CAPACITANCE AND INDUCTANCE

Introduces two passive, energy storing devices: Capacitors and Inductors

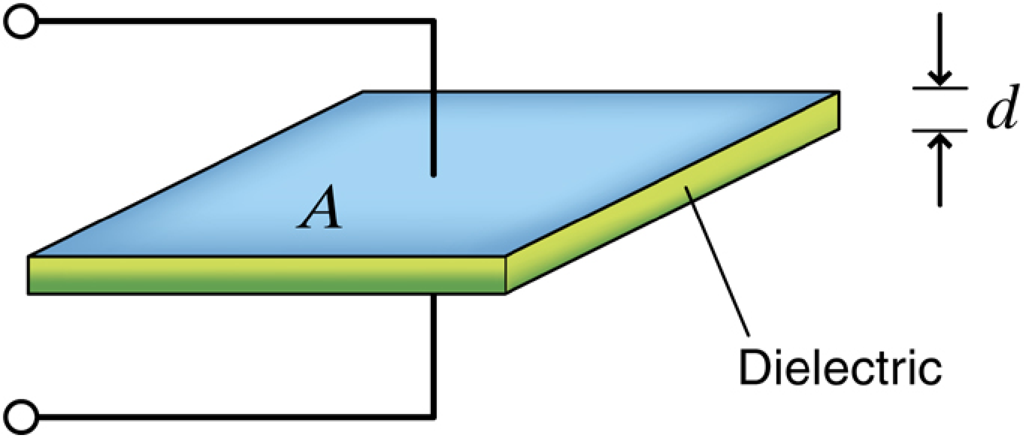
CAPACITORS
Store energy in their electric field (electrostatic energy)
Model as circuit element

INDUCTORS
Store energy in their magnetic field
Model as circuit element

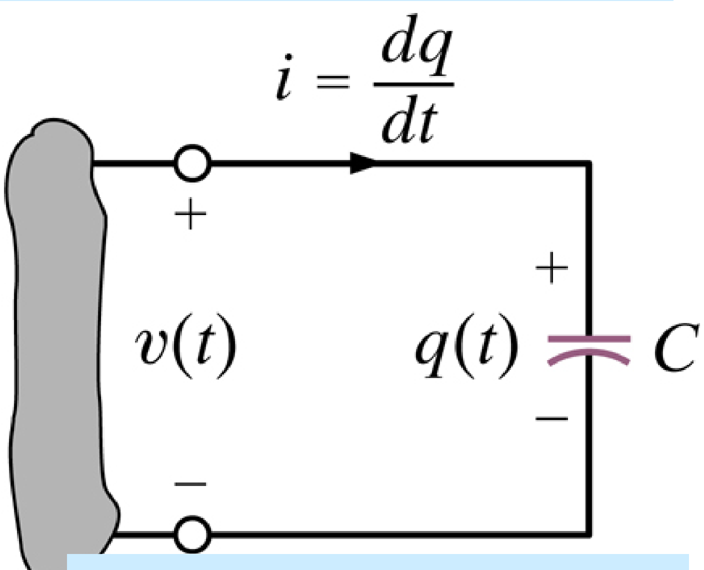
CAPACITOR AND INDUCTOR COMBINATIONS
Series/parallel combinations of elements

