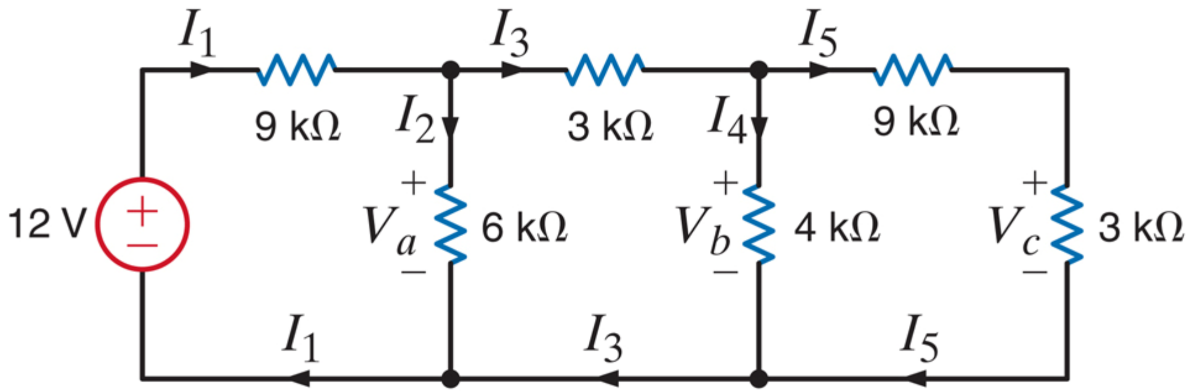
NODE ANALYSIS

- One of the systematic ways to determine every voltage and current in a circuit

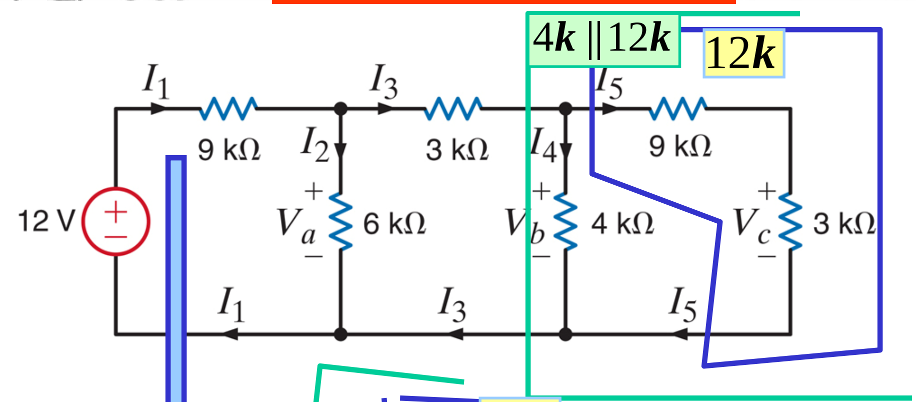
The variables used to describe the circuit will be "Node Voltages"
-- The voltages of each node with respect to a pre-selected reference node

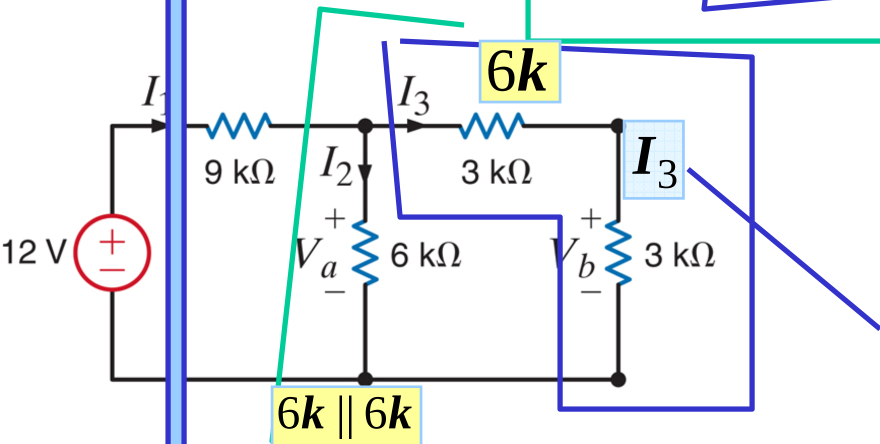
IT IS INSTRUCTIVE TO START THE PRESENTATION WITH A RECAP OF A PROBLEM SOLVED BEFORE USING SERIES/ PARALLEL RESISTOR COMBINATIONS



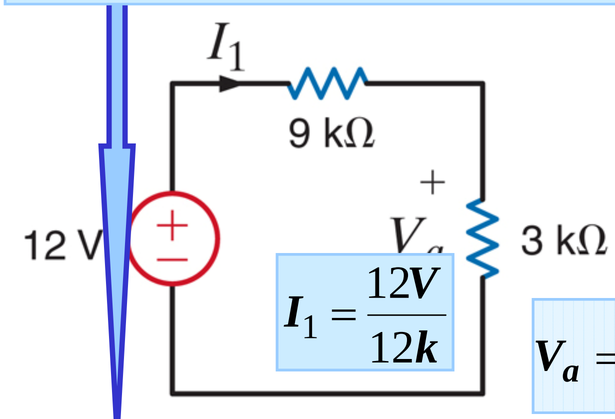
COMPUTE ALL THE VOLTAGES AND CURRENTS IN THIS CIRCUIT

We wish to find all the currents and voltages labeled in the ladder network shown





