AC STEADY-STATE ANALYSIS

SINUSOIDAL AND COMPLEX FORCING FUNCTIONS
Behavior of circuits with sinusoidal independent sources and modeling of sinusoids in terms of complex exponentials

PHASORS
Representation of complex exponentials as vectors. It facilitates steady-state analysis of circuits.

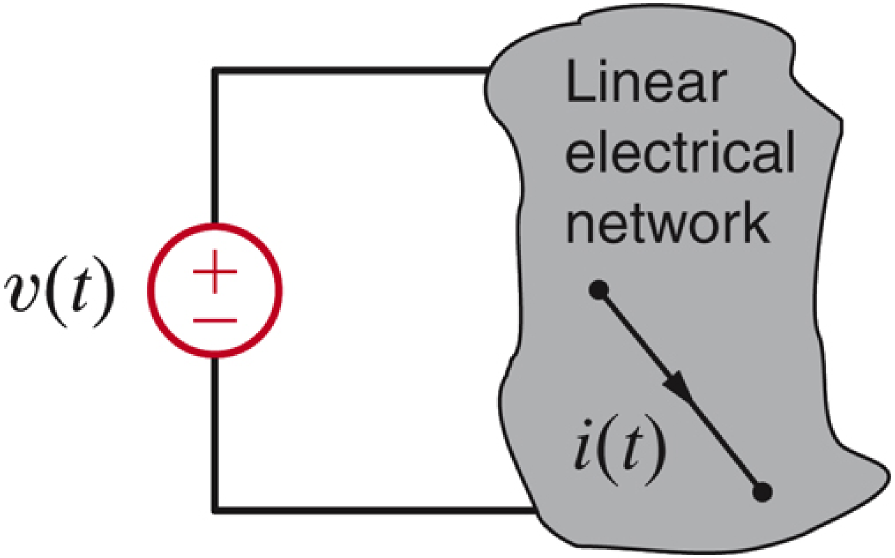
IMPEDANCE AND ADMITANCE
Generalization of the familiar concepts of resistance and conductance to describe AC steady state circuit operation

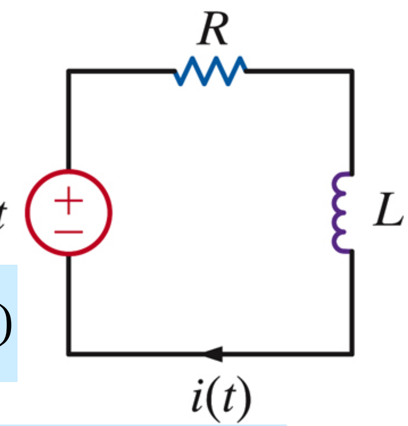
PHASOR DIAGRAMS
Representation of AC voltages and currents as complex vectors

BASIC AC ANALYSIS USING KIRCHHOFF LAWS

ANALYSIS TECHNIQUES
Extension of node, loop, Thevenin and other techniques

SINUSOIDAL AND COMPLEX FORCING FUNCTIONS





KVL : $L\frac{di}{dt}(t) + Ri(t) = v(t)$

In steady state $i(t) = A\cos(\omega t + \phi)$, or

$i(t) = A_1 \cos \omega t + A_2 \sin \omega t$ $\quad */R$

$\frac{di}{dt}(t) = -A_1 \omega \sin \omega t + A_2 \omega \cos \omega t$ $\quad */L$

$(-L\omega A_1 + RA_2)\sin \omega t + (L\omega A_2 + RA_1)\cos \omega t =$
$= V_M \cos \omega t$

$-L\omega A_1 + RA_2 = 0$
$L\omega A_2 + RA_1 = V_M$

algebraic problem

$$A_1 = \frac{RV_M}{R^2 + (\omega L)^2}, \quad A_2 = \frac{\omega L V_M}{R^2 + (\omega L)^2}$$

If the independent sources are sinusoids of the same frequency then for any variable in the linear circuit the steady state response will be sinusoidal and of the same frequency

$$v(t) = A\sin(\omega t + \theta) \Rightarrow i_{SS}(t) = B\sin(\omega t + \phi)$$

To determine the steady state solution we only need to determine the parameters B, ϕ

Determining the steady state solution can be accomplished with only algebraic tools!

FURTHER ANALYSIS OF THE SOLUTION

The solution is $\boldsymbol{i}(\boldsymbol{t}) = \boldsymbol{A}_1 \cos \omega \boldsymbol{t} + \boldsymbol{A}_2 \sin \omega \boldsymbol{t}$

The applied voltage is $\boldsymbol{v}(\boldsymbol{t}) = \boldsymbol{V}_{\boldsymbol{M}} \cos \omega \boldsymbol{t}$

For comparison purposes one can write $\boldsymbol{i}(\boldsymbol{t}) = \boldsymbol{A} \cos(\omega \boldsymbol{t} + \phi)$

$$\boldsymbol{A}_1 = \boldsymbol{A} \cos \phi, \quad \boldsymbol{A}_2 = -\boldsymbol{A} \sin \phi$$

$$\boldsymbol{A} = \sqrt{\boldsymbol{A}_1^2 + \boldsymbol{A}_2^2}, \ \tan \phi = -\frac{\boldsymbol{A}_2}{\boldsymbol{A}_1}$$

$$\boldsymbol{A}_1 = \frac{\boldsymbol{R} \boldsymbol{V}_{\boldsymbol{M}}}{\boldsymbol{R}^2 + (\omega \boldsymbol{L})^2}, \quad \boldsymbol{A}_2 = \frac{\omega \boldsymbol{L} \boldsymbol{V}_{\boldsymbol{M}}}{\boldsymbol{R}^2 + (\omega \boldsymbol{L})^2}$$

$$A = \frac{V_M}{\sqrt{R^2 + (\omega L)^2}}, \quad \phi = -\tan^{-1} \frac{\omega L}{R}$$

$$i(t) = \frac{V_M}{\sqrt{R^2 + (\omega L)^2}} \cos(\omega t - \tan^{-1} \frac{\omega L}{R})$$

For $\boldsymbol{L} \neq 0$ the current ALWAYS lags the voltage

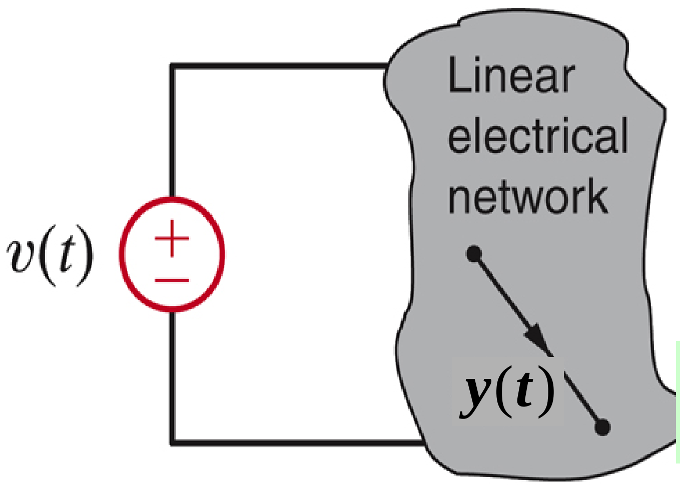
If $\boldsymbol{R} = 0$ (pure inductor) the current lags the voltage by 90°

SOLVING A SIMPLE ONE LOOP CIRCUIT CAN BE VERY LABORIOUS IF ONE USES SINUSOIDAL EXCITATIONS

TO MAKE ANALYSIS SIMPLER ONE RELATES SINUSOIDAL SIGNALS TO COMPLEX NUMBERS. THE ANALYSIS OF STEADY STATE WILL BE CONVERTED TO SOLVING SYSTEMS OF ALGEBRAIC EQUATIONS ...

... WITH COMPLEX VARIABLES

ESSENTIAL IDENTITY: $e^{j\theta} = \cos\theta + j\sin\theta$ (Euler identity)



$$v(t) = V_M \cos\omega t \rightarrow y(t) = A\cos(\omega t + \phi)$$

$$v(t) = V_M \sin\omega t \rightarrow y(t) = A\sin(\omega t + \phi)$$

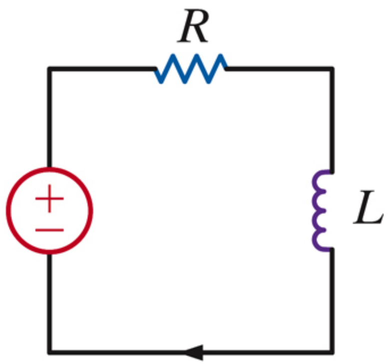
$*/ j$ (and add)

$$V_M e^{j\omega t} \rightarrow Ae^{j(\omega t + \phi)} = Ae^{j\theta}e^{j\omega t}$$

If everybody knows the frequency of the sinusoid then one can skip the term *exp(jwt)*

$$V_M \rightarrow Ae^{j\theta}$$

Example



$v(t) = V_M \cos \omega t$

$i(t)$

$v(t) = V_M e^{j\omega t}$

Assume $i(t) = I_M e^{(j\omega t+\phi)}$

KVL : $L\dfrac{di}{dt}(t) + Ri(t) = v(t)$

$$\frac{di}{dt}(t) = j\omega I_M e^{(j\omega t+\phi)}$$

$$L\frac{di}{dt}(t) + Ri(t) = j\omega L I_M e^{(j\omega t+\phi)} + R I_M e^{(j\omega t+\phi)}$$
$$= (j\omega L + R) I_M e^{(j\omega t+\phi)}$$
$$= (j\omega L + R) I_M e^{j\phi} e^{j\omega t}$$

$$(j\omega L + R) I_M e^{j\phi} e^{j\omega t} = V_M e^{j\omega t}$$

$$I_M e^{j\phi} = \frac{V_M}{j\omega L + R} \quad */ \frac{R - j\omega L}{R - j\omega L}$$

$$I_M e^{j\phi} = \frac{V_M (R - j\omega L)}{R^2 + (\omega L)^2}$$

$$R - j\omega L = \sqrt{R^2 + (\omega L)^2}\, e^{-\tan^{-1}\frac{\omega L}{R}}$$

$$I_M e^{j\phi} = \frac{V_M}{\sqrt{R^2 + (\omega L)^2}} e^{-\tan^{-1}\frac{\omega L}{R}}$$

$$I_M = \frac{V_M}{\sqrt{R^2 + (\omega L)^2}}, \quad \phi = -\tan^{-1}\frac{\omega L}{R}$$

$$v(t) = V_M \cos \omega t = \mathrm{Re}\{V_M e^{j\omega t}\}$$
$$\Rightarrow i(t) = \mathrm{Re}\{I_M e^{(j\omega t - \phi)}\} = I_M \cos(\omega t - \phi)$$