CAPACITORS

First of the energy storage devices to be discussed



Basic parallel-plates capacitor

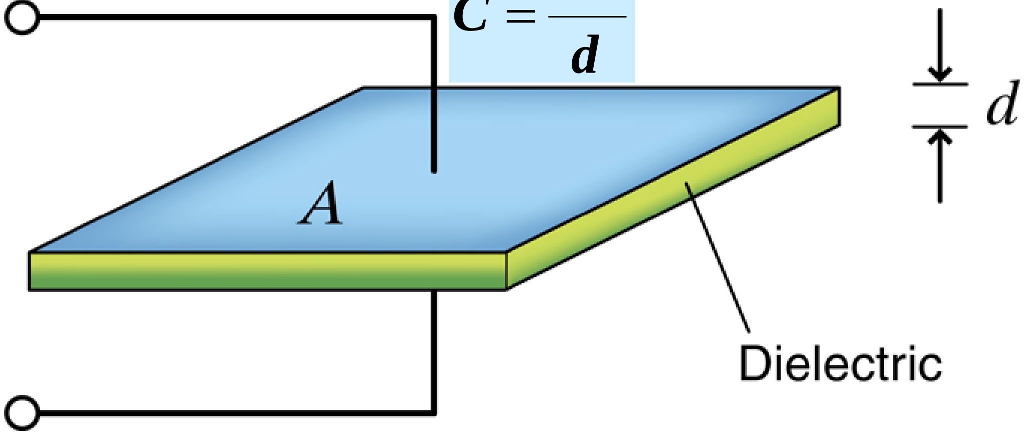


CIRCUIT REPRESENTATION

NOTICE USE OF PASSIVE SIGN CONVENTION

Typical Capacitors

$$C = \frac{\varepsilon A}{d}$$



ε Dielectric constant of material in gap

PLATE SIZE FOR EQUIVALENT AIR-GAP CAPACITOR

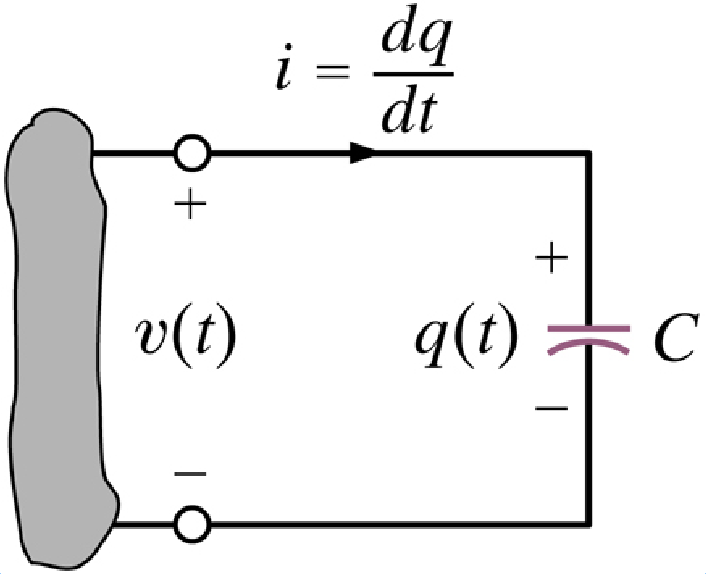
$$55F = \frac{8.85 \times 10^{-12} A}{1.016 \times 10^{-4}} \Rightarrow A = 6.3141 \times 10^{8} m^{2}$$

Normal values of capacitance are small.
Microfarads is common.
For integrated circuits nano or pico farads are not unusual

Basic capacitance law $Q = f(V_C)$

Linear capacitors obey Coulomb's law $Q = CV_C$

C is called the CAPACITANCE of the device and has units of $\frac{\text{charge}}{\text{voltage}}$



Linear capacitor circuit representation

One Farad(F)is the capacitance of a device that can store one Coulomb of charge at one Volt.

$$\text{Farad} = \frac{\text{Coulomb}}{\text{Volt}}$$

EXAMPLE **Voltage across a capacitor of 2 micro Farads holding 10mC of charge**

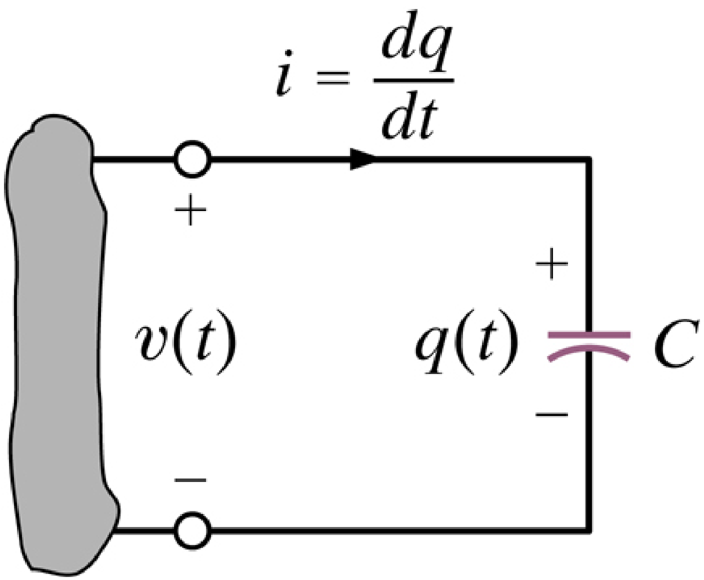
$$V_C = \frac{1}{C}Q = \frac{1}{2*10^{-6}}10*10^{-3} = 5000 \text{ V}$$

Capacitance in Farads, charge in Coulombs result in voltage in Volts

Capacitors can be dangerous!!!

Capacitors only store and release ELECTROSTATIC energy. They do not "create"

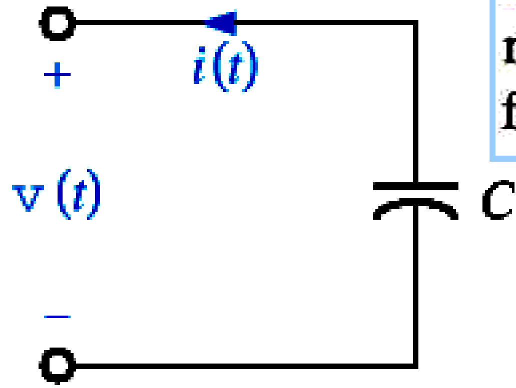
The capacitor is a passive element and follows the passive sign convention



Linear capacitor circuit representation

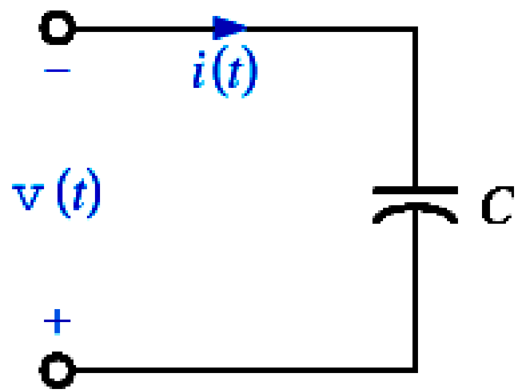
$$i(t) = C\frac{dv}{dt}(t)$$

Write the i-v relationship for the following capacitors.



