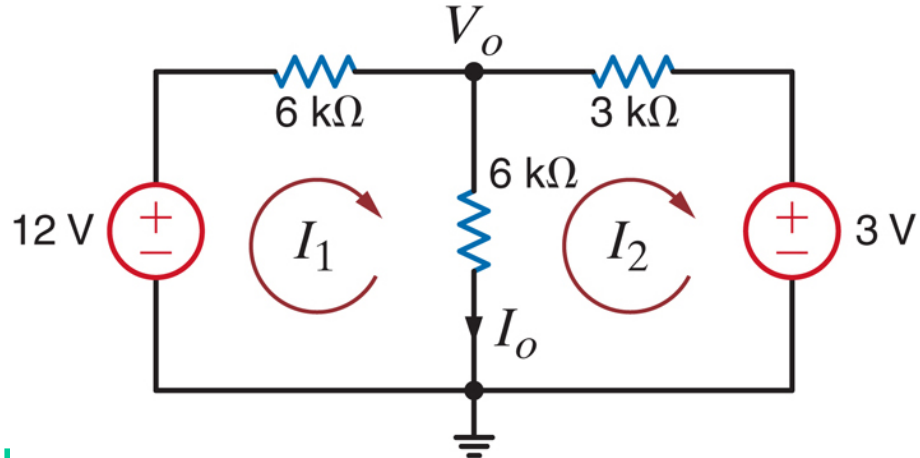
NOW WRITE KVL FOR EACH MESH AND APPLY OHM'S LAW TO EVERY RESISTOR.