$C \leftrightarrow P$

$$x + jy = re^{j\theta}$$

$$r = \sqrt{x^2 + y^2}, \quad \theta = \tan^{-1}\frac{y}{x}$$

$$x = r\cos\theta, \quad y = r\sin\theta$$

PHASORS

ESSENTIAL CONDITION
ALL INDEPENDENT SOURCES ARE SINUSOIDS OF THE SAME FREQUENCY

BECAUSE OF SOURCE SUPERPOSITION ONE CAN CONSIDER A SINGLE SOURCE

$$u(t) = U_M \cos(\omega t + \theta)$$

THE STEADY STATE RESPONSE OF ANY CIRCUIT VARIABLE WILL BE OF THE FORM

$$y(t) = Y_M \cos(\omega t + \phi)$$

SHORTCUT 1

$$u(t) = U_M e^{j(\omega t + \theta)} \Rightarrow y(t) = Y_M e^{j(\omega t + \phi)}$$

$$\mathrm{Re}\{U_M e^{j(\omega t + \theta)}\} \Rightarrow \mathrm{Re}\{Y_M e^{j(\omega t + \phi)}\}$$

NEW IDEA: $U_M e^{j(\omega t + \theta)} = U_M e^{j\theta} e^{j\omega t}$ $\quad u = U_M e^{j\theta} \Rightarrow y = Y_M e^{j\phi}$

SHORTCUT IN NOTATION

INSTEAD OF WRITING $u = U_M e^{j\theta}$ WE WRITE $u = U_M \angle \theta$

... AND WE ACCEPT ANGLES IN DEGREES

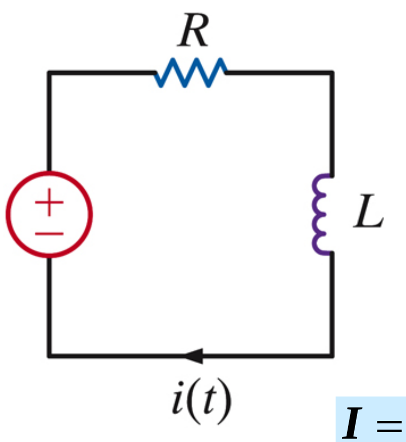
$U_M \angle \theta$ IS THE PHASOR REPRESENTATION FOR $U_M \cos(\omega t + \theta)$

$$u(t) = U_M \cos(\omega t + \theta) \rightarrow U = U_M \angle \theta \Rightarrow Y = Y_M \angle \phi \rightarrow y(t) = Y_M \cos(\omega t + \phi)$$

SHORTCUT 2: DEVELOP EFFICIENT TOOLS TO DETERMINE THE PHASOR OF THE RESPONSE GIVEN THE INPUT PHASOR(S)

Example

$v(t) = V_M \cos \omega t$



$$\boldsymbol{V} = V_M \angle 0$$
$$\boldsymbol{v} = \boldsymbol{V} e^{j\omega t}$$

$$\boldsymbol{I} = I_M \angle \phi$$
$$\boldsymbol{i} = \boldsymbol{I} e^{j\omega t}$$

$$L\frac{di}{dt}(t) + Ri(t) = v$$
$$L(j\omega \boldsymbol{I} e^{j\omega t}) + R\boldsymbol{I} e^{j\omega t} = \boldsymbol{V} e^{j\omega t}$$

In terms of phasors one has

$$j\omega L\boldsymbol{I} + R\boldsymbol{I} = \boldsymbol{V}$$
$$\boldsymbol{I} = \frac{\boldsymbol{V}}{R + j\omega L}$$

The phasor can be obtained using only complex algebra

We will develop a phasor representation for the circuit that will eliminate the need of writing the differential equation

It is essential to be able to move from sinusoids to phasor representation

$$A\cos(\omega t \pm \theta) \leftrightarrow A\angle \pm \theta$$
$$A\sin(\omega t \pm \theta) \leftrightarrow A\angle \pm \theta - 90°$$

$$v(t) = 12\cos(377t - 425°) \leftrightarrow 12\angle -425°$$
$$y(t) = 18\sin(2513t + 4.2°) \leftrightarrow 18\angle -85.8°$$

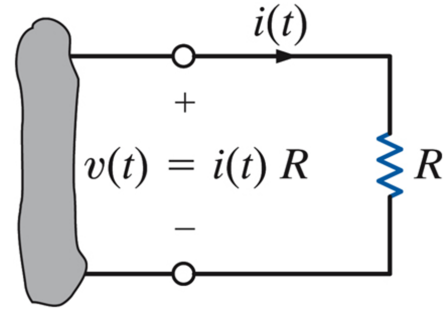
Given $f = 400Hz$

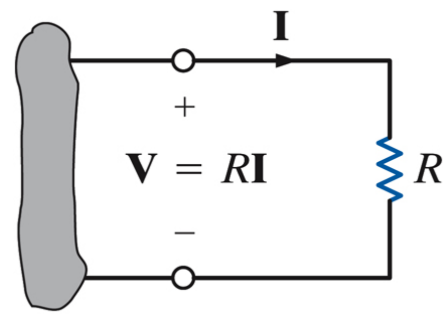
$$\boldsymbol{V}_1 = 10\angle 20° \leftrightarrow v_1(t) = 10\cos(800\pi t + 20°)$$
$$\boldsymbol{V}_2 = 12\angle -60° \leftrightarrow v_2(t) = 12\cos(800\pi t - 60°)$$

Phasors can be combined using the rules of complex algebra

$$(V_1\angle\theta_1)(V_2\angle\theta_2) = V_1V_2\angle(\theta_1 + \theta_2)$$
$$\frac{V_1\angle\theta_1}{V_2\angle\theta_2} = \frac{V_1}{V_2}\angle(\theta_1 - \theta_2)$$

PHASOR RELATIONSHIPS FOR CIRCUIT ELEMENTS





RESISTORS

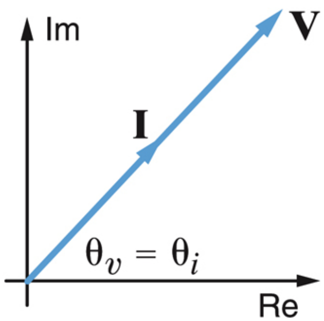
$$v(t) = Ri(t)$$

$$V_M e^{(j\omega t+\theta)} = RI_M e^{(j\omega t+\theta)}$$

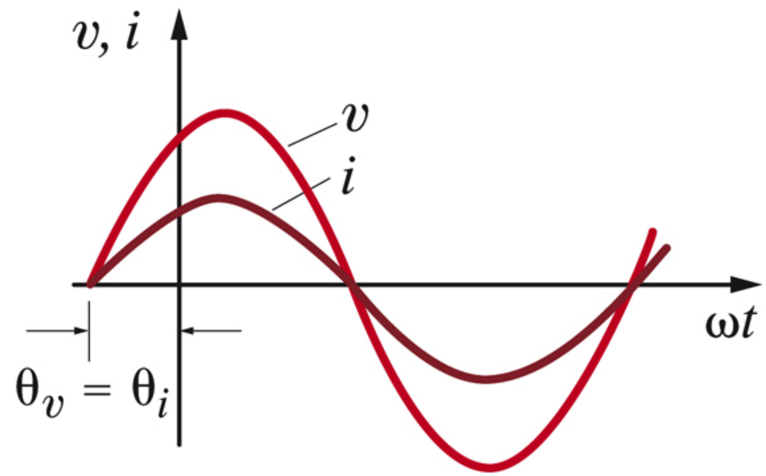
$$V_M e^{j\theta} = RI_M e^{j\theta}$$

$V = RI$ **Phasor representation for a resistor**

Phasors are complex numbers. The resistor model has a geometric interpretation



The voltage and current phasors are colineal

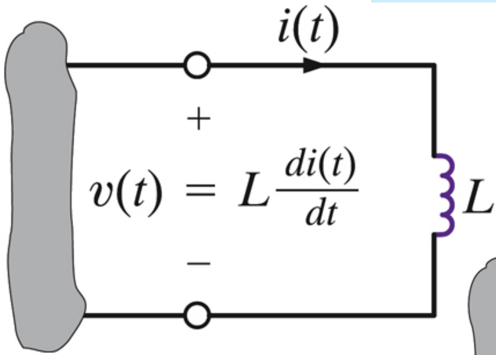


In terms of the sinusoidal signals this geometric representation implies that the two sinusoids are "in phase"

INDUCTORS

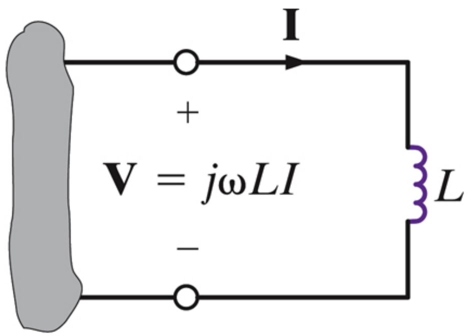
$$V_M e^{(j\omega t+\theta)} = L\frac{d}{dt}\left(I_M e^{(j\omega t+\phi)}\right)$$