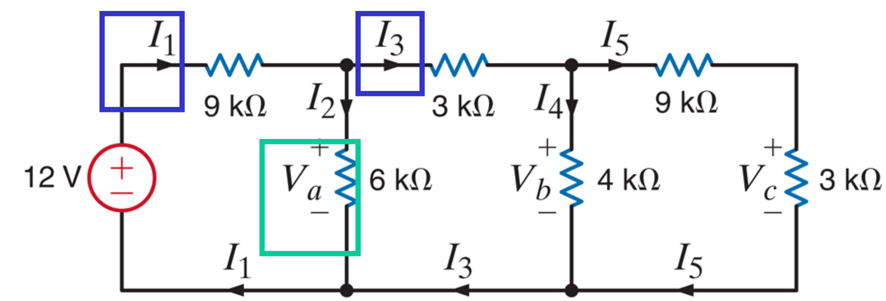
FIRST REDUCE TO A SINGLE LOOP CIRCUIT



$$I_1 = \frac{12V}{12k}$$

$$V_a = \frac{3}{3+9}(12)$$

SECOND: "BACKTRACK" USING KVL, KCL OHM'S



OHM'S: $I_2 = \frac{V_a}{6k}$

KCL: $I_1 - I_2 - I_3 = 0$

OHM'S: $V_b = 3k * I_3$

…OTHER OPTIONS...

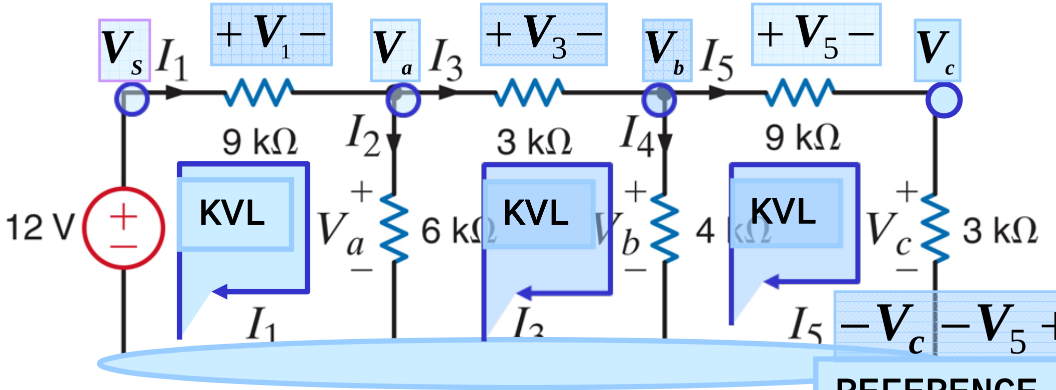
$$I_4 = \frac{12}{4+12} I_3$$

$$V_b = 4k * I_4$$

KCL: $I_5 + I_4 - I_3 = 0$

OHM'S: $V_C = 3k * I_5$

THE NODE ANALYSIS PERSPECTIVE



$$-V_c - V_5 + V_b = 0$$

THERE ARE FIVE NODES.
IF ONE NODE IS SELECTED AS REFERENCE THEN THERE ARE FOUR VOLTAGES WITH RESPECT TO THE REFERENCE NODE

$$V_5 = V_b - V_c$$

$$-V_s + V_1 + V_a = 0$$

$$V_1 = V_s - V_a$$

$$-V_a + V_3 + V_b = 0$$

$$V_3 = V_a - V_b$$

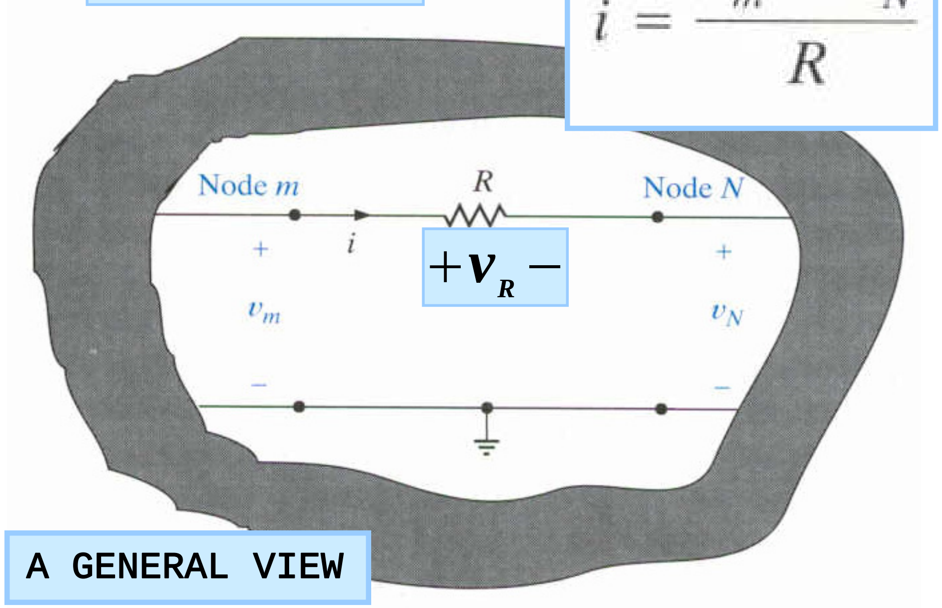
WHAT IS THE PATTERN???