AT EACH LOOP FOLLOW THE PASSIVE SIGN CONVENTION USING LOOP CURRENT REFERENCE DIRECTION

$$-V_1 + I_1R_1 + (I_1 - I_2)R_2 + I_1R_5 = 0$$

$$V_2 + I_2R_3 + I_2R_4 + (I_2 - I_1)R_2 = 0$$

EXAMPLE: FIND Io USING LOOP ANALYSII



SHORTCUT: POLARITIES ARE NOT NEEDED. APPLY OHM'S LAW TO EACH ELEMENT AS KVL IS BEING WRITTEN

KVL @ I_1: $-12 + 6kI_1 + 6k(I_1 - I_2) = 0$

KVL @ I_2: $6k(I_2 - I_1) + 3kI_2 + 3 = 0$

REARRANGE

$12kI_1 - 6kI_2 = 12$

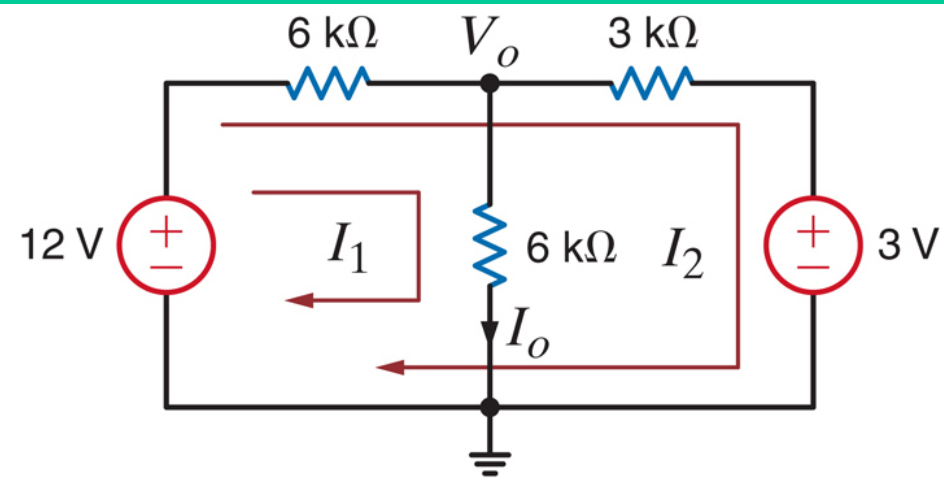
$-6kI_1 + 9kI_2 = -3$ */2 and add

$12kI_2 = 6 \Rightarrow I_2 = 0.5mA$

$12kI_1 = 12 + 6kI_2 \Rightarrow I_1 = \frac{5}{4}mA$

EXPRESS VARIABLE OF INTEREST AS FUNCTION OF LOOP CURRENTS $I_O = I_1 - I_2$

AN ALTERNATIVE SELECTION OF LOOP CURRENTS



KVL @ I_1: $-12 + 6k(I_1 + I_2) + 6kI_1 = 0$

KVL @ I_2: $-12 + 6k(I_1 + I_2) + 3kI_2 + 3 = 0$

NOW $I_O = I_1$

THIS SELECTION IS MORE EFFICIENT

REARRANGE

$12kI_1 + 6kI_2 = 12$ */3

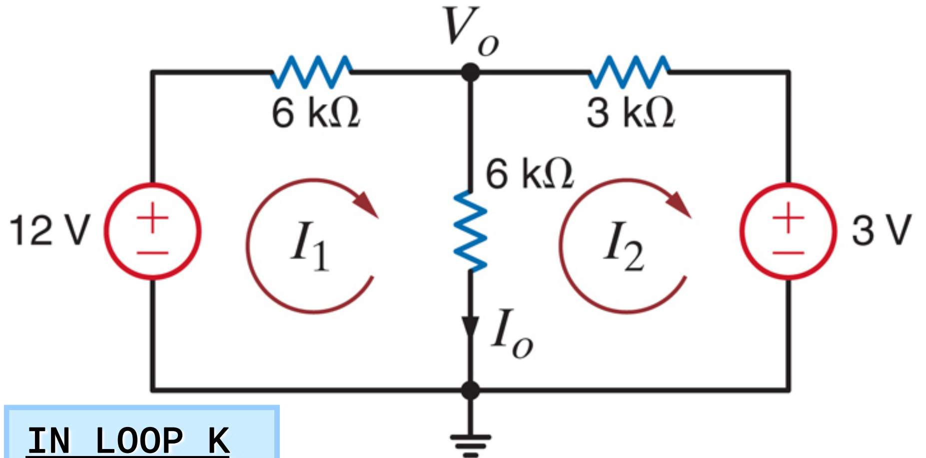
$6kI_1 + 9kI_2 = 9$ */2 and substract

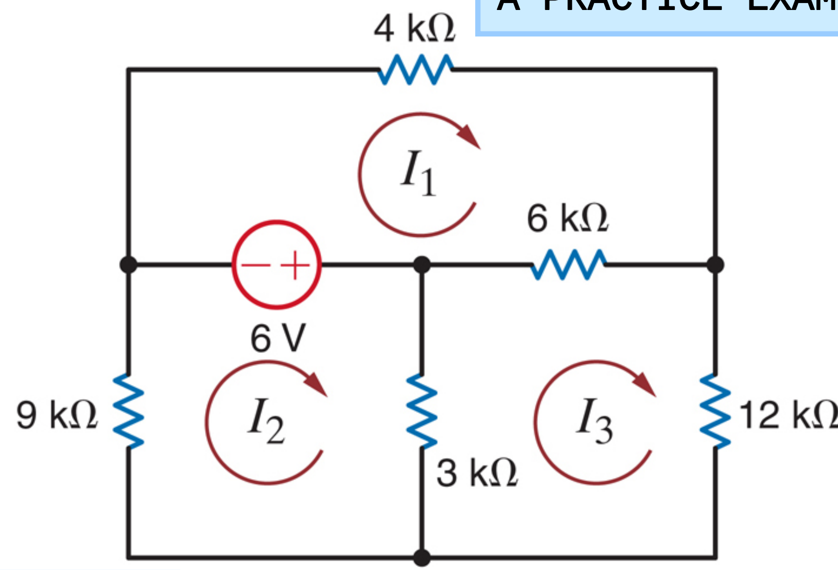
$24kI_1 = 18 \Rightarrow I_1 = \frac{3}{4}mA$

IF THE CIRCUIT CONTAINS ONLY INDEPENDENT SOURCE THE MESH EQUATIONS CAN BE WRITTEN "BY INSPECTION"

MUST HAVE ALL MESH CURRENTS WITH THE SAME ORIENTATION

A PRACTICE EXAMPLE





IN LOOP K

THE COEFFICENT OF Ik IS THE SUM OF RESISTANCES AROUND THE LOOP.