$$= j\omega L I_M e^{(j\omega t+\phi)}$$

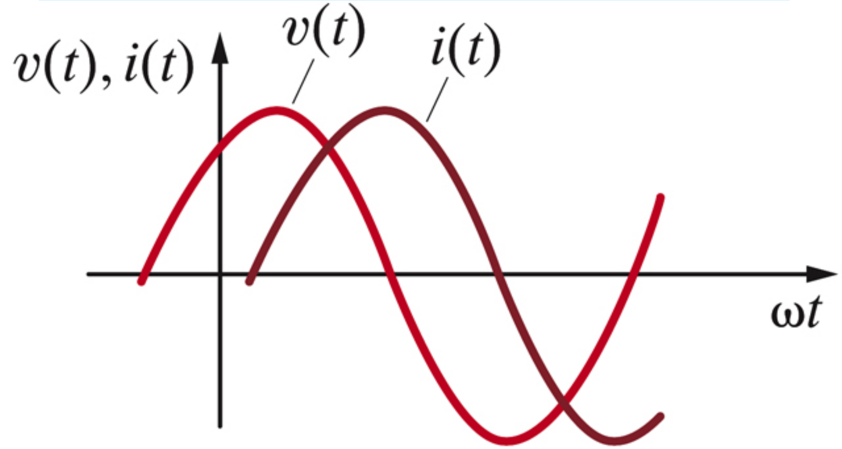


$$V_M e^{j\theta} = j\omega L I_M e^{j\phi}$$

$$V = j\omega L I$$



Relationship between sinusoids

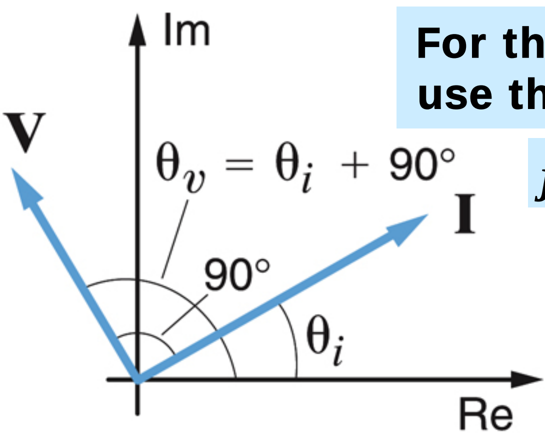


The relationship between phasors is algebraic

For the geometric view use the result

$$j = 1\angle 90° = e^{j90°}$$

$$V = \omega L I \angle 90°$$



The voltage leads the current by 90 deg
The current lags the voltage by 90 deg

Example

$L = 20mH$, $v(t) = 12\cos(377t + 20°)$. Find $i(t)$

$$\omega = 377$$

$$V = 12\angle 20°$$

$$I = \frac{V}{j\omega L}$$

$$I = \frac{12\angle 20°}{\omega L\angle 90°}(A)$$

$$I = \frac{12}{377\times 20\times 10^{-3}}\angle -70°(A)$$

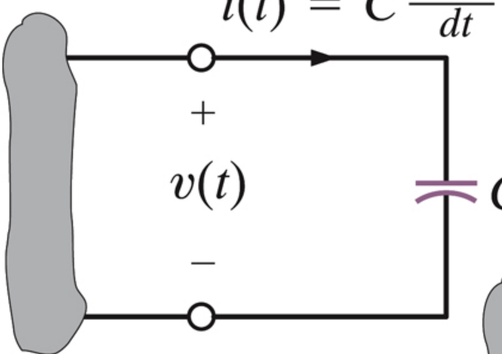
$$i(t) = \frac{12}{377\times 20\times 10^{-3}}\cos(377t - 70°)$$

CAPACITORS

$$I_M e^{(j\omega t+\phi)} = C\frac{d}{dt}\left(V_M e^{(j\omega t+\theta)}\right)$$

$$i(t) = C\frac{dv(t)}{dt}$$

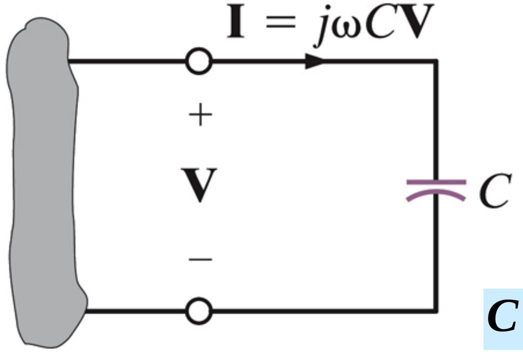
$$I_M e^{j\phi} = j\omega C e^{j\theta}$$



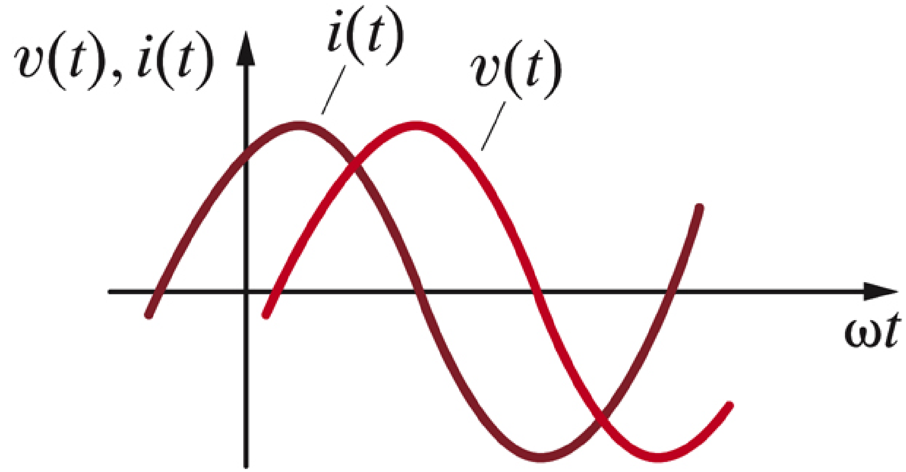
$$\boldsymbol{I} = j\omega C\boldsymbol{V}$$

$$\boldsymbol{I} = \omega C\boldsymbol{V}\angle 90^\circ$$

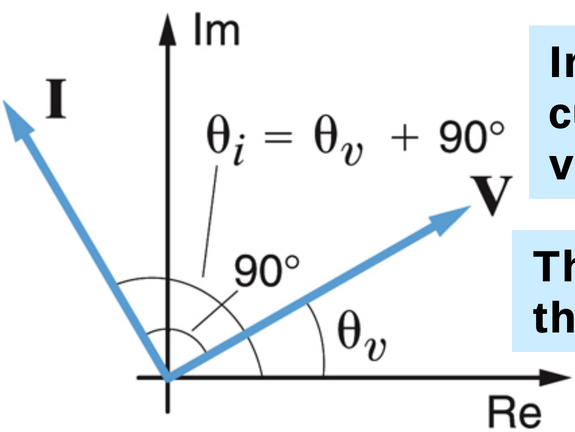
$$\mathbf{I} = j\omega C\mathbf{V}$$



Relationship between sinusoids



The relationship between phasors is algebraic



In a capacitor the current leads the voltage by 90 deg

The voltage lags the current by 90 deg

$C = 100\mu F, v(t) = 100\cos(314t + 15^\circ)$. Find $i(t)$

$$\omega = 314$$

$$\boldsymbol{V} = 100\angle 15^\circ$$

$$\boldsymbol{I} = j\omega C\boldsymbol{V}$$

$$\boldsymbol{I} = \omega C \times 1\angle 90^\circ \times 100\angle 15^\circ$$

$$\boldsymbol{I} = 314\times 100\times 10^{-6}\times 100\angle 105^\circ(A)$$

$$i(t) = 3.14\cos(314t + 105^\circ)(A)$$

$$\boldsymbol{L} = 0.05\boldsymbol{H},\ \boldsymbol{I} = 4\angle -30°(\boldsymbol{A}),\ \boldsymbol{f} = 60\boldsymbol{Hz}$$

Find the voltage across the inductor

$$\omega = 2\pi \boldsymbol{f} = 120\pi$$

$$\boldsymbol{V} = \boldsymbol{j}\omega\boldsymbol{LI}$$

$$\boldsymbol{V} = 120\pi \times 0.05 \times 1\angle 90° \times 4\angle -30°$$

$$\boldsymbol{V} = 24\pi\angle 60°$$

$$\boldsymbol{v}(\boldsymbol{t}) = 24\pi\cos(120\pi + 60°)$$

Now an example with capacitors

$$C = 150\mu F, I = 3.6\angle -145°,\ f = 60Hz$$

Find the voltage across the capacitor

$$\omega = 2\pi \boldsymbol{f} = 120\pi$$

$$\boldsymbol{I} = \boldsymbol{j}\omega\boldsymbol{CV} \Rightarrow \boldsymbol{V} = \frac{\boldsymbol{I}}{\boldsymbol{j}\omega\boldsymbol{C}}$$

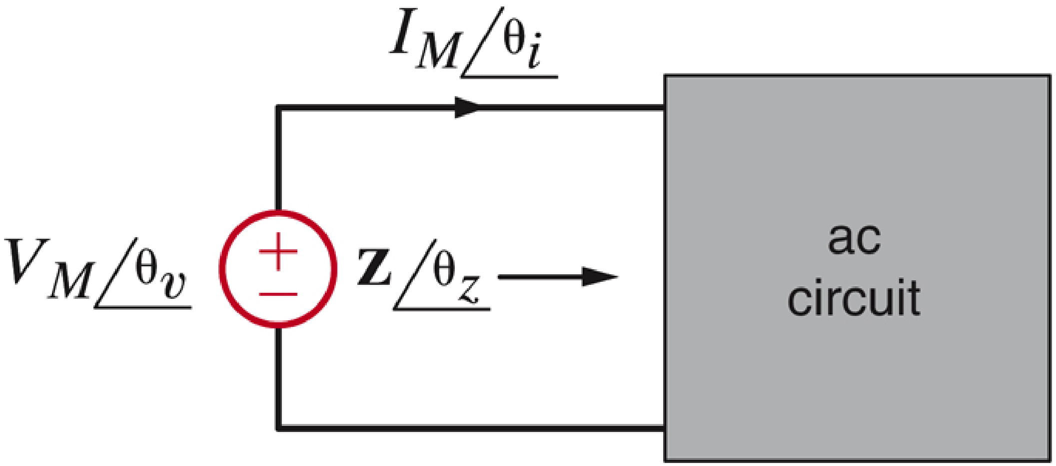
$$\boldsymbol{V} = \frac{3.6\angle -145°}{120\pi \times 150 \times 10^{-6} \times 1\angle 90°}$$

$$\boldsymbol{V} = \frac{200}{\pi}\angle -235°$$

$$\boldsymbol{v}(\boldsymbol{t}) = \frac{200}{\pi}\cos(120\pi \boldsymbol{t} - 235°)$$

IMPEDANCE AND ADMITTANCE

For each of the passive components the relationship between the voltage phasor and the current phasor is algebraic. We now generalize for an arbitrary 2-terminal element



$$Z(\omega) = R(\omega) + jX(\omega)$$

$R(\omega)$=Resistive component

$X(\omega)$=Reactive component

$$|Z| = \sqrt{R^2 + X^2}$$

$$\theta_z = \tan^{-1}\frac{X}{R}$$

(INPUT) IMPEDANCE

$$Z = \frac{V}{I} = \frac{V_M\angle\theta_v}{I_M\angle\theta_i} = \frac{V_M}{I_M}\angle(\theta_v - \theta_i) = |Z|\angle\theta_z$$

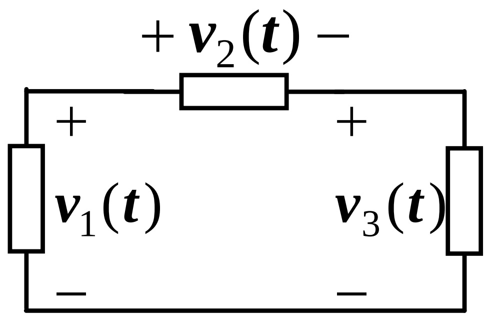
(DRIVING POINT IMPEDANCE)