ONCE THE VOLTAGES ARE KNOWN THE CURRENTS CAN BE COMPUTED USING OHM'S LAW

THEOREM: IF ALL NODE VOLTAGES WITH RESPECT TO A COMMON REFERENCE NODE ARE KNOWN THEN ONE CAN DETERMINE ANY OTHER ELECTRICAL VARIABLE FOR THE CIRCUIT

$$v_R = v_m - v_N$$

$$i = \frac{v_m - v_N}{R}$$

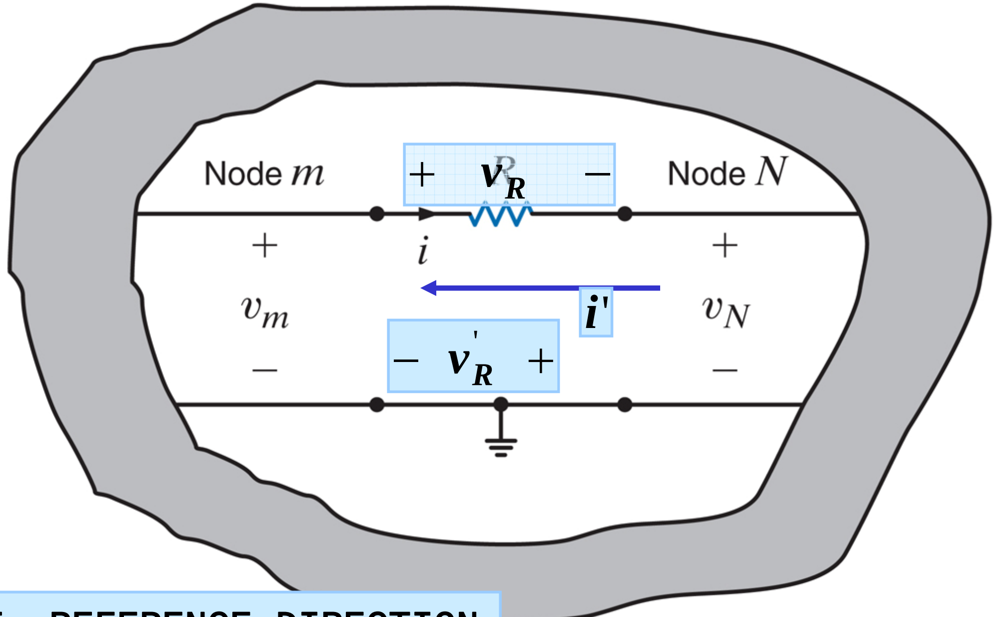


A GENERAL VIEW

DRILL QUESTION

$V_{ca} =$ ______

THE REFERENCE DIRECTION FOR CURRENTS IS IRRELEVANT



USING THE LEFT-RIGHT REFERENCE DIRECTION THE VOLTAGE DROP ACROSS THE RESISTOR MUST HAVE THE POLARITY SHOWN

OHM'S LAW $i = \dfrac{v_m - v_N}{R}$

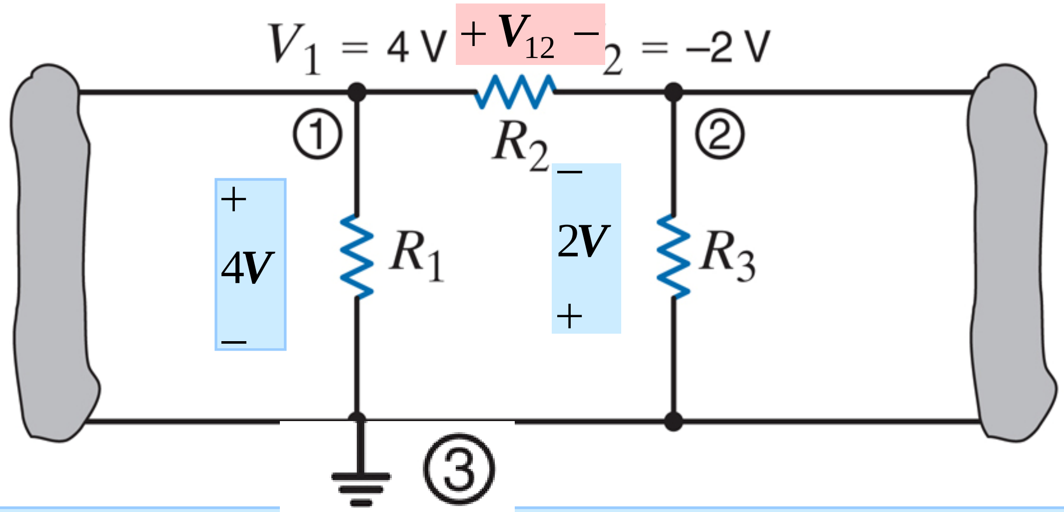
IF THE CURRENT REFERENCE DIRECTION IS REVERSED ...

THE PASSIVE SIGN CONVENTION WILL ASSIGN THE REVERSE REFERENCE POLARITY TO THE VOLTAGE ACROSS THE RESISTOR

OHM'S LAW $i' = \dfrac{v_N - v_m}{R}$

$i = -i'$ PASSIVE SIGN CONVENTION RULES!