THE RIGHT HAND SIDE IS THE ALGEBRAIC SUM OF VOLTAGE SOURCES AROUND THE LOOP (VOLTAGE RISES - VOLTAGE DROPS)

THE COEFFICIENT OF Ij IS THE SUM OF RESISTANCES COMMON TO BOTH k AND j AND WITH A NEGATIVE SIGN.

LOOP 1 $12kI_1 - 6kI_2 = 12$

LOOP 2 $-6kI_1 + 9kI_2 = -3$

LOOP 1
coefficient of $I_1 = 4k + 6k$
coefficient of $I_2 = 0$
coefficient of $I_3 = -6k$
RHS $= -6[V]$

$$(4k + 6k)I_1 - (0)I_2 - (6k)I_3 = -6$$

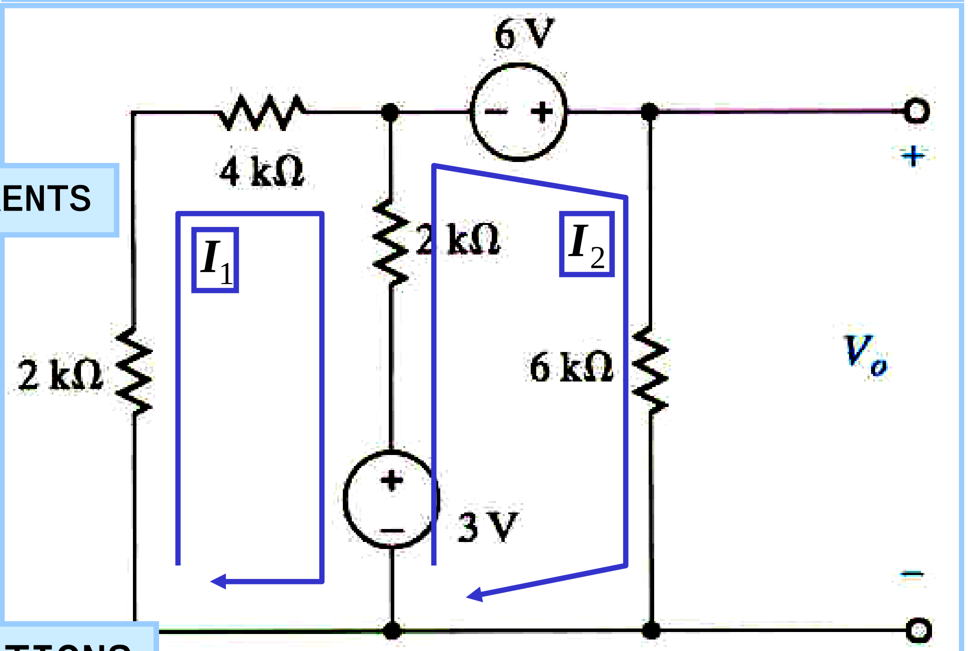
LOOP 2
coefficient of $I_1 = 0$
coefficient of $I_2 = 9k + 3k$
coefficient of $I_3 = -3k$
RHS $= 6[V]$