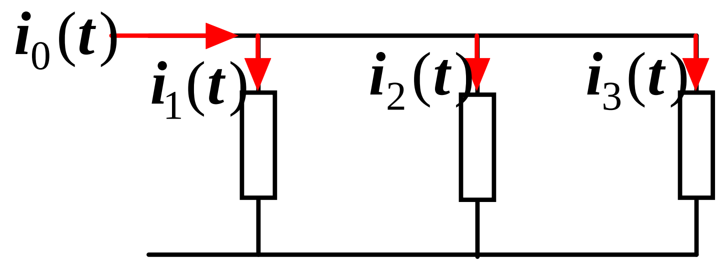
The units of impedance are OHMS

Element	Phasor Eq.	Impedance
R	$V = RI$	$Z = R$
L	$V = j\omega LI$	$Z = j\omega L$
C	$V = \frac{1}{j\omega C}I$	$Z = \frac{1}{j\omega C}$

Impedance is NOT a phasor but a complex number that can be written in polar or Cartesian form. In general its value depends on the frequency

KVL AND KCL HOLD FOR PHASOR REPRESENTATIONS





KVL: $v_1(t)+v_2(t)+v_3(t)=0$

KCL: $-i_0(t)+i_1(t)+i_2(t)+i_3(t)=0$

$$v_i(t)=V_{Mi}e^{j(\omega t+\theta_i)},\ i=1,2,3$$

$$i_k(t)=I_{Mk}e^{j(\omega t+\phi_k)},\ k=0,1,2,3$$

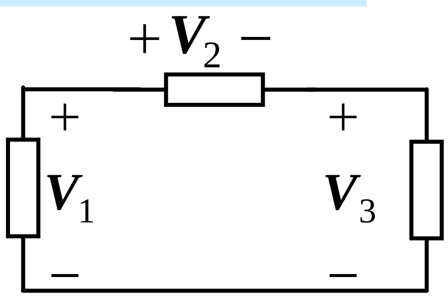
KVL: $(V_{M1}e^{j\theta_1}+V_{M2}e^{j\theta_2}+V_{M3}e^{j\theta_3})e^{j\omega t}=0$

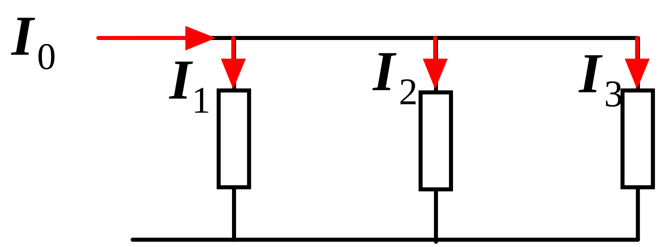
In a similar way, one shows ...

$$V_{M1}\angle\theta_1+V_{M2}\angle\theta_2+V_{M3}\angle\theta_3=0$$

$V_1+V_2+V_3=0$ Phasors!

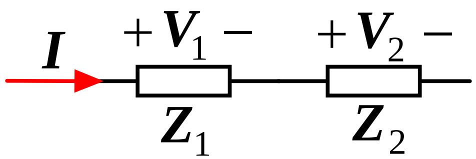
$$-I_0+I_1+I_2+I_3=0$$

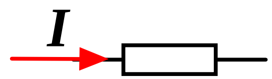




The components will be represented by their impedances and the relationships will be entirely algebraic!!

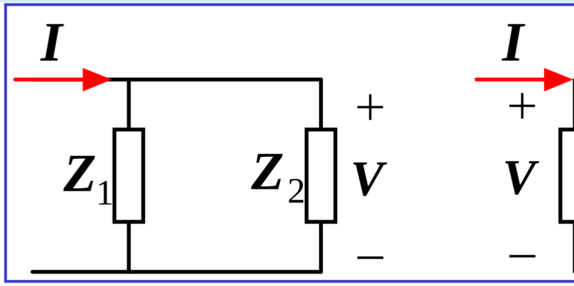
SPECIAL APPLICATION:
IMPEDANCES CAN BE COMBINED USING THE SAME RULES DEVELOPED FOR RESISTORS





$$Z_s = Z_1 + Z_2$$

$$Z_s = \sum_k Z_k$$



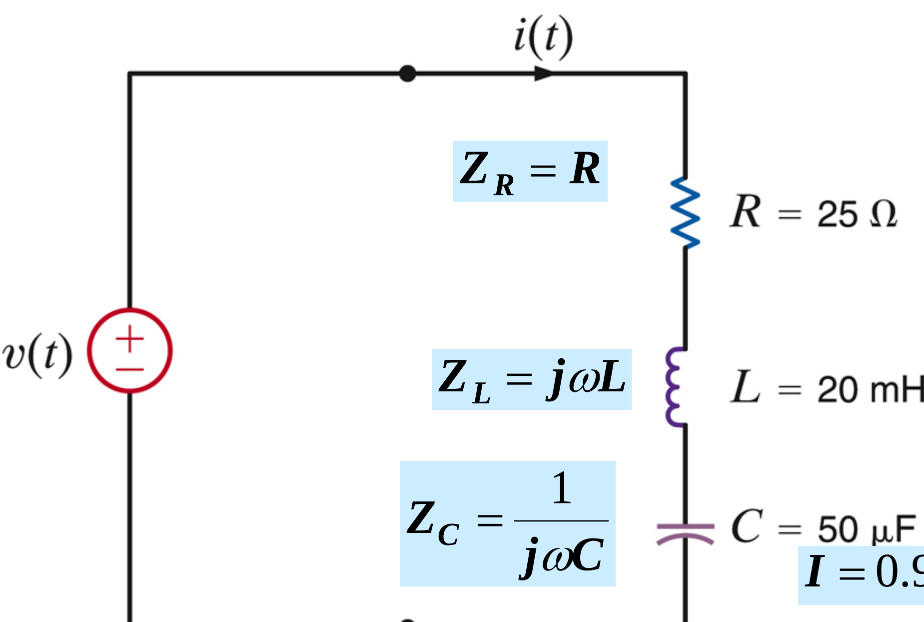
$$Z_p = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

$$\frac{1}{Z_p} = \sum_k \frac{1}{Z_k}$$

LEARNING EXAMPLE

$f = 60Hz, v(t) = 50\cos(\omega t + 30°)$

Compute equivalent impedance and current



$$Z_R = R$$

$$Z_L = j\omega L$$

$$Z_C = \frac{1}{j\omega C}$$

$$\omega = 120\pi, V = 50\angle 30°, Z_R = 25\Omega$$

$$Z_L = j120\pi \times 20 \times 10^{-3}\Omega, \; Z_C = \frac{1}{j120\pi \times 50 \times 10^{-6}}$$

$$Z_L = j7.54\Omega, \; Z_C = -j53.05\Omega$$

$$Z_s = Z_R + Z_L + Z_C = 25 - j45.51\Omega$$

$$I = \frac{V}{Z_s} = \frac{50\angle 30°}{25 - j45.51}(A) = \frac{50\angle 30°}{51.93\angle -61.22°}(A)$$

$$I = 0.96\angle 91.22°(A) \Rightarrow i(t) = 0.96\cos(120\pi t + 91.22°)(A)$$

(COMPLEX) ADMITTANCE

$$Y = \frac{1}{Z} = G + jB \text{ (Siemens)}$$

G = conductanc e

B = Suceptanc e

$$\frac{1}{Z} = \frac{1}{R + jX} \times \frac{R - jX}{R - jX} = \frac{R - jX}{R^2 + X^2}$$

$$G = \frac{R}{R^2 + X^2}$$

$$B = \frac{-X}{R^2 + X^2}$$

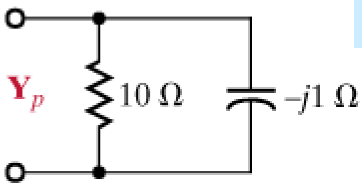
Element	Phasor Eq.	Impedance	Admittance
R	$V = RI$	$Z = R$	$Y = \frac{1}{R} = G$
L	$V = j\omega LI$	$Z = j\omega L$	$Y = \frac{1}{j\omega L}$
C	$V = \frac{1}{j\omega C} I$	$Z = \frac{1}{j\omega C}$	$Y = j\omega C$

Parallel Combination of Admittances

$$Y_p = \sum_k Y_k$$

$$Y_R = 0.1S$$

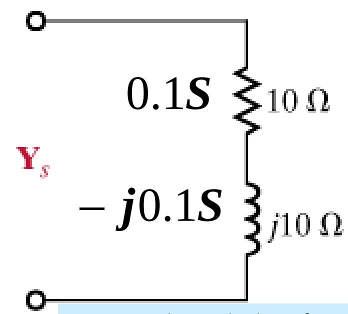
$$Y_C = \frac{1}{-j1} = j1(S)$$



$$Y_p = 0.1 + j1(S)$$

Series Combination of Admittances

$$\frac{1}{Y_s} = \sum_k \frac{1}{Y_k}$$



$$\frac{1}{Y_s} = \frac{1}{0.1} + \frac{1}{-j0.1}$$

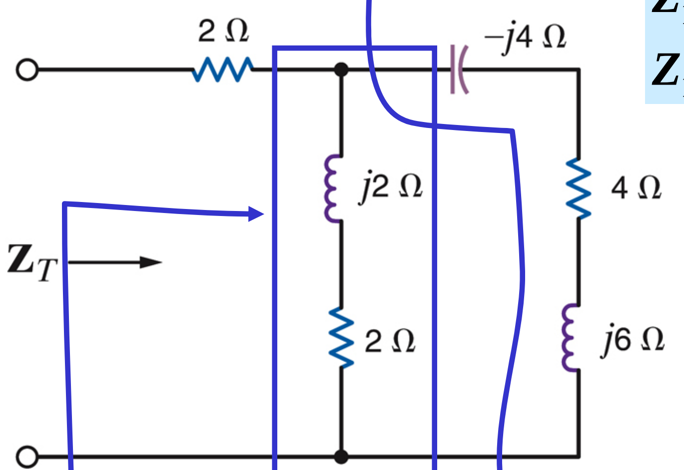
$$= 10 + j10$$

$$Y_s = \frac{(0.1)(-j0.1)}{0.1 - j0.1} \times \frac{0.1 + j0.1}{0.1 + j0.1}$$

$$Y_s = \frac{1}{10 + j10} = \frac{10 - j10}{200}$$

$$Y_s = 0.05 - j0.05S$$

FIND THE IMPEDANCE Z_T



$$Z_1 = 4 + j6 - j4$$

$$Z_1 = 4 + j2$$

$$(R \to P) Z_1 = 4.472\angle 26.565^\circ$$

$$Y_1 = 0.224\angle -26.565^\circ$$

$$(P \to R) Y_1 = 0.200 - j0.100$$

$$Z_2 = 2 + j2$$

$$(R \to P) Z_2 = 2.828\angle 45^\circ$$

$$Y_2 = 0.354\angle -45^\circ$$

$$(P \to R) Y_2 = 0.250 - j0.250$$

$$Y_{12} = Y_1 + Y_2$$

$$Z_{12} = \frac{1}{Y_{12}}$$

$$Y_{12} = Y_1 + Y_2 = 0.45 - j0.35$$

$$(R \to P) Y_{12} = 0.570\angle -37.875^\circ$$

$$Z_{12} = 1.754\angle 37.875^\circ$$

$$(P \to R) Z_{12} = 1.384 + j1.077$$

$$Z_T = 2 + (1.384 + j1077) = 3.383 + j1.077$$

$$Y_1 = \frac{1}{4 + j2} = \frac{4 - j2}{(4)^2 + (2)^2}$$

$$Y_2 = \frac{1}{2 + j2} = \frac{2 - j2}{(2)^2 + (2)^2}$$

$$Z_{12} = \frac{1}{Y_{12}} = \frac{1}{0.45 - j0.35} = \frac{0.45 + j0.35}{0.325}$$