(a)

$$i(t) = -C\frac{dv(t)}{dt}$$



(b)

$$i(t) = -C\frac{dv(t)}{dt}$$

$$Q_C = CV_C$$

Capacitance Law

If the voltage varies the charge varies and there is a displacement current

One can also express the voltage across in terms of the current

... Or one can express the current through in terms of the voltage across

$$V_C(t) = \frac{1}{C}Q = \frac{1}{C}\int_{-\infty}^{t} i_C(x)dx$$

Integral form of Capacitance law

$$i_C = \frac{dQ}{dt} = C\frac{dV_C}{dt}$$

Differential form of Capacitance law

The mathematical implication of the integral form is ...

$$V_C(t-) = V_C(t+);\ \forall t$$

Voltage across a capacitor MUST be continuous

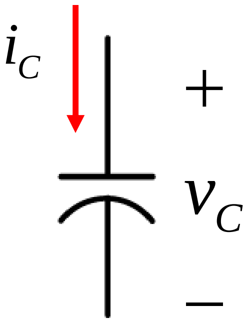
Implications of differential form??

$$V_C = Const \Rightarrow i_C = 0$$

DC or steady state behavior

A capacitor in steady state acts as an OPEN CIRCUIT

CAPACITOR AS CIRCUIT ELEMENT



$$i_C(t) = C\frac{dv_c}{dt}(t)$$

$$v_C(t) = \frac{1}{C}\int_{-\infty}^{t} i_C(x)dx$$

$$\int_{-\infty}^{t} = \int_{-\infty}^{t_0} + \int_{t_0}^{t}$$

$$i_R = \frac{1}{R}v_R$$

$$v_R = Ri_R$$

Ohm's Law

$$\boldsymbol{v_c(t_O)}$$

$$\boldsymbol{v_C(t) = \frac{1}{C}\int_{-\infty}^{t_0} i_C(x)dx + \frac{1}{C}\int_{t_0}^{t} i_C(x)dx}$$

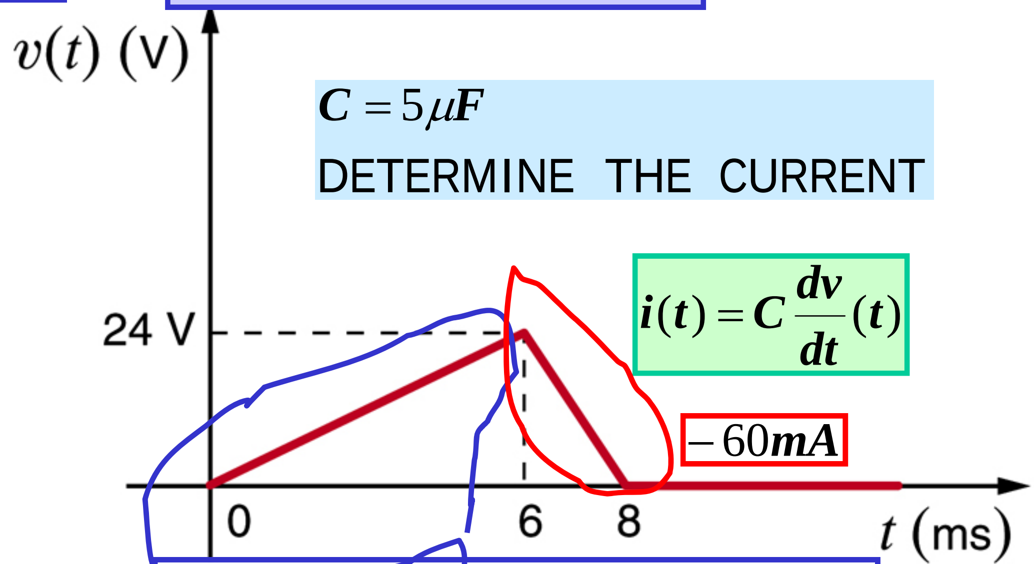
$$v_C(t) = v_C(t_0) + \frac{1}{C}\int_{t_0}^{t} i_C(x)dx$$

The fact that the voltage is defined through an integral has important implications...