$$-(0)I_1 + (9k + 3k)I_2 - (3k)I_3 = 6$$

Loop 3 $-(6k)I_1 - (3k)I_2 + (3k + 6k + 12k)I_3 = 0$

LEARNING EXTENSION

Use mesh equations to find V_o



1. DRAW THE MESH CURRENTS

2. WRITE MESH EQUATIONS

MESH 1 $(2k + 4k + 2k)I_1 - 2kI_2 = -3[V]$

MESH 2 $-2kI_1 + (2k + 6k)I_2 = (6V + 3V)$

DIVIDE BY 1k. GET NUMBERS FOR COEFFICIENTS ON THE LEFT AND mA ON THE RHS

3. SOLVE EQUATIONS

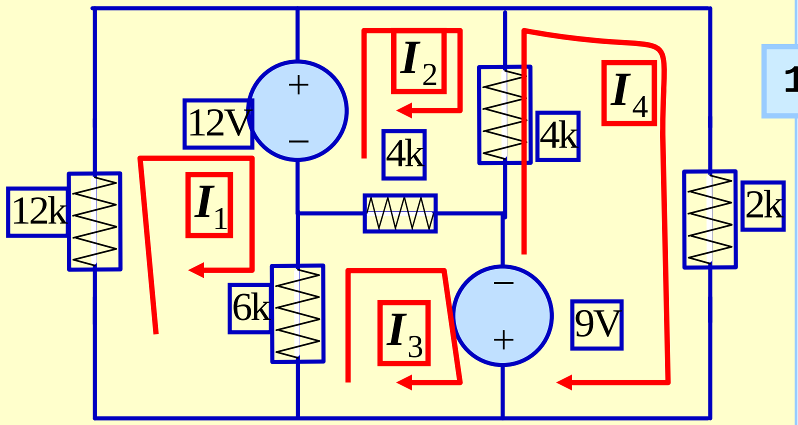
$$8I_1 - 2I_2 = -3[mA]$$

$$-2I_1 + 8I_2 = 9[mA]$$ $*/4$ and add

$$30I_2 = 33[mA]$$

$$V_O = 6kI_2 = \frac{33}{5}[V]$$

WRITE THE MESH EQUATIONS



1. DRAW MESH CURRENTS

BOOKKEEPING: B = 7, N = 4

2. WRITE MESH EQUATIONS. USE KVL

MESH 1: $12kI_1 + 12V + 6k(I_1 - I_3) = 0$

MESH 2: $-12V + 4k(I_2 - I_4) + 4k(I_2 - I_3) = 0$

MESH 3: $-9V + 6k(I_3 - I_1) + 4k(I_3 - I_2) = 0$

MESH 4: $9V + 4k(I_4 - I_2) + 2kI_4 = 0$