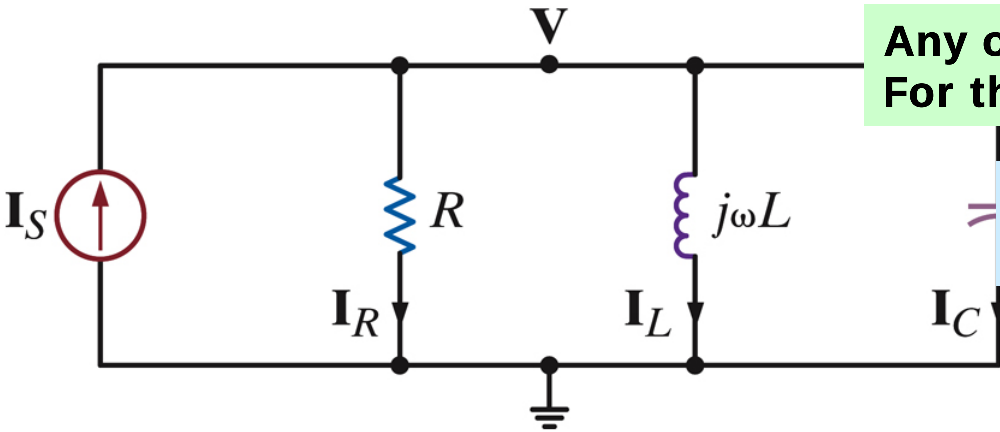
PHASOR DIAGRAMS

Display all relevant phasors on a common reference frame

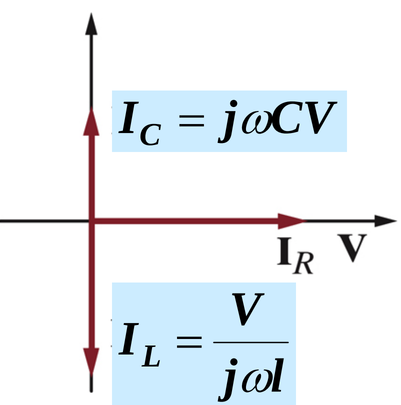
Very useful to visualize phase relationships among variables. Especially if some variable, like the frequency, can change

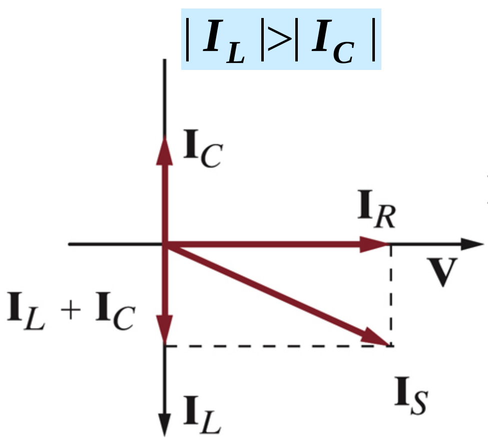
SKETCH THE PHASOR DIAGRAM FOR THE CIRCUIT

Any one variable can be chosen as reference. For this case select the voltage V

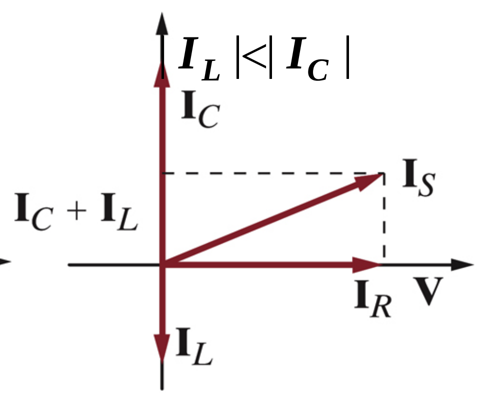


$$\text{KCL}: \; I_S = \frac{V}{R} + \frac{V}{j\omega L} + j\omega CV$$

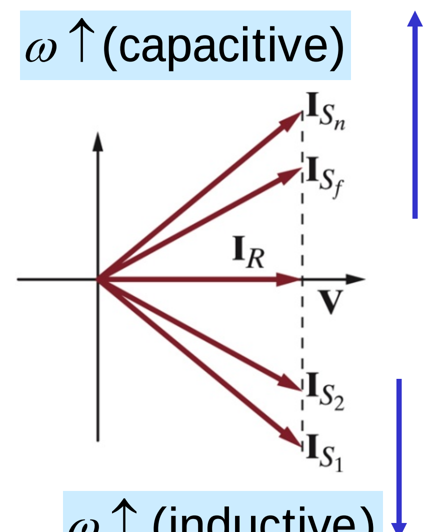




INDUCTIVE CASE



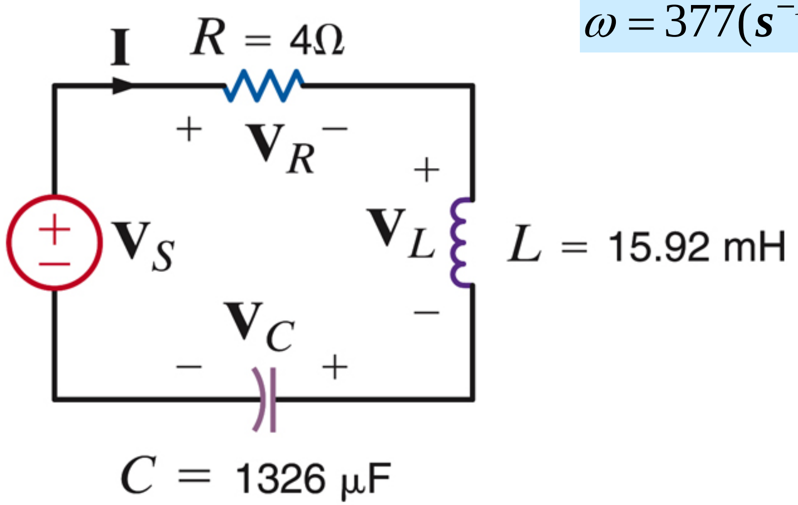
CAPACITIVE CASE



LEARNING EXAMPLE

DO THE PHASOR DIAGRAM FOR THE CIRCUIT

$\omega = 377(s^{-1})$



$V_R = RI$

$V_L = j\omega L I$

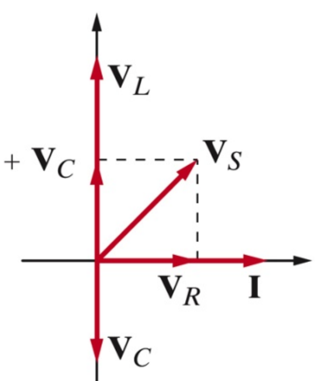
$V_C = \frac{1}{j\omega C} I$

$V_S = V_R + V_L + V_C$

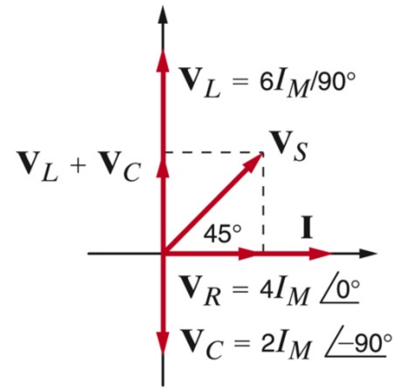
It is convenient to select the current as reference

1. DRAW ALL THE PHASORS

$|V_L| > |V_C|$



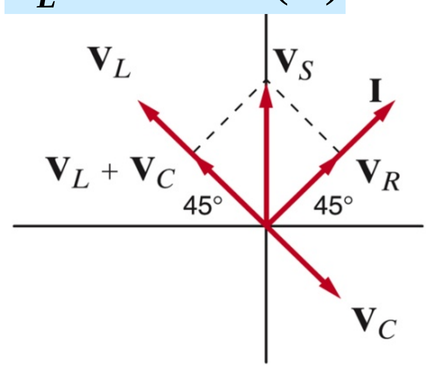
2. PUT KNOWN NUMERICAL VALUES



$|V_L - V_C| = |V_R|$

DIAGRAM WITH REFERENCE $V_S = 12\sqrt{2}\angle 90°$

$V_L = 18\angle 135°(V)$



Read values from diagram!

$\therefore I = 3\angle 45°(A)$

$V_R = 12\angle 45°(V)$

(Pythagoras)

$V_C = 6\angle -45°$

BASIC ANALYSIS USING KIRCHHOFF'S LAWS

PROBLEM SOLVING STRATEGY

For relatively simple circuits use

Ohm's law for AC analysis; i.e., $\mathbf{V} = \mathbf{I}\mathbf{Z}$

The rules for combining $\mathbf{Z}$ and $\mathbf{Y}$

KCL AND KVL

Current and voltage divider

For more complex circuits use

Node analysis

Loop analysis

Superposition

Thevenin's and Norton's theorems

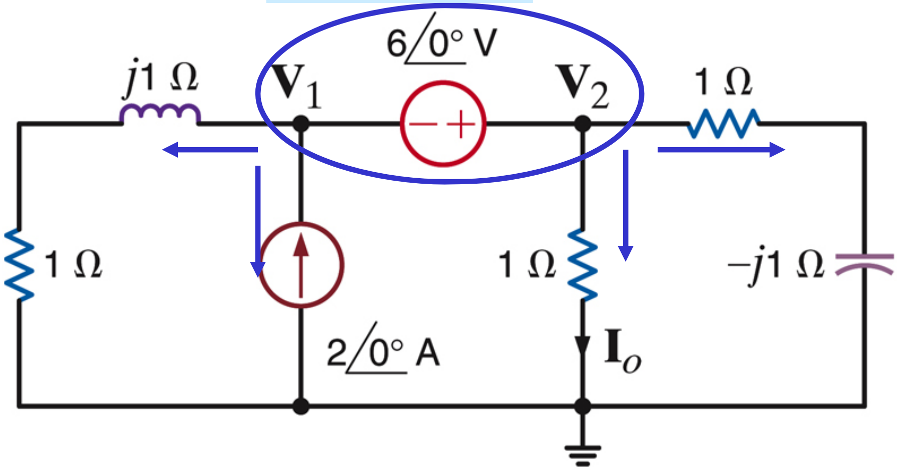
MATLAB

PSPICE

ANALYSIS TECHNIQUES

PURPOSE: TO REVIEW ALL CIRCUIT ANALYSIS TOOLS DEVELOPED FOR RESISTIVE CIRCUITS; I.E., NODE AND LOOP ANALYSIS, SOURCE SUPERPOSITION, SOURCE TRANSFORMATION, THEVENIN'S AND NORTON'S THEOREMS.