DEFINING THE REFERENCE NODE IS VITAL



THE STATEMENT $V_1 = 4V$ IS MEANINGLES

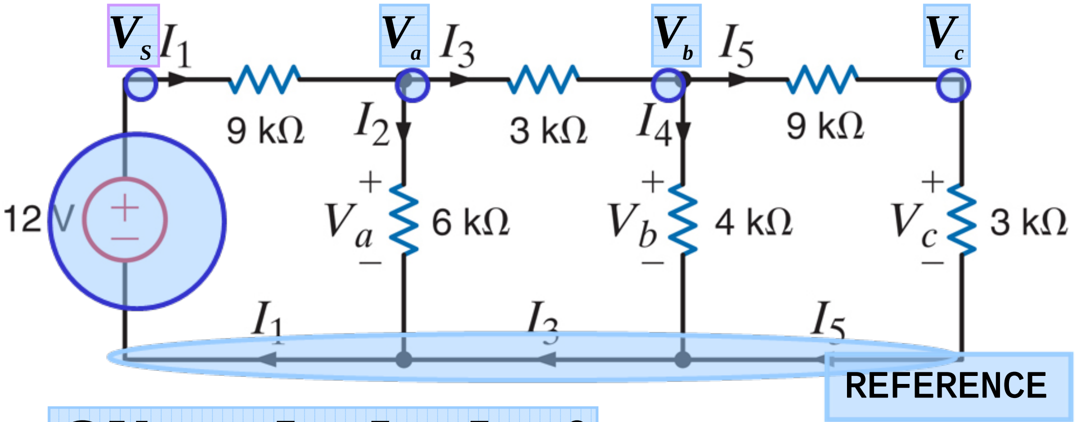
UNTIL THE REFERENCE POINT IS DEFINED

BY CONVENTION THE GROUND SYMBOL SPECIFIES THE REFERENCE POINT.

<u>ALL NODE VOLTAGES ARE MEASURED WITH RESPECT TO THAT REFERENCE POINT</u>

$V_{12} = \_\_\_\_\_\_\_?$

THE STRATEGY FOR NODE ANALYSIS



1. IDENTIFY ALL NODES AND SELECT A REFERENCE NODE

2. IDENTIFY KNOWN NODE VOLTAGES

3. AT EACH NODE WITH UNKNOWN VOLTAGE WRITE A KCL EQUATION (e.g.,SUM OF CURRENT LEAVING =0)

4. REPLACE CURRENTS IN TERMS OF NODE VOLTAGES

AND GET ALGEBRAIC EQUATIONS IN THE NODE VOLTAGES ...

$$@V_a: \quad -I_1 + I_2 + I_3 = 0$$

$$\frac{V_a - V_s}{9k} + \frac{V_a}{6k} + \frac{V_a - V_b}{3k} = 0$$

$$@V_b: \quad -I_3 + I_4 + I_5 = 0$$

$$\frac{V_b - V_a}{3k} + \frac{V_b}{4k} + \frac{V_b - V_c}{9k} = 0$$

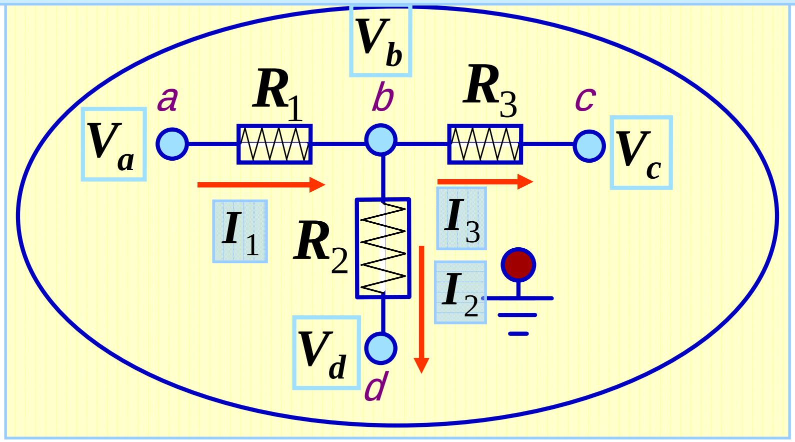
$$@V_c: \quad -I_5 + I_6 = 0$$

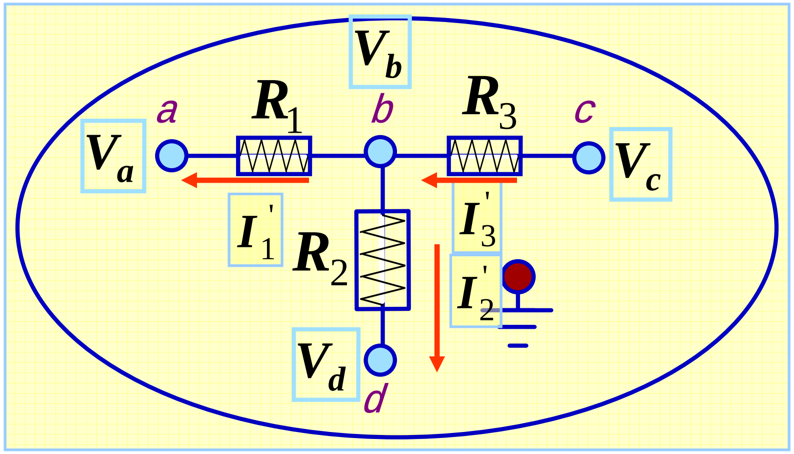
$$\frac{V_c - V_b}{9k} + \frac{V_c}{3k} = 0$$

<u>SHORTCUT:</u> SKIP WRITING THESE EQUATIONS...