EQUATIONS BY INSPECTION

$18kI_1 - 6kI_3 = -12V$

$8kI_2 - 4kI_3 - 4kI_4 = 12V$

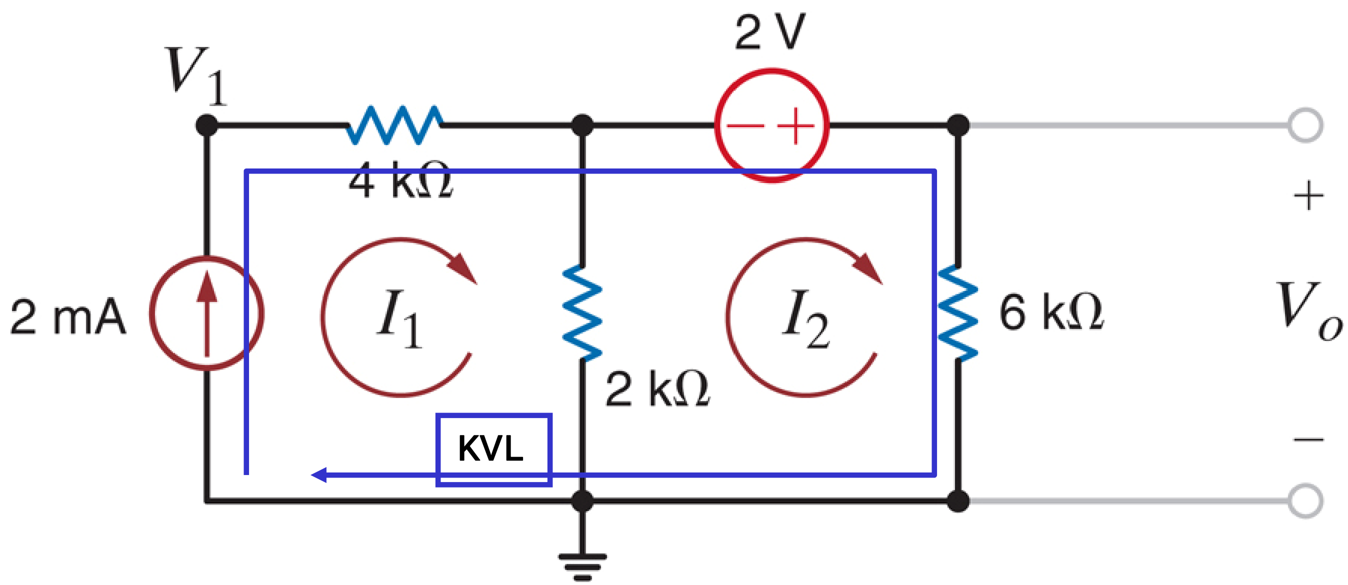
$-6kI_1 - 4kI_2 + 10kI_3 = 9V$

$-4kI_2 + 6kI_4 = -9V$

CHOOSE YOUR FAVORITE TECHNIQUE TO SOLVE THE SYSTEM OF EQUATIONS

CIRCUITS WITH INDEPENDENT CURRENT SOURCES

find both V_o and V_1 in the circuit



THERE IS NO RELATIONSHIP BETWEEN V1 AND THE SOURCE CURRENT! HOWEVER ...

MESH 1 CURRENT IS CONSTRAINED

MESH 1 EQUATION $I_1 = 2mA$

MESH 2 $2\text{k}(I_2 - I_1) - 2 + 6\text{k}I_2 = 0$

"BY INSPECTION" $-2kI_1 + 8kI_2 = 2V$

$$I_2 = \frac{2k \times (2mA) + 2V}{8k} = \frac{3}{4} mA \Rightarrow V_O = 6kI_2 = \frac{9}{2}[V]$$

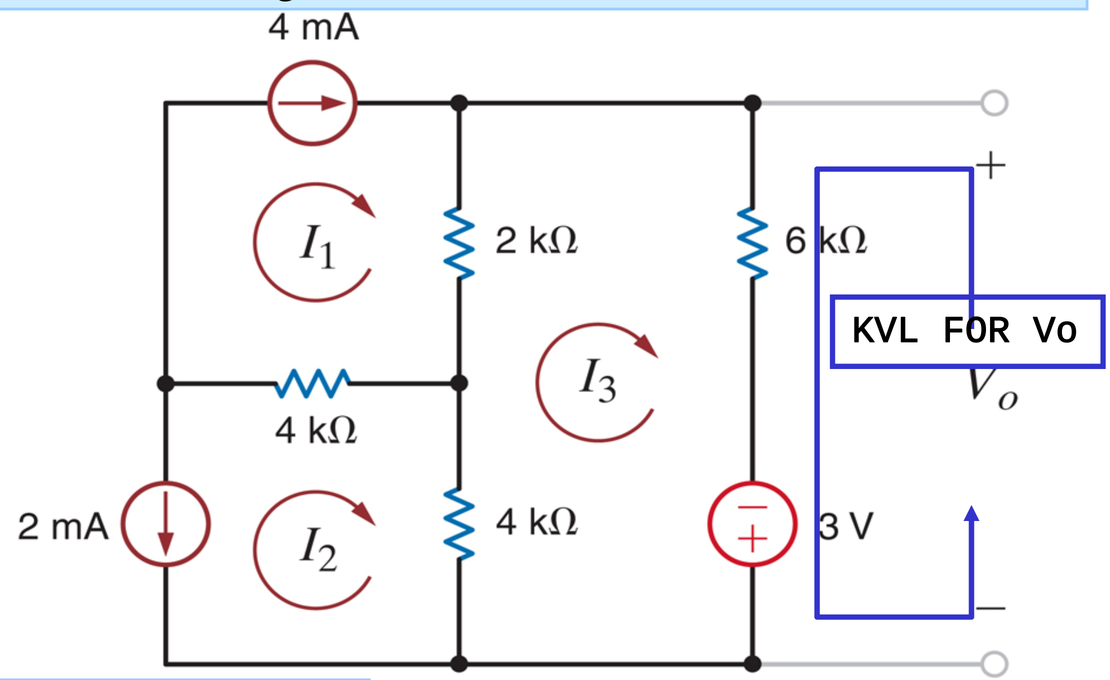
CURRENT SOURCES THAT ARE NOT SHARED BY OTHER MESHES (OR LOOPS) SERVE TO DEFINE A MESH (LOOP) CURRENT AND REDUCE THE NUMBER OF REQUIRED EQUATIONS