COMPUTE I_0



1. NODE ANALYSIS

$$\frac{V_1}{1+j1} - 2\angle 0° + \frac{V_2}{1} + \frac{V_2}{1-j1} = 0$$

$$V_1 - V_2 = -6\angle 0°$$

$$I_0 = \frac{V_2}{1}(A)$$

$$\frac{V_2 - 6\angle 0°}{1+j1} - 2\angle 0° + V_2 + \frac{V_2}{1-j1} = 0$$

$$V_2\left[\frac{1}{1+j1} + 1 + \frac{1}{1-j1}\right] = 2 + \frac{6}{1+j1}$$

$$V_2\frac{(1-j1)+(1+j1)(1-j1)+(1+j1)}{(1+j1)(1-j1)} = \frac{2(1+j1)+6}{1+j1}$$

$$V_2\frac{4}{1-j} = 8 + j2$$

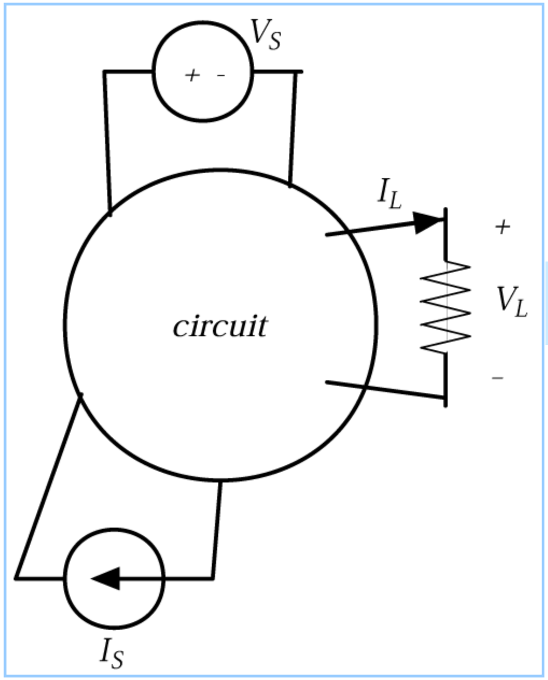
$$V_2 = \frac{(4+j)(1-j)}{2}$$

$$I_0 = \left(\frac{5}{2} - j\frac{3}{2}\right)(A)$$

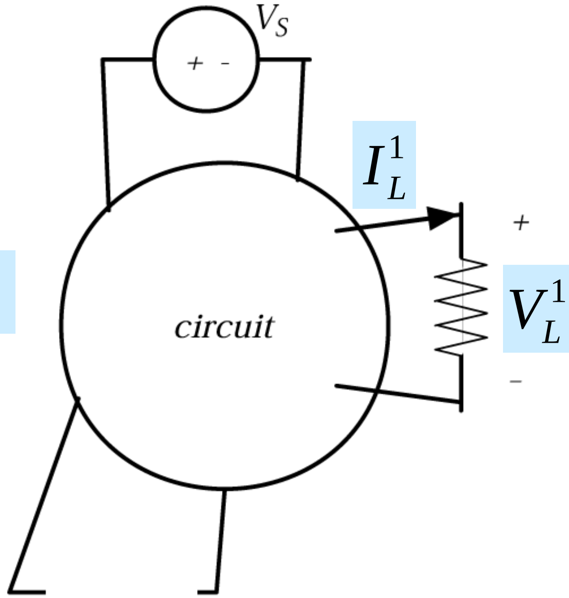
$$I_0 = 2.92\angle -30.96°$$

SOURCE SUPERPOSITION

Circuit with voltage source set to zero (SHORT CIRCUITED)

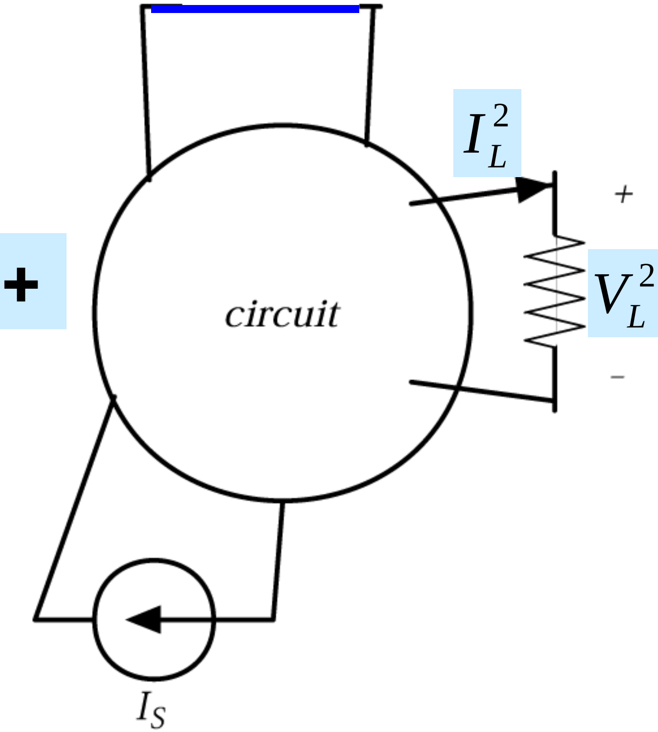


=



Circuit with current source set to zero(OPEN)

+



Due to the linearity of the models we must have

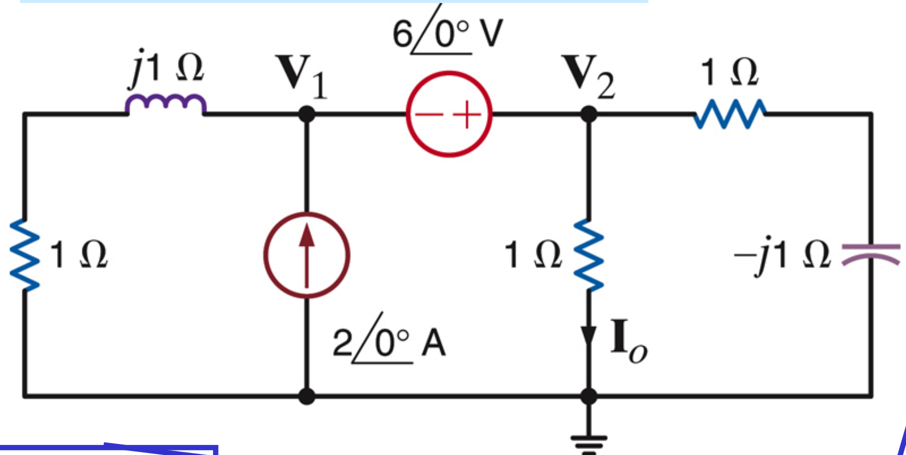
$$I_L = I_L^1 + I_L^2 \qquad V_L = V_L^1 + V_L^2$$

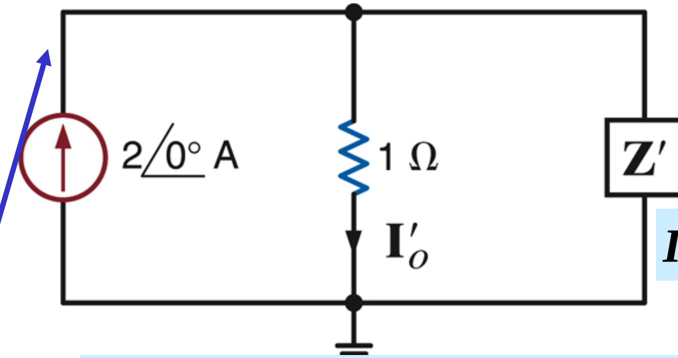
Principle of Source Superposition

The approach will be useful if solving the two circuits is simpler, or more convenient, than solving a circuit with two sources

We can have any combination of sources. And we can partition any way we find convenient

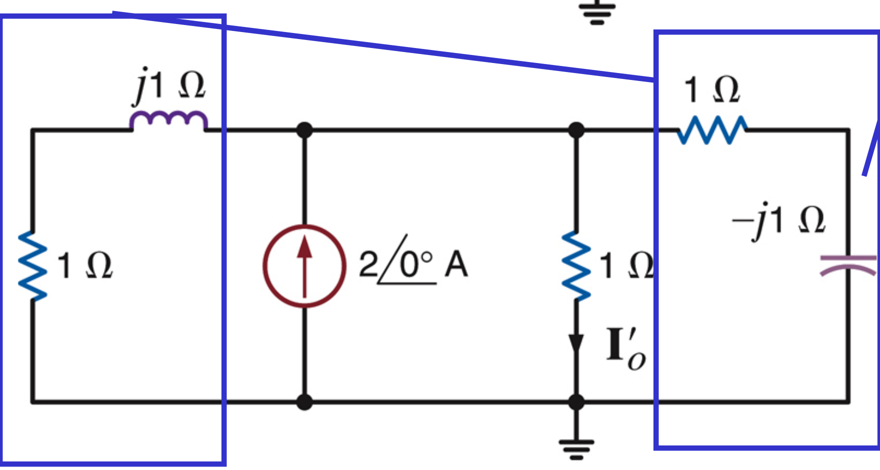
3. SOURCE SUPERPOSITION





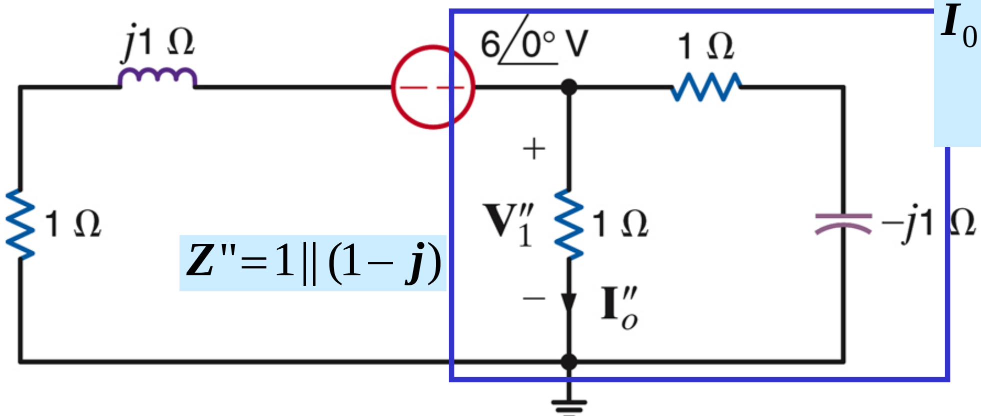
$I_0^{'} = 1\angle 0°(A)$

$$Z' = (1+j)\,||\,(1-j) = \frac{(1+j)(1-j)}{(1+j)-(1-j)} = 1$$



COULD USE SOURCE TRANSFORMATION TO COMPUTE $I_0^{''}$

$$V_1^{''} = \frac{Z^{''}}{Z^{''}+1+j}6\angle 0°(V) \qquad I_0^{''} = \frac{Z^{''}}{Z^{''}+1+j}6\angle 0°(A)$$