LEARNING EXAMPLE

$C = 5\mu F$

DETERMINE THE CURRENT

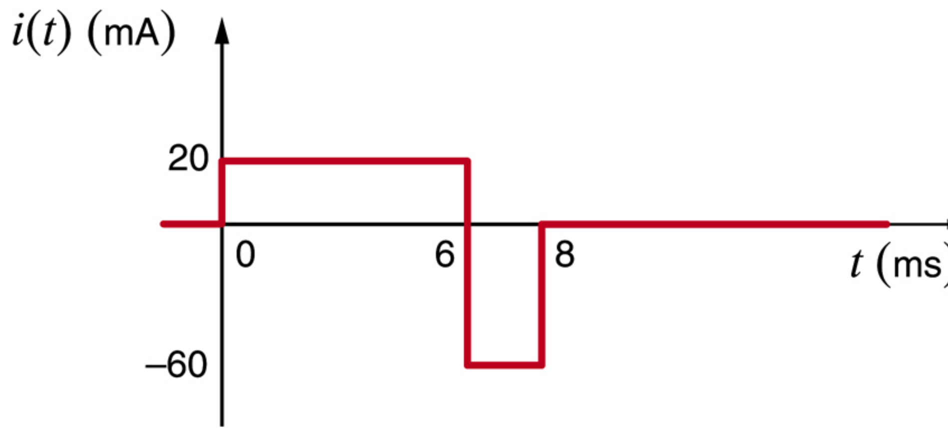


$$\boldsymbol{i(t) = C\frac{dv}{dt}(t)}$$

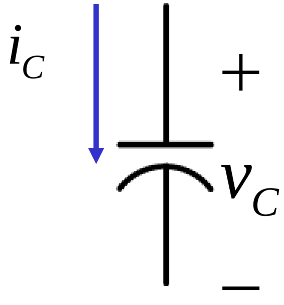
$$-60\boldsymbol{mA}$$

$$\boldsymbol{i} = 5\times10^{-6}[\boldsymbol{F}]\times\frac{24}{6\times10^{-3}}\left[\frac{\boldsymbol{V}}{\boldsymbol{s}}\right] = 20\boldsymbol{mA}$$

$\boldsymbol{i(t)} = 0$ ***elsewhere***



CAPACITOR AS ENERGY STORAGE DEVICE



Instantaneous power

$$p_C(t) = v_C(t) i_C(t) \quad \text{W}$$

$$i_C(t) = C \frac{dv_c}{dt}(t)$$

$$p_C(t) = C v_C(t) \frac{dv_c}{dt}$$

$$\boldsymbol{p_C(t) = C \frac{d}{dt}\left(\frac{1}{2} v_C^2(t)\right)}$$

$$v_C(t) = \frac{1}{C}\int_{-\infty}^{t} i_C(x)dx = \frac{q_C(t)}{C}$$

$$p_C(t) = \frac{1}{C} q_C(t) \frac{dq_C}{dt}(t)$$

$$\boldsymbol{p_C(t) = \frac{1}{C}\frac{d}{dt}\left(\frac{1}{2} q_c^2(t)\right)}$$

Energy is the integral of power

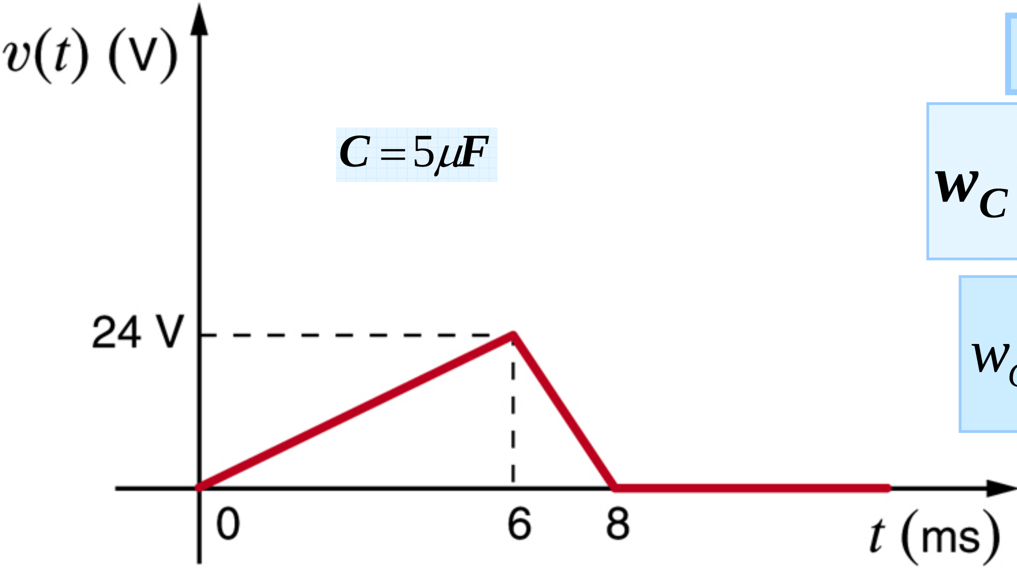
$$\boldsymbol{w_C(t_2, t_1) = \int_{t_1}^{t_2} p_C(x)dx}$$

If t1 is minus infinity we talk about "energy stored at time t2."

If both limits are infinity then we talk about the "total energy stored."

$$\boldsymbol{w_C(t_2, t_1) = \frac{1}{2} C v_C^2(t_2) - \frac{1}{2} C v_C^2(t_1)}$$