TO OBTAIN V1 APPLY KVL TO ANY CLOSED PATH THAT INCLUDES V1

$$-V_1 + 4\text{k}I_1 - 2 + 6\text{k}I_2 = 0$$

EXAMPLE

COMPUTE V_O USING MESH ANALYSIS



TWO MESH CURRENTS ARE DEFINED BY CURRENT SOURCES

$I_1 = 4mA \qquad I_2 = -2mA$

MESH 3

$$4\text{k}(I_3 - I_2) + 2\text{k}(I_3 - I_1) + 6\text{k}I_3 - 3 = 0$$

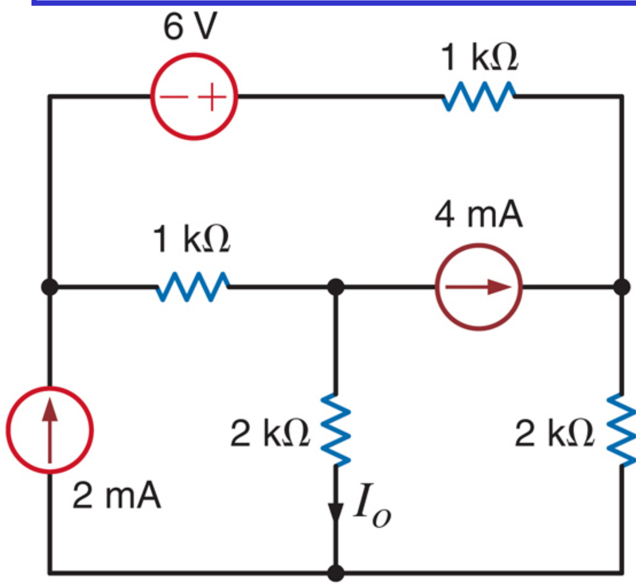
"BY INSPECTION" $-2kI_1 - 4kI_2 + 12kI_3 = 3V$