$Z'' = 1\,||\,(1-j)$

Z''

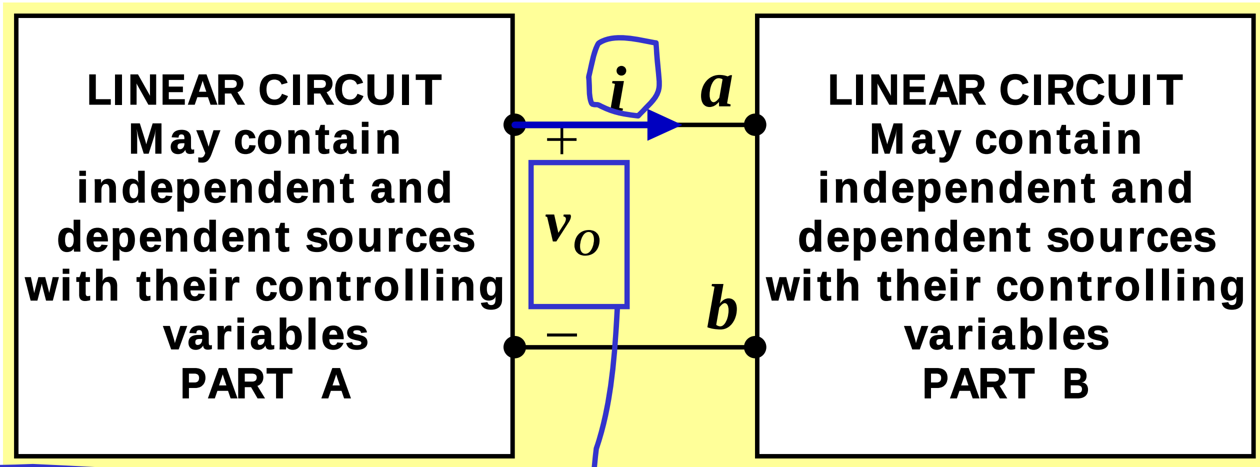
$$I_0^{''} = \frac{\frac{1-j}{2-j}}{\frac{1-j}{2-j}+1+j}6(A)$$