AND PRACTICE WRITING THESE DIRECTLY

WHEN WRITING A NODE EQUATION...
AT EACH NODE ONE CAN CHOSE ARBITRARY DIRECTIONS FOR THE CURRENTS





AND SELECT ANY FORM OF KCL.
WHEN THE CURRENTS ARE REPLACED IN TERMS OF THE NODE VOLTAGES THE NODE EQUATIONS THAT RESULT ARE THE SAME OR EQUIVALENT

$\sum$ CURRENTS LEAVING = 0

$$-I_1 + I_2 + I_3 = 0 \Rightarrow -\frac{V_a - V_b}{R_1} + \frac{V_b - V_d}{R_2} + \frac{V_b - V_c}{R_3} = 0$$

$\sum$ CURRENTS INTO NODE = 0

$$I_1 - I_2 - I_3 = 0 \Rightarrow \frac{V_a - V_b}{R_1} - \frac{V_b - V_d}{R_2} - \frac{V_b - V_c}{R_3} = 0$$

$\sum$ CURRENTS LEAVING = 0

$$I_1' + I_2' - I_3' = 0 \Rightarrow \frac{V_b - V_a}{R_1} + \frac{V_b - V_d}{R_2} - \frac{V_c - V_b}{R_3} = 0$$

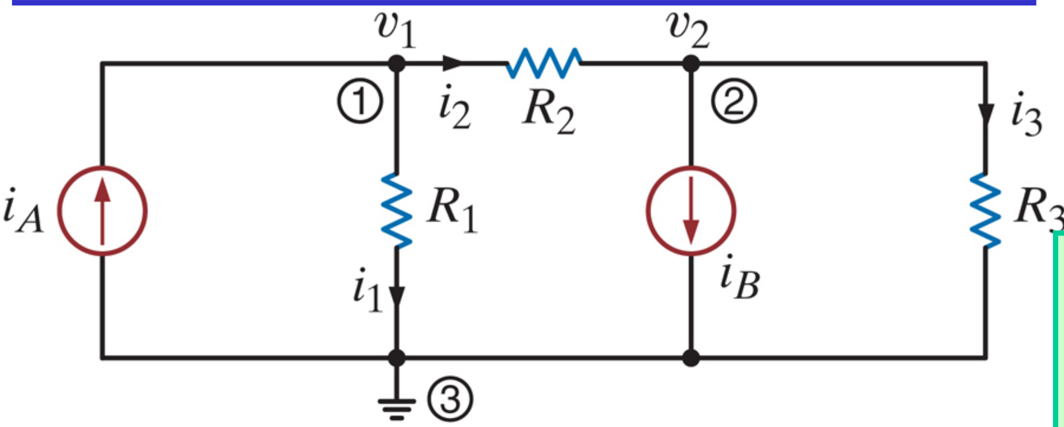
$\sum$ CURRENTS INTO NODE = 0

$$-I_1' - I_2' + I_3' = 0 \Rightarrow -\frac{V_b - V_a}{R_1} - \frac{V_b - V_d}{R_2} + \frac{V_c - V_b}{R_3} = 0$$

WHEN WRITING THE NODE EQUATIONS WRITE THE EQUATION DIRECTLY IN TERMS OF THE NODE VOLTAGES.
BY DEFAULT USE KCL IN THE FORM SUM-OF-CURRENTS-LEAVING = 0

THE REFERENCE DIRECTION FOR THE CURRENTS DOES NOT AFFECT THE NODE EQUATION

CIRCUITS WITH ONLY INDEPENDENT SOURCES



HINT: THE **FORMAL MANIPULATION** OF EQUATIONS MAY BE SIMPLER IF ONE USES CONDUCTANCES INSTEAD OF RESISTANCES.

@ NODE 1

$$-i_A + i_1 + i_2 = 0$$

USING RESISTANCES $-i_A + \frac{v_1}{R_1} + \frac{v_1 - v_2}{R_2} = 0$

WITH CONDUCTANCES $-i_A + G_1 v_1 + G_2(v_1 - v_2) = 0$

REORDERING TERMS $(G_1 + G_2)v_1 - G_2 v_2 = i_A$

@ NODE 2

$$-G_2(v_1 - v_2) + i_B + G_3(v_2 - 0) = 0$$

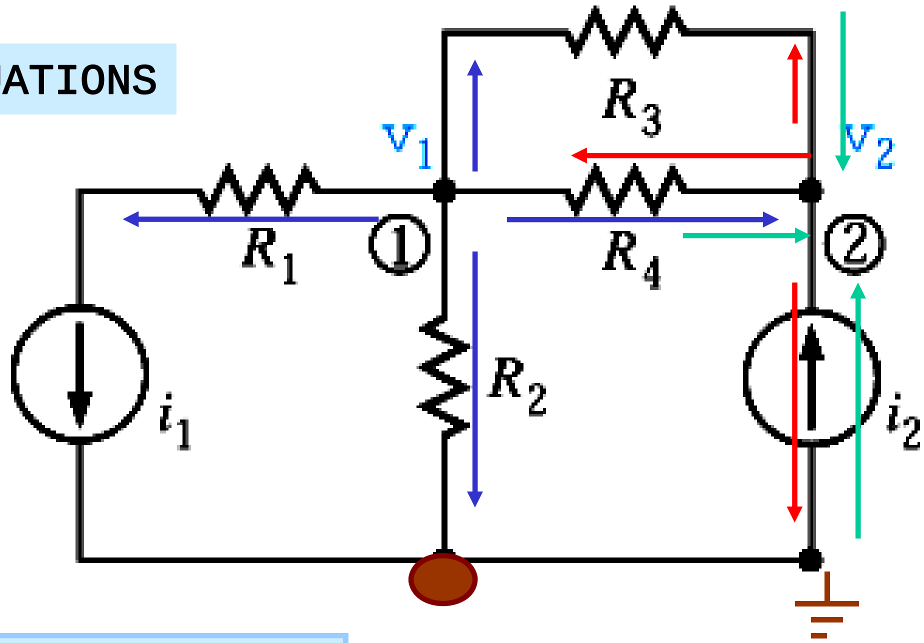
REORDERING TERMS $-G_2 v_1 + (G_2 + G_3)v_2 = -i_B$