$$I_3 = \frac{3V + 2k(4mA) + 4k(-2mA)}{12k} = \frac{1}{4}mA$$

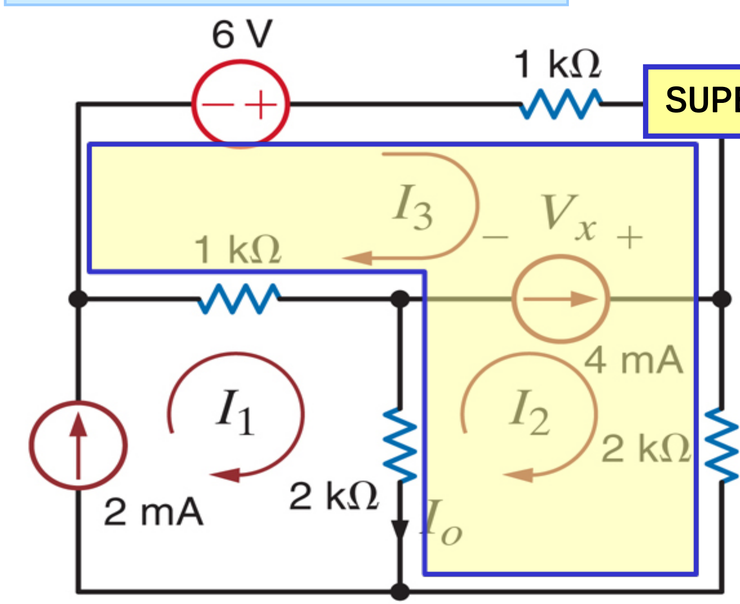
USE KVL TO COMPUTE Vo

$$V_o = 6\text{k}I_3 - 3 = \frac{-3}{2}\text{V}$$

CURRENT SOURCES SHARED BY LOOPS - THE SUPERMESH APPROACH



1. SELECT MESH CURRENTS



2. WRITE CONSTRAINT EQUATION DUE TO MESH CURRENTS SHARING CURRENT SOURCES

$$I_2 - I_3 = 4mA$$

3. WRITE EQUATIONS FOR THE OTHER MESHES

$$I_1 = 2mA$$

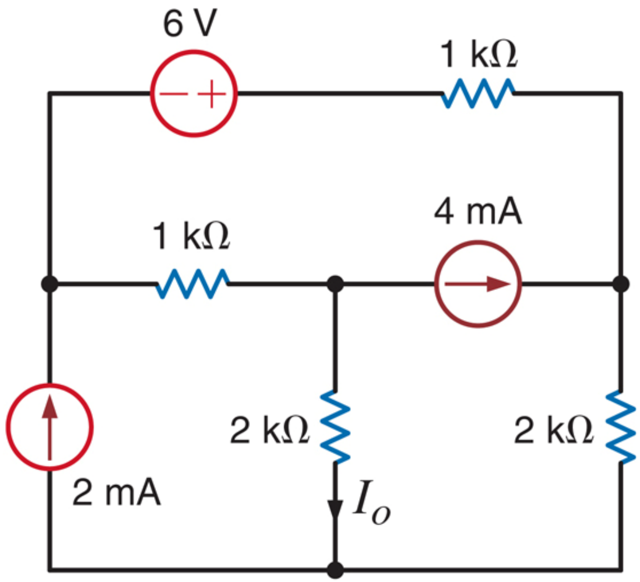
4. DEFINE A <u>SUPERMESH</u> BY (MENTALLY) REMOVING THE SHARED CURRENT SOURCE

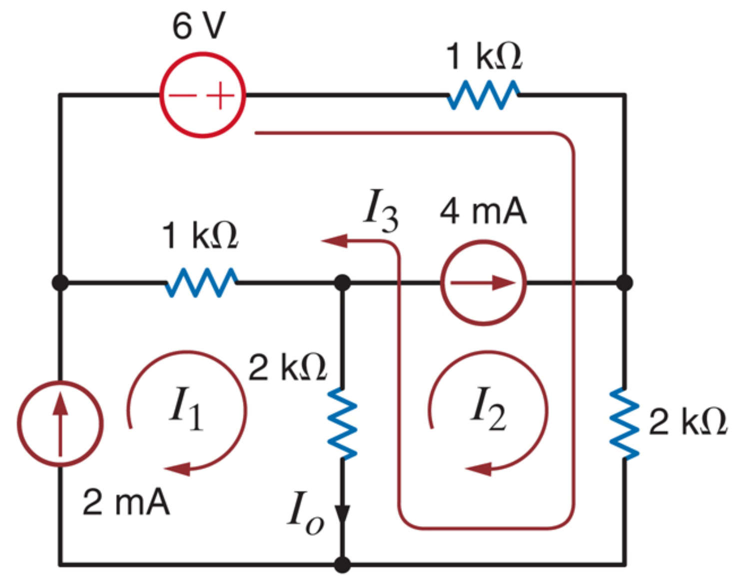
5. WRITE KVL FOR THE SUPERMESH

$$-6 + 1kI_3 + 2kI_2 + 2k(I_2 - I_1) + 1k(I_3 - I_1) = 0$$

NOW WE HAVE THREE EQUATIONS IN THREE UNKNOWNS. THE MODEL IS COMPLETE

CURRENT SOURCES SHARED BY MESHES - THE GENERAL LOOP APPROACH





THE STRATEGY IS TO DEFINE LOOP CURRENTS THAT DO NOT SHARE CURRENT SOURCES -

EVEN IF IT MEANS ABANDONING MESHES

THE LOOP EQUATIONS FOR THE LOOPS WITH CURRENT SOURCES ARE

$$I_1 = 2mA$$

$$I_2 = 4mA$$

FOR CONVENIENCE START USING MESH CURRENTS UNTIL REACHING A SHARED SOURCE. AT THAT POINT DEFINE A NEW LOOP.