$$\boldsymbol{w_C(t_2, t_1) = \frac{1}{C} q_C^2(t_2) - \frac{1}{C} q_C^2(t_1)}$$



EXAMPLE

Energy stored in 0 - 6 msec

$$w_C(0,6) = \frac{1}{2}Cv_C^2(6) - \frac{1}{2}Cv_C^2(0)$$

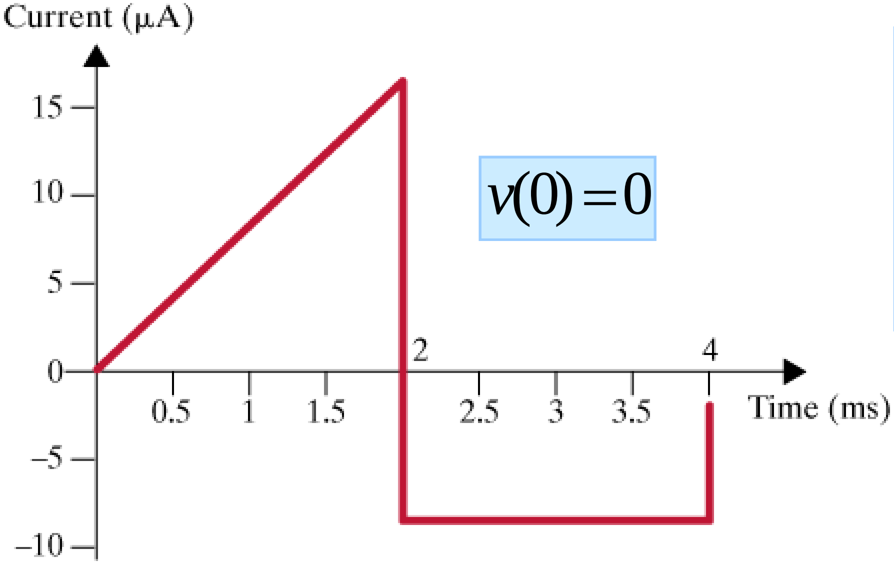
$$w_C(0,6) = \frac{1}{2}5*10^{-6}[F]*(6)^2[V^2]$$

Charge stored at 3msec

$$q_C(3) = Cv_C(3)$$

$$q_C(3) = 5*10^{-6}[F]*12[V] = 60\mu C$$

$C = 4\mu F$. FIND THE VOLTAGE



$v(0) = 0$

$$i(t) = \frac{16 \times 10^{-6} t}{2 \times 10^{-3}} \quad 0 \le t \le 2 \text{ ms}$$
$$= -8 \times 10^{-6} \quad 2 \text{ ms} \le t \le 4 \text{ ms}$$
$$= 0 \quad 4 \text{ ms} < t$$

$$v(t) = v(0) + \frac{1}{C}\int_0^t i(x)dx; \; t > 0$$

$$v(t) = \frac{1}{(4)(10^{-6})}\int_0^t 8(10^{-3})x\,dx = 10^3 t^2$$

$0 \le t \le 2$

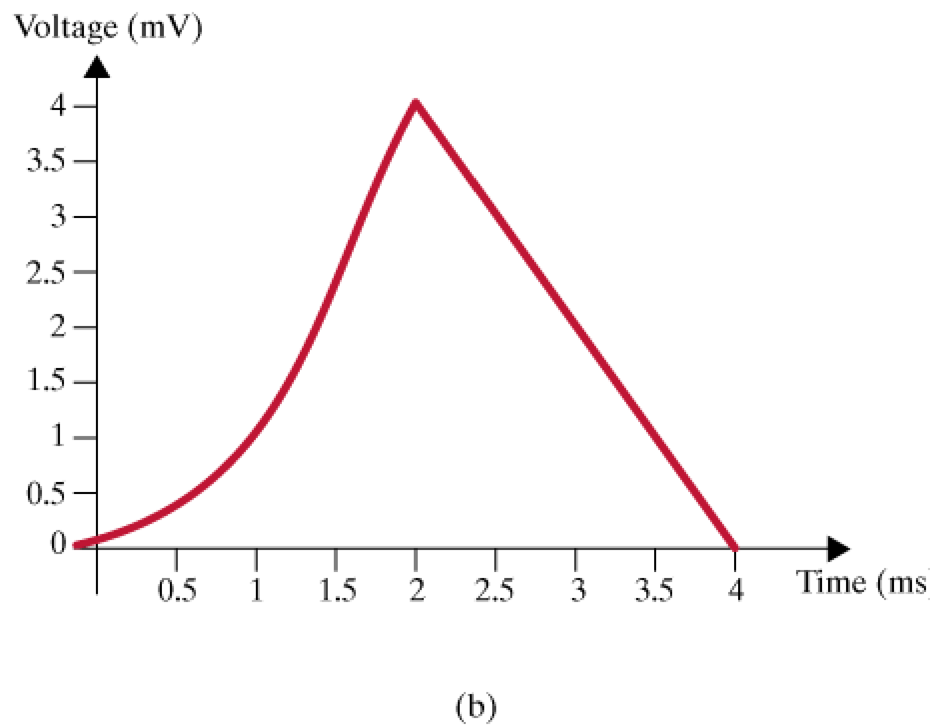
$$v(t) = v(2) + \frac{1}{C}\int_2^t i(x)dx; \; t > 2$$

$$v(2 \text{ ms}) = 10^3(2 \times 10^{-3})^2 = 4 \text{ mV}$$

$$v(t) = \frac{1}{(4)(10^{-6})}\int_{2(10^{-3})}^t -(8)(10^{-6})\,dx + (4)(10^{-3})$$