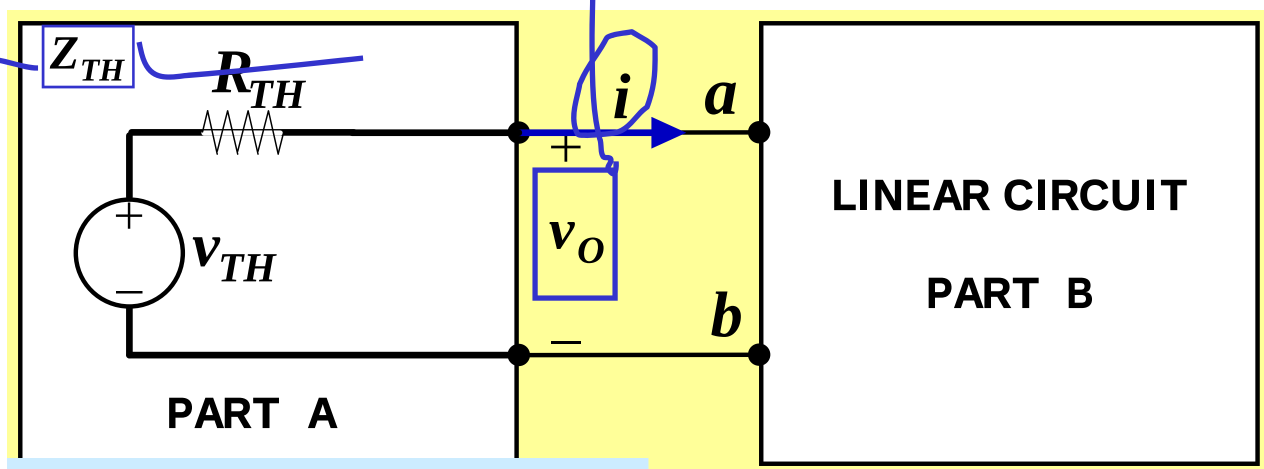
$$I_0^{''} = \frac{1-j}{(1-j)+3+j}6$$

$$I_0^{''} = \frac{6}{4} - \frac{6}{4}j(A)$$

$$I_0 = I_0^{'} + I_0^{''} = \left(\frac{5}{2} - \frac{3}{2}j\right)(A)$$

THEVENIN'S EQUIVALENCE THEOREM





Thevenin Equivalent Circuit for PART A

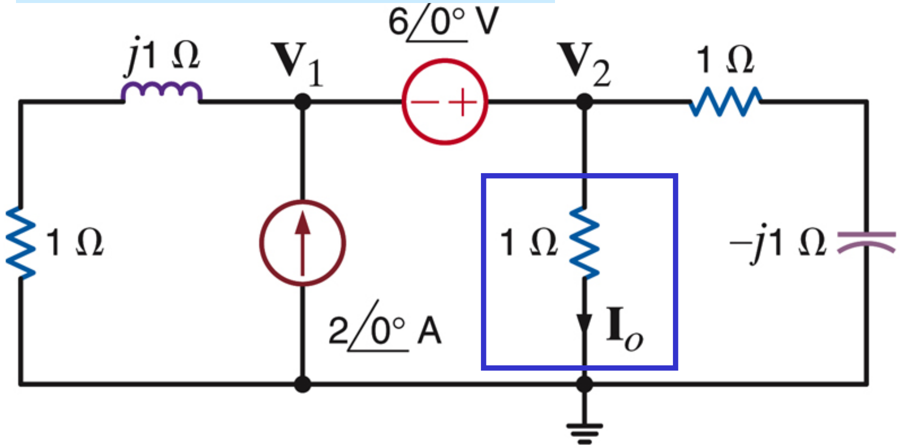
Phasor

v_{TH} Thevenin Equivalent Source

R_{TH} Thevenin Equivalent Resistance

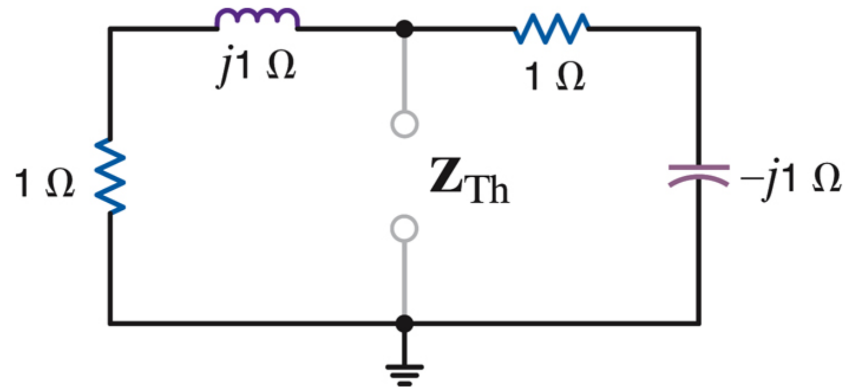
Impedance

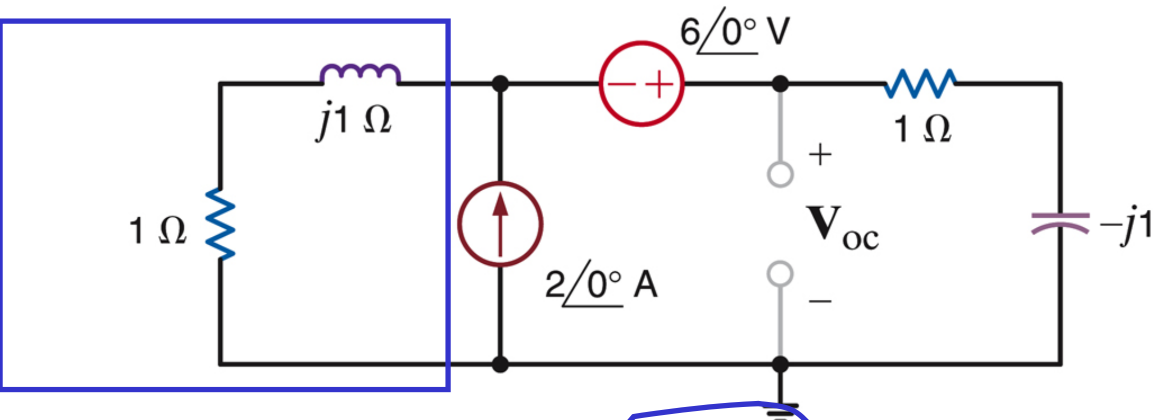
5. THEVENIN ANALYSIS



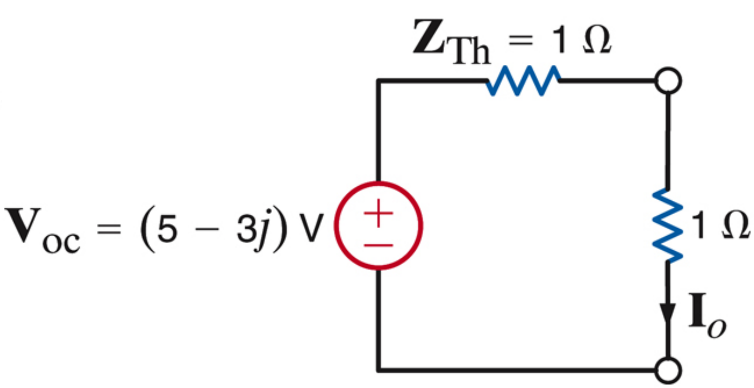
Voltage Divider

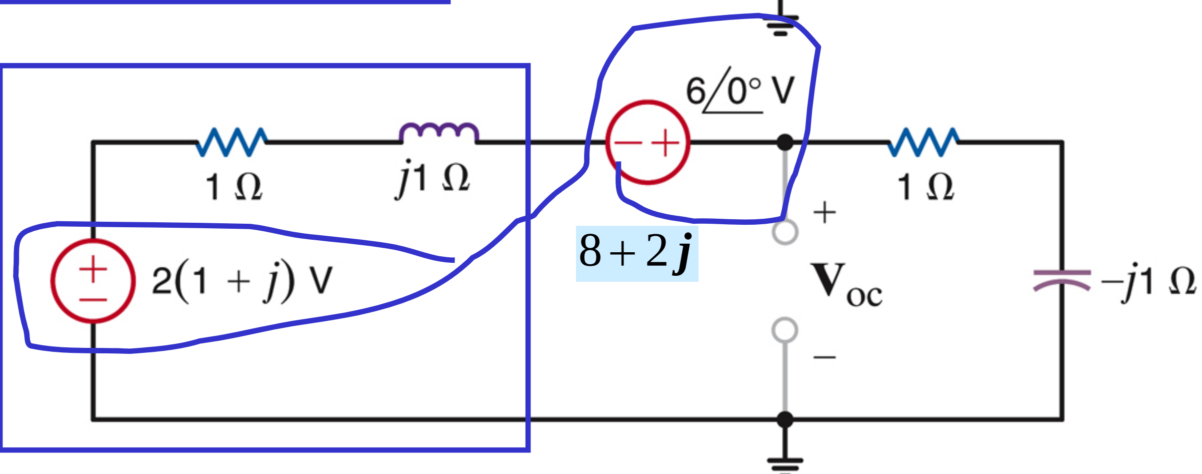
$$V_{OC} = \frac{1-j}{(1+j)+(1-j)}(8+2j) = \frac{10-6j}{2}$$





$$Z_{TH} = (1+j)\,\|\,(1-j) = 1\Omega$$

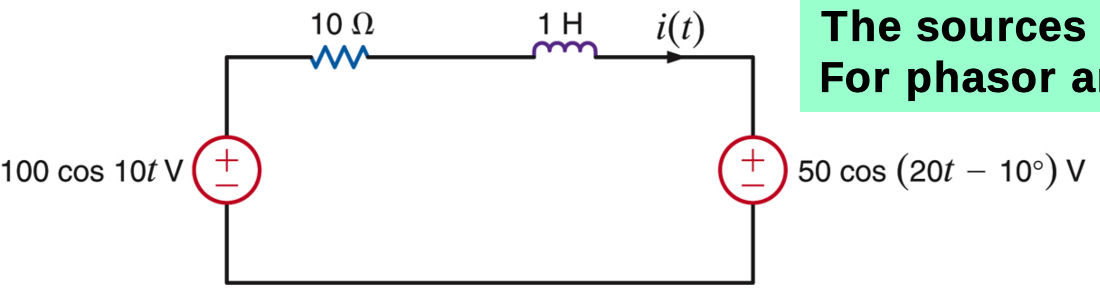




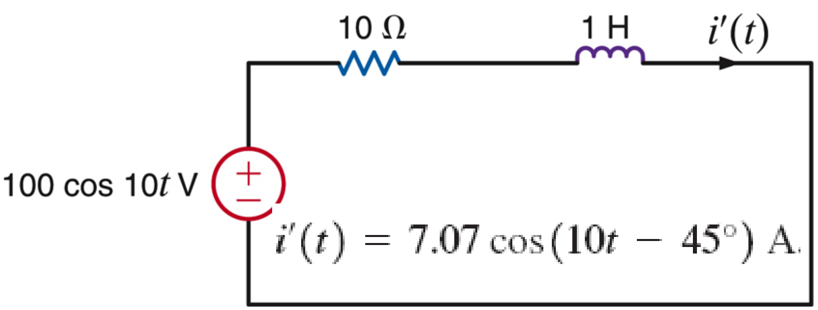
$$I_0 = \frac{5-3j}{2}(A)$$

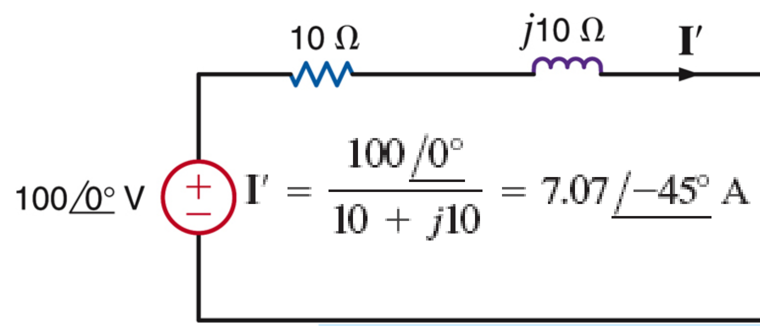
EXAMPLE

Find the current i(t) in steady state

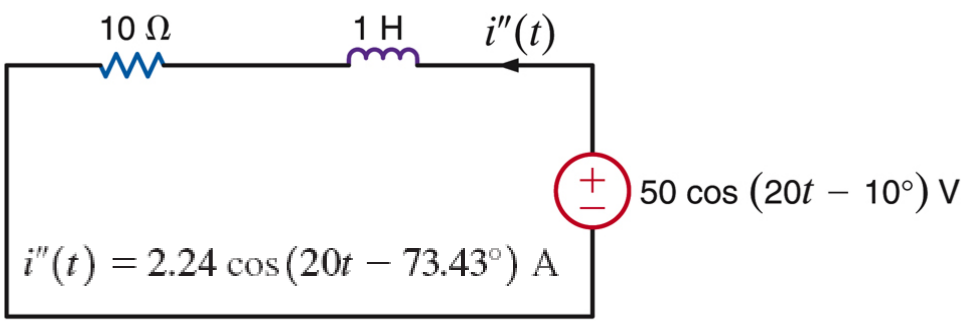


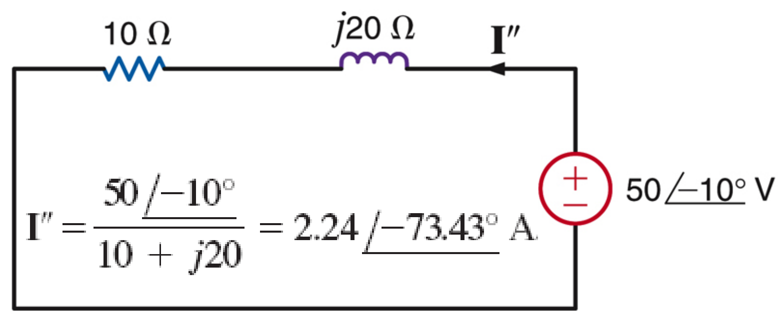
The sources have different frequencies!
For phasor analysis MUST use source superpositio





Frequency domain





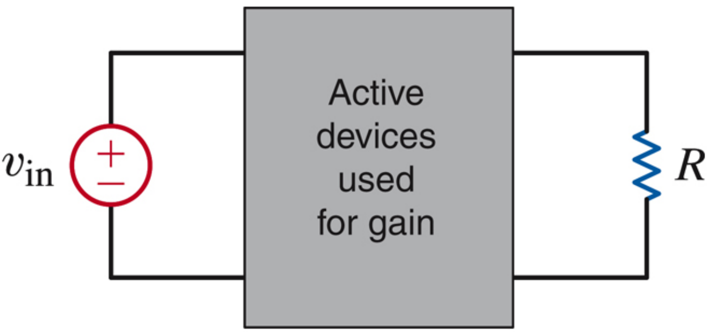
SOURCE 2: FREQUENCY 20r/s

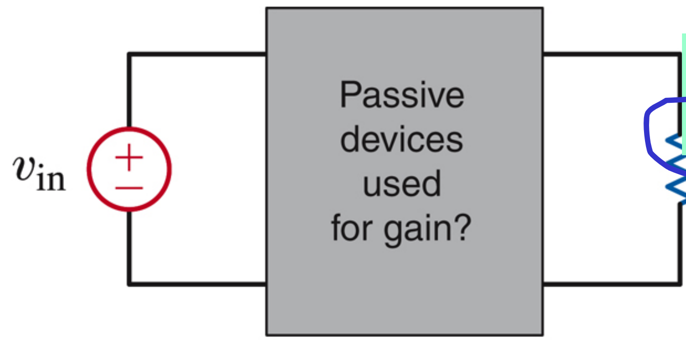
$$i'(t) - i''(t) = 7.07\cos(10t - 45^\circ) - 2.24\cos(20t - 73.43^\circ)\ \text{A}$$

Principle of superposition

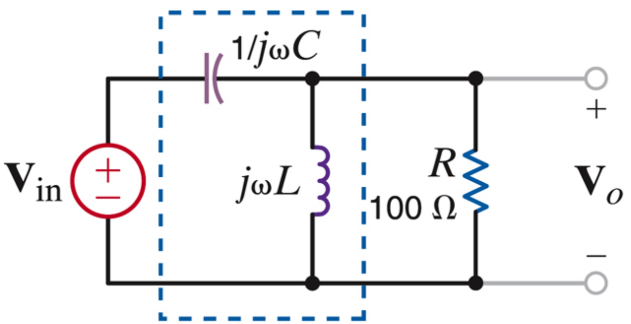
LEARNING BY DESIGN

USING PASSIVE COMPONENTS TO CREATE GAINS LARGER THAN ONE





PRODUCE A GAIN=10 AT 1KhZ WHEN R=100



$$\frac{\mathbf{V}_o}{\mathbf{V}_{in}} = \left[\frac{\mathbf{Z}}{\mathbf{Z} + \frac{1}{j\omega C}} \right]$$

$$\frac{\mathbf{V}_o}{\mathbf{V}_{in}} = \left[\frac{j\omega L}{j\left[\omega L - \frac{1}{\omega C}\right] + \frac{L}{CR}} \right]$$

$$\mathbf{Z} = \frac{(j\omega L)R}{j\omega L + R}$$

$$\omega^2 LC = 1 \Rightarrow \frac{\mathbf{V}_o}{\mathbf{V}_{in}} = j\omega RC$$

$$\Rightarrow C = 15.9\mu F$$

$$\Rightarrow L = 1.59mH$$