THE MODEL FOR THE CIRCUIT IS A SYSTEM OF ALGEBRAIC EQUATIONS

$$(G_1 + G_2)v_1 - G_2 v_2 = i_A$$
$$-G_2 v_1 + (G_2 + G_3)v_2 = -i_B$$

EXAMPLE

WRITE THE KCL EQUATIONS



@ NODE 1 WE VISUALIZE THE CURRENTS LEAVING AND WRITE THE KCL EQUATION

$$i_1 + \frac{v_1}{R_2} + \frac{v_1 - v_2}{R_3} + \frac{v_1 - v_2}{R_4} = 0$$

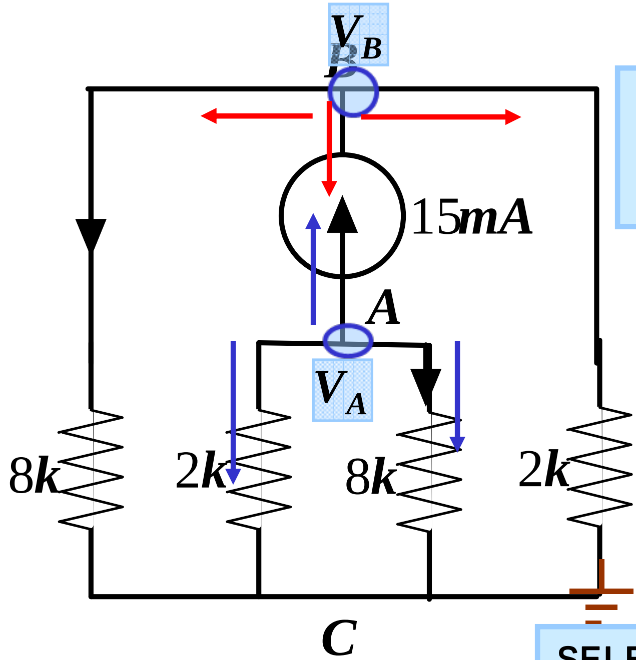
REPEAT THE PROCESS AT NODE 2

$$-i_2 + \frac{v_2 - v_1}{R_4} + \frac{v_2 - v_1}{R_3} = 0$$

OR VISUALIZE CURRENTS GOING INTO NODE

$$i_2 + \frac{v_1 - v_2}{R_3} + \frac{v_1 - v_2}{R_4} = 0$$

ANOTHER EXAMPLE OF WRITING NODE EQUATIONS



MARK THE NODES (TO INSURE THAT NONE IS MISSING)

SELECT AS REFERENCE

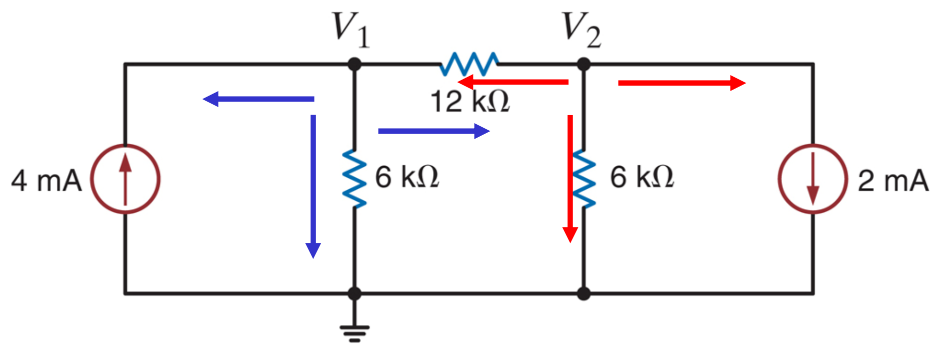
WRITE KCL AT EACH NODE IN TERMS OF NODE VOLTAGES