$2 < t \le 4ms$

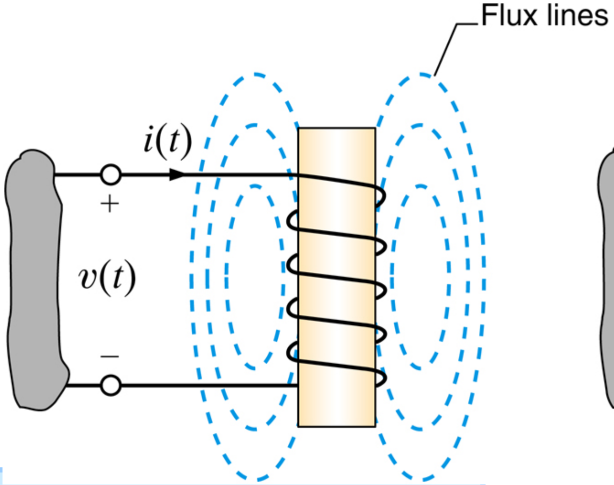
$$v(t) = -2t + 8 \times 10^{-3} [V]$$

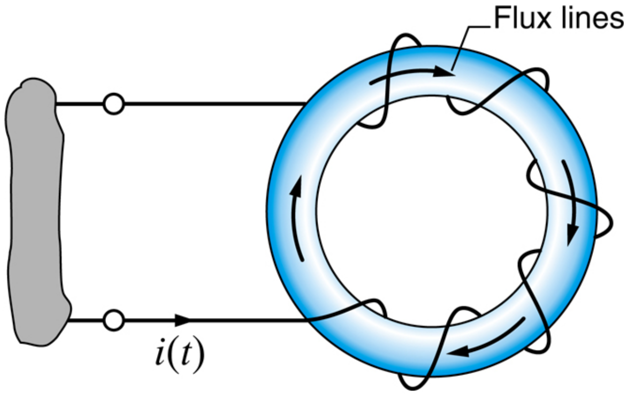


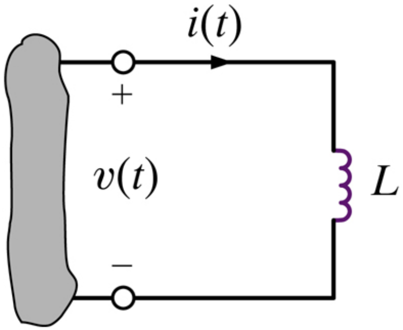
(b)

INDUCTORS

NOTICE USE OF PASSIVE SIGN CONVENTION







Flux lines may extend beyond inductor creating stray inductance effects

Circuit representation for an inductor

A TIME VARYING FLUX CREATES A COUNTER EMF AND CAUSES A VOLTAGE TO APPEAR AT THE TERMINALS OF THE DEVICE

A TIME VARYING MAGNETIC FLUX INDUCES A VOLTAGE

$$v_L = \frac{d\phi}{dt}$$

Induction law

FOR A LINEAR INDUCTOR THE FLUX IS PROPORTIONAL TO THE CURRENT

$$\phi = L i_L \Rightarrow v_L = L\frac{di_L}{dt}$$

DIFFERENTIAL FORM OF INDUCTION LAW

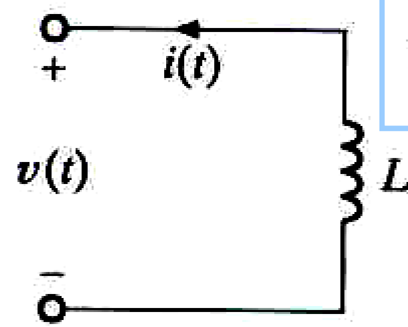
THE PROPORTIONALITY CONSTANT, L, IS CALLED THE INDUCTANCE OF THE COMPONENT

INDUCTANCE IS MEASURED IN UNITS OF henry (H). DIMENSIONALLY

$$\text{HENRY} = \frac{\text{Volt}}{\text{Amp}/\text{sec}}$$

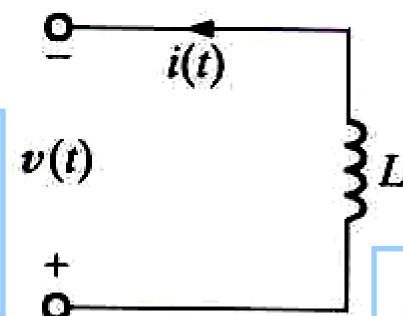
INDUCTORS STORE ELECTROMAGNETIC ENERGY. THEY MAY SUPPLY STORED ENERGY BACK TO THE CIRCUIT BUT THEY CANNOT CREATE ENERGY. THEY MUST ABIDE BY THE PASSIVE SIGN CONVENTION

Write the i-v relationship for the following inductors.



(a)

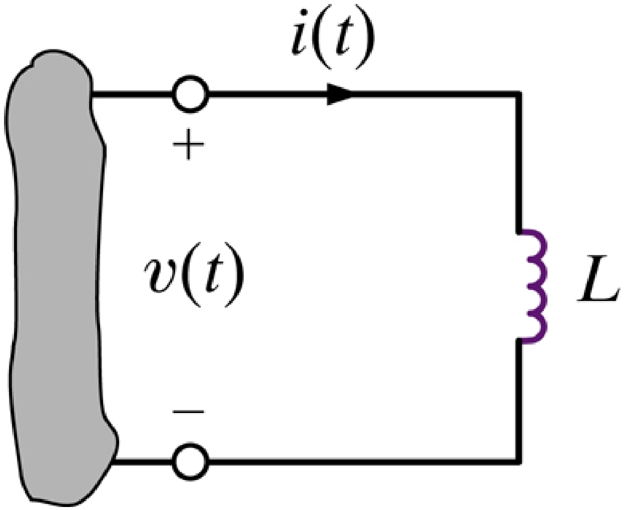
$$v(t) = -L\frac{di(t)}{dt}$$



(b)

$$v(t) = L\frac{di(t)}{dt}$$

Follow passive sign convention



$$v_L = L\frac{di_L}{dt}$$

Differential form of induction law

$$i_L(t) = \frac{1}{L}\int_{-\infty}^{t} v_L(x)dx$$

Integral form of induction law

$$i_L(t) = i_L(t_0) + \frac{1}{L}\int_{t_0}^{t} v_L(x)dx;\ \ t \geq t_0$$

A direct consequence of integral form $i_L(t-) = i_L(t+);\ \ \forall t$ Current MUST be continuous