THE LOOP EQUATION FOR THE THIRD LOOP IS

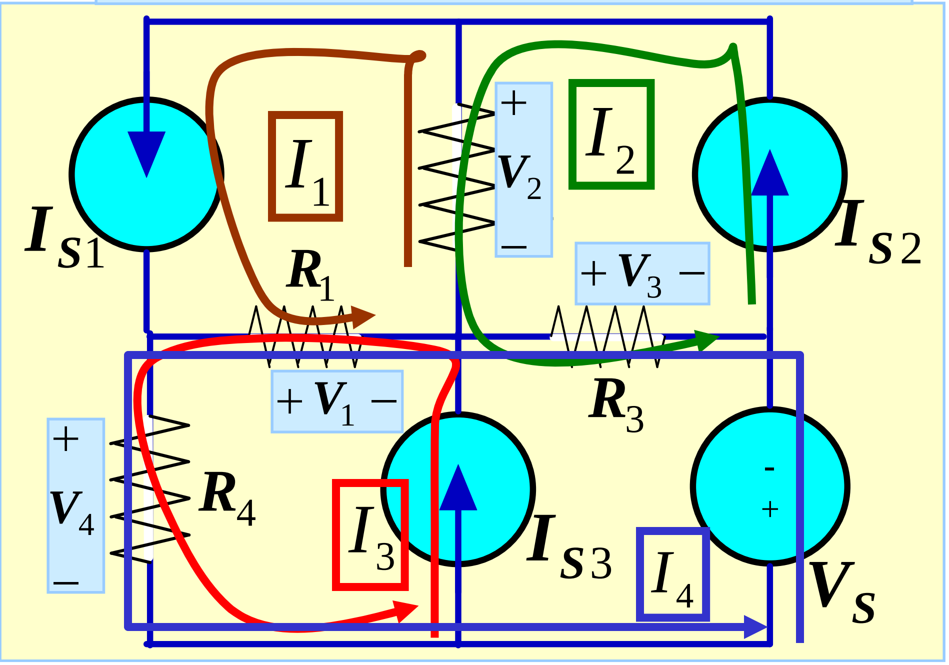
$$-6[V] + 1kI_3 + 2k(I_3 + I_2) + 2k(I_3 + I_2 - I_1) + 1k(I_3 - I_1) = 0$$

IN ORDER TO GUARANTEE THAT IF GIVES AN INDEPENDENT EQUATION ONE MUST MAKE SURE THAT THE LOOP INCLUDES COMPONENTS THAT ARE NOT PART OF PREVIOUSLY DEFINED LOOPS

A POSSIBLE STRATEGY IS TO CREATE A LOOP BY OPENING THE CURRENT SOURCE

THE MESH CURRENTS OBTAINED WITH THIS METHOD ARE DIFFERENT FROM THE ONES OBTAINED WITH A SUPERMESH. EVEN FOR THOSE DEFINED USING MESHES.

FIND VOLTAGES ACROSS RESISTORS



Now we need a loop current that does not go over any current source and passes through all unused components.

HINT: IF ALL CURRENT SOURCES ARE REMOVED THERE IS ONLY ONE LOOP LEFT

MESH EQUATIONS FOR LOOPS WITH CURRENT SOURCES

$$I_1 = I_{s1}$$

$$I_2 = I_{S2}$$

$$I_3 = I_{S3}$$

KVL OF REMAINING LOOP

$$V_S + R_3(I_4 - I_2) + R_1(I_4 + I_3 - I_1) + R_4(I_4 + I_3) = 0$$

For loop analysis we notice...

Three independent current sources.
Four meshes.
One current source shared by two meshes.

Careful choice of loop currents should make only one loop equation necessary. Three loop currents can be chosen using meshes and not sharing any source.

SOLVE FOR THE CURRENT I4.
USE OHM'S LAW TO COMPUTE REQUIRED VOLTAGES

$$\boldsymbol{V}_1 = \boldsymbol{R}_1(\boldsymbol{I}_1 - \boldsymbol{I}_3 - \boldsymbol{I}_4)$$

$$\boldsymbol{V}_2 = \boldsymbol{R}_2(\boldsymbol{I}_2 - \boldsymbol{I}_1)$$

$$\boldsymbol{V}_3 = \boldsymbol{R}_3(\boldsymbol{I}_2 - \boldsymbol{I}_4)$$