$$@A \quad \frac{V_A}{2k} + \frac{V_A}{8k} + 15mA = 0$$

$$@B \quad \frac{V_B}{8k} + \frac{V_B}{2k} - 15mA = 0$$

LEARNING EXTENSION

Write the node equations



$$@V_1: \; -4mA + \frac{V_1}{6k} + \frac{V_1 - V_2}{12k}$$

USING KCL

$$@V_2: \; 2mA + \frac{V_2}{6k} + \frac{V_2 - V_1}{12k} = 0$$

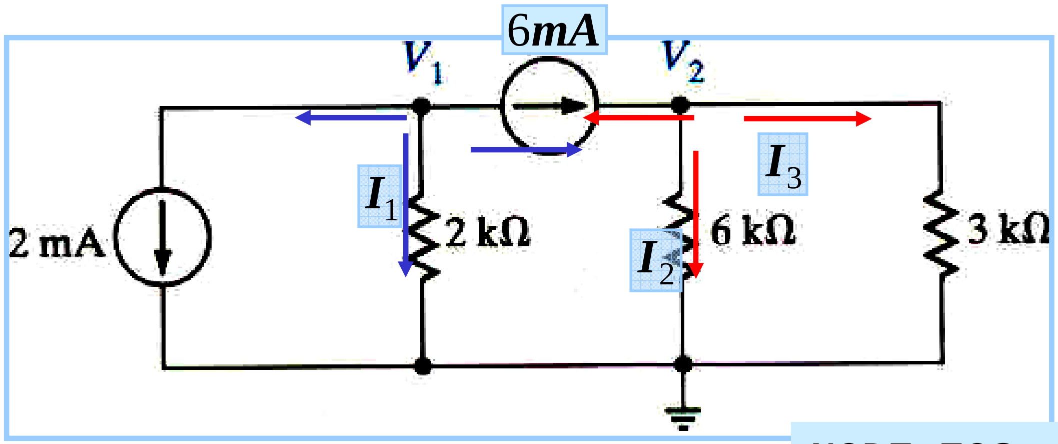
BY "INSPECTION"

$$\left(\frac{1}{6k} + \frac{1}{12k}\right)V_1 - \frac{1}{12k}V_2 = 4mA$$

$$-\frac{1}{12k} + \left(\frac{1}{6k} + \frac{1}{12k}\right)V_2 = -2mA$$

LEARNING EXTENSION

Find all the branch currents



Node analysis

$$@\,V_1:\ \frac{V_1}{2k} + 2mA + 6mA = 0 \quad \Rightarrow V_1 = -16V$$

$$@V_2:\quad -6mA + \frac{V_2}{6k} + \frac{V_2}{3k} = 0 \Rightarrow \quad V_2 = 12V$$

NODE EQS. BY INSPECTION

$$\frac{1}{2k}V_1 + (0)V_2 = -(2+6)mA$$

$$(0)V_1 + \left(\frac{1}{6k} + \frac{1}{3k}\right)V_2 = 6mA$$

Once node voltages are known

$$I_1 = \frac{V_1}{2k} \qquad I_2 = \frac{V_2}{6k} \qquad I_3 = \frac{V_2}{3k}$$

IN MOST CASES THERE ARE SEVERAL DIFFERENT WAYS OF SOLVING A PROBLEM

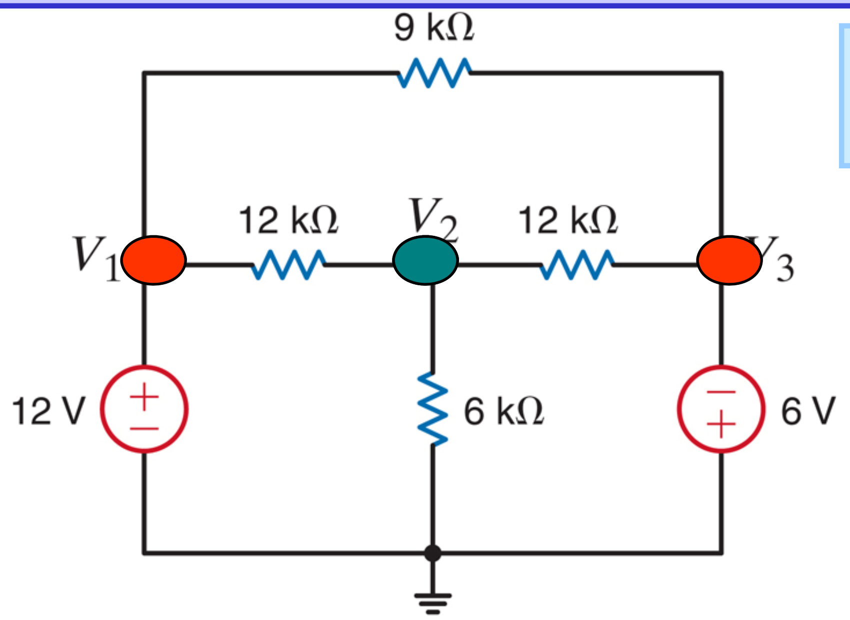
$$I_1 = -8mA$$

$$I_2 = \frac{3k}{3k+6k}(6mA) = 2mA$$

$$I_3 = \frac{6k}{3k+6k}(6mA) = 4mA$$

CURRENTS COULD BE COMPUTED DIRECTLY USING KCL AND CURRENT DIVIDER!!

CIRCUITS WITH INDEPENDENT VOLTAGE SOURCES



3 nodes plus the reference. In principle one needs 3 equations...

…but two nodes are connected to the reference through voltage sources. Hence those node voltages are known!!!