A direct consequence of differential form $i_L = Const. \Rightarrow v_L = 0$ DC (steady state) behavior

Power and Energy stored

$$p_L(t) = v_L(t)i_L(t)$$ W

$$p_L(t) = L\frac{di_L}{dt}(t)i_L(t) = \frac{d}{dt}\left(\frac{1}{2}Li_L^2(t)\right)$$

$$w_L(t_2,t_1) = \int_{t_1}^{t_2} \frac{d}{dt}\left(\frac{1}{2}Li_L^2(x)\right)dx$$ J

Current in Amps, Inductance in Henrys yield energy in Joules

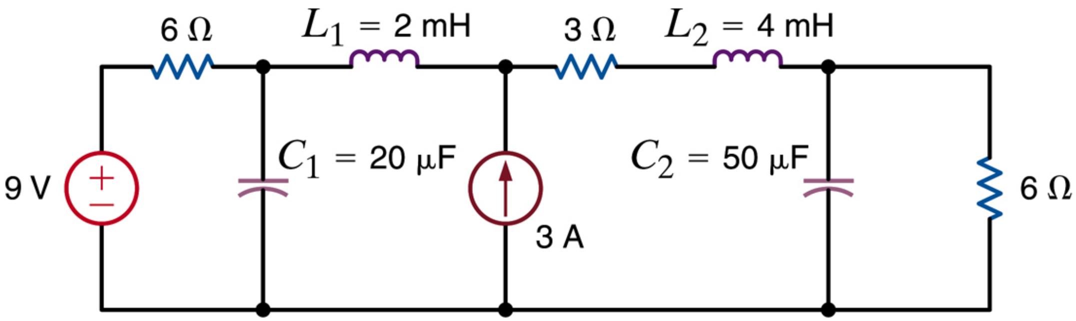
$$w(t_2,t_1) = \frac{1}{2}Li_L^2(t_2) - \frac{1}{2}Li_L^2(t_1)$$

Energy stored on the interval
Can be positive or negative

$$w_L(t) = \frac{1}{2}Li_L^2(t)$$

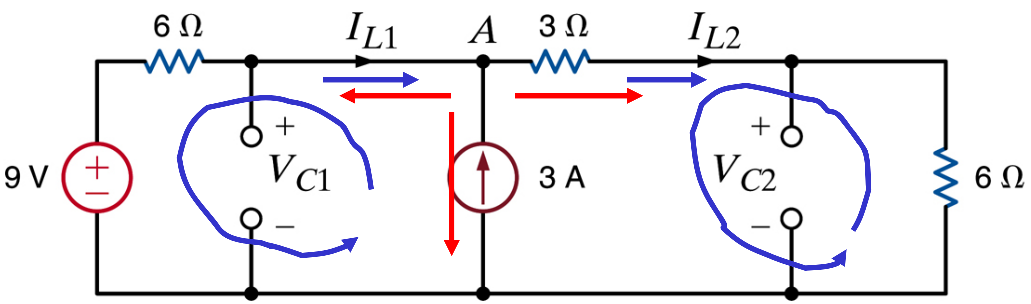
EXAMPLE

FIND THE TOTAL ENERGY STORED IN THE CIRCUIT



In steady state inductors act as short circuits and capacitors act as open circuits