…Only one KCL is necessary

$$\frac{V_2}{6k} + \frac{V_2 - V_3}{12k} + \frac{V_2 - V_1}{12k} = 0$$

$$V_1 = 12[V]$$

$$V_3 = -6[V]$$

THESE ARE THE REMAINING TWO NODE EQUATIONS

Hint: Each voltage source connected to the reference node saves one node equation

SOLVING THE EQUATIONS

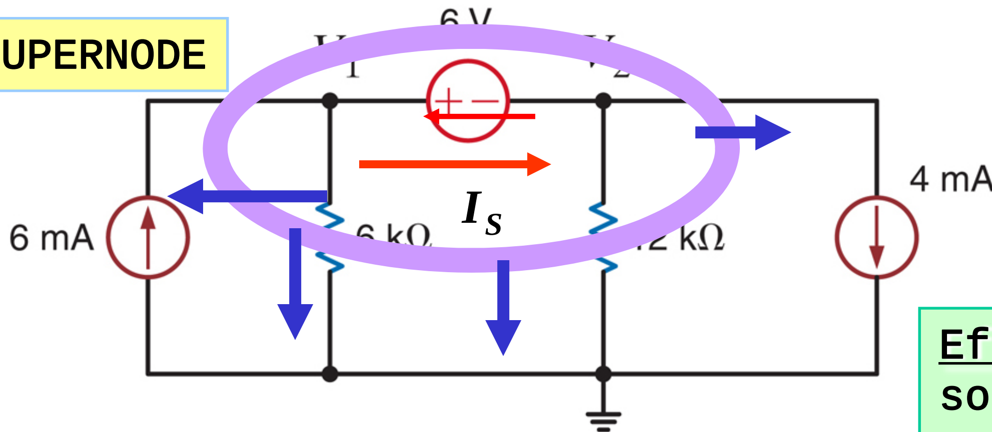
$$2V_2 + (V_2 - V_3) + (V_2 - V_1) = 0$$

$$4V_2 = 6[V] \Rightarrow V_2 = 1.5[V]$$

THE SUPERNODE TECHNIQUE

We will use this example to introduce the concept of a SUPERNODE

SUPERNODE



Efficient solution: enclose the source, and all elements in parallel, inside a surface.

Apply KCL to the surface!!!

$$-6mA + \frac{V_1}{6k} + \frac{V_2}{12k} + 4mA = 0$$

Conventional node analysis requires all currents at a node

@V_1

$$-6mA + \frac{V_1}{6k} + I_S = 0$$

@V_2

$$-I_S + 4mA + \frac{V_2}{12k} = 0$$

The source current is interior to the surface and is not required

2 eqs, 3 unknowns...Panic!!
The current through the source is not related to the voltage of the source

We STILL need one more equation

$$V_1 - V_2 = 6[V]$$

Math solution: add one equation

$$V_1 - V_2 = 6[V]$$

Only 2 eqs in two unknowns!!!

ALGEBRAIC DETAILS

The Equations

(1) $$\frac{V_1}{6k} + \frac{V_2}{12k} - 6mA + 4mA = 0$$ *\12k

(2) $$V_1 - V_2 = 6[V]$$

Solution

1. Eliminate denominators in Eq(1). Multiply by …

$$2V_1 + V_2 = 24[V]$$

$$V_1 - V_2 = 6[V]$$

2. Add equations to eliminate V_2

$$3V_1 = 30[V] \Rightarrow V_1 = 10[V]$$

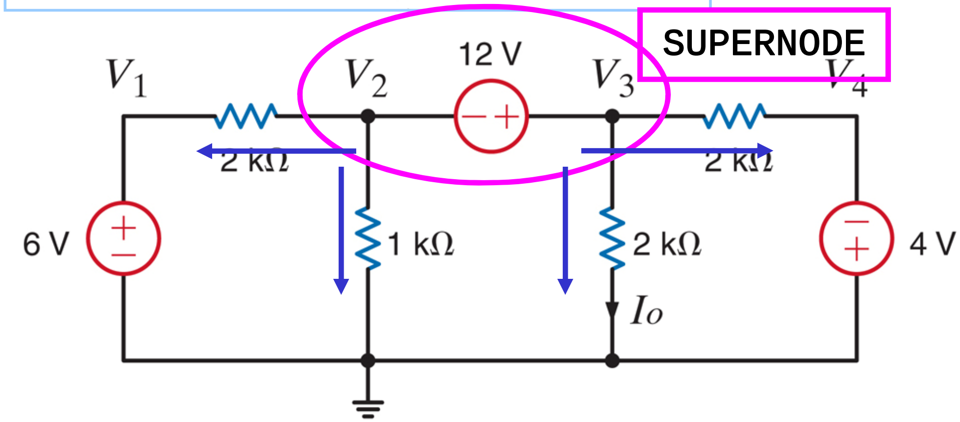
3. Use Eq(2) to compute V_2

$$V_2 = V_1 - 6[V] = 4[V]$$

The supernode technique

- Used when a branch between two nonreference nodes contains a voltage source.
- First encircle the voltage source and the two connecting nodes to form the supernode.
- Write the equation that defines the voltage relationship between the two nonreference nodes as a result of the presence of the voltage source.
- Write the KCL equation for the supernode.
- If the voltage source is dependent, then the controlling equation for the dependent source is also needed.

Use nodal analysis to find I_o



$$V_1 = 6V$$
$$V_4 = -4V$$

SOURCES CONNECTED TO THE REFERENCE

CONSTRAINT EQUATION $V_3 - V_2 = 12V$

KCL @ SUPERNODE

$$\frac{V_2 - 6}{2k} + \frac{V_2}{1k} + \frac{V_3}{2k} + \frac{V_3 - (-4)}{2k} = 0 \quad */2k$$

V_2 IS NOT NEEDED FOR I_o

$$3V_2 + 2V_3 = 2V$$
$$-V_2 + V_3 = 12V \quad */3 \text{ and add}$$
$$5V_3 = 38V$$

OHM'S LAW $I_o = \frac{V_3}{2k} = 3.8mA$