$$W_C = \frac{1}{2} C V_C^2 \quad W_L = \frac{1}{2} L I_L^2$$



$$@A: -3A + \frac{V_A}{9} + \frac{V_A - 9}{6} = 0$$

$$V_A = \frac{81}{5}[V]$$

$$I_{L1} + 3A = I_{L2} \Rightarrow I_{L1} = -1.2A$$

$$V_{C1} = 9 - 6 I_{L1} \Rightarrow V_{C1} = 16.2V$$

$$V_{C2} = \frac{6}{6+3} V_A = 10.8V$$

$$I_{L2} = \frac{V_A}{9} = 1.8A$$

$$w_{L1} = \frac{1}{2}(2 \times 10^{-3})(-1.2)^2 = 1.44 \text{ mJ}$$

$$w_{L2} = \frac{1}{2}(4 \times 10^{-3})(1.8)^2 = 6.48 \text{ mJ}$$

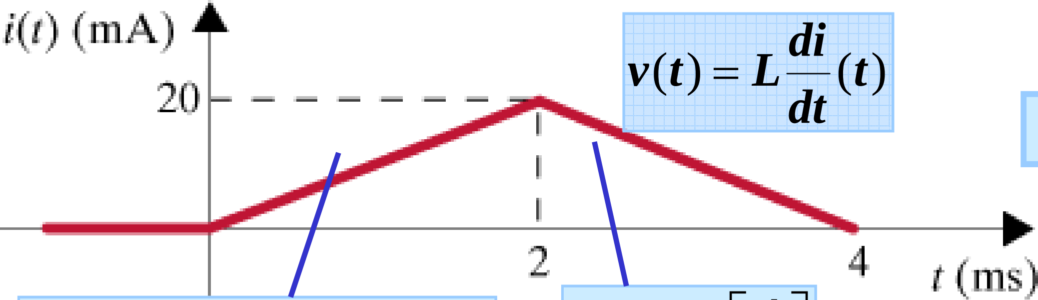
$$w_{C1} = \frac{1}{2}(20 \times 10^{-6})(16.2)^2 = 2.62 \text{ mJ}$$

$$w_{C2} = \frac{1}{2}(50 \times 10^{-6})(10.8)^2 = 2.92 \text{ mJ}$$

EXAMPLE

L=10mH. FIND THE VOLTAGE

$$v(t) = L\frac{di}{dt}(t)$$



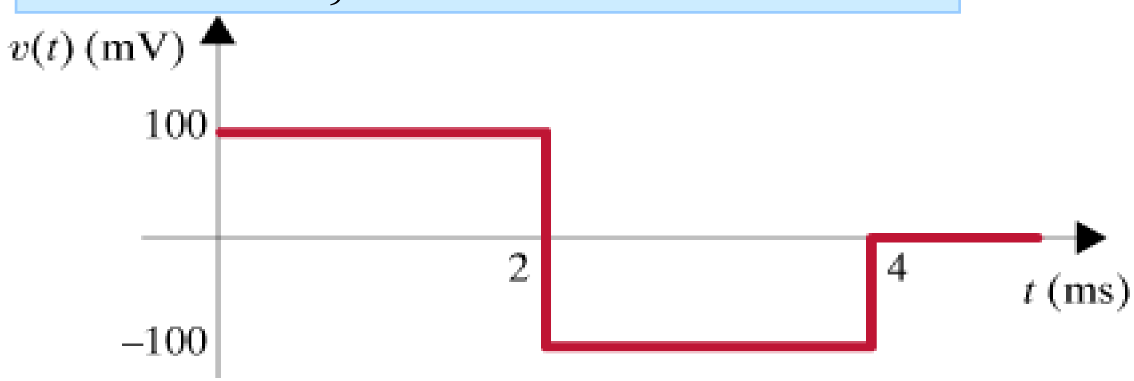
$$m = \frac{20\times10^{-3}A}{2\times10^{-3}s} = 10\left[\frac{A}{s}\right]$$

$$m = -10\left[\frac{A}{s}\right]$$

THE DERIVATIVE OF A STRAIGHT LINE IS ITS SLOPE

$$\frac{di}{dt} = \begin{cases} 10(A/s) & 0 \le t \le 2ms \\ -10(A/s) & 2 < t \le 4ms \\ 0 & elsewhere \end{cases}$$

$$\left.\begin{array}{l} \frac{di}{dt}(t) = 10(A/s) \\ L = 10\times10^{-3}H \end{array}\right\} \Rightarrow v(t) = 100\times10^{-3}V = 100mV$$



ENERGY STORED BETWEEN 2 AND 4 ms

$$w(4,2) = \frac{1}{2}Li_L^2(4) - \frac{1}{2}Li_L^2(2)$$

$$w(4,2) = 0 - 0.5*10*10^{-3}(20*10^{-3})^2 \text{ J}$$

THE VALUE IS NEGATIVE BECAUSE THE INDUCTOR IS SUPPLYING ENERGY PREVIOUSLY STORED

CAPACITOR SPECIFICATIONS

CAPACITANCE RANGE $pF \approx C \approx 50mF$

IN STANDARD VALUES

STANDARD CAPACITOR RATINGS

$6.3V \quad - \quad 500V$

STANDARD TOLERANCE

$\pm 5\%, \pm 10\%, \pm 20\%$

INDUCTOR SPECIFICATIONS

INDUCTANCE RANGES $\approx 1nH \leq L \leq \approx 100mH$

IN STANDARD VALUES

STANDARD INDUCTOR RATINGS

$\approx mA \quad - \quad \approx 1A$

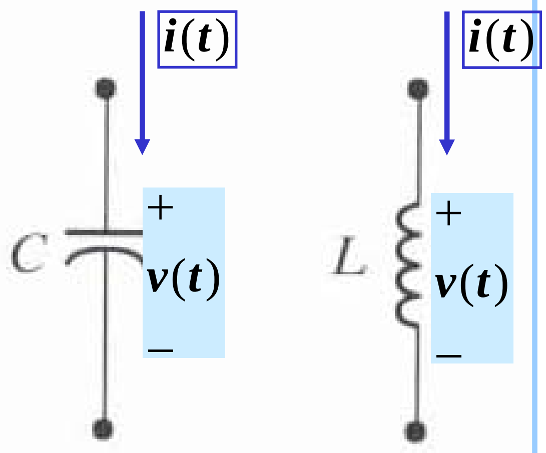
STANDARD TOLERANCE

$\pm 5\%, \pm 10\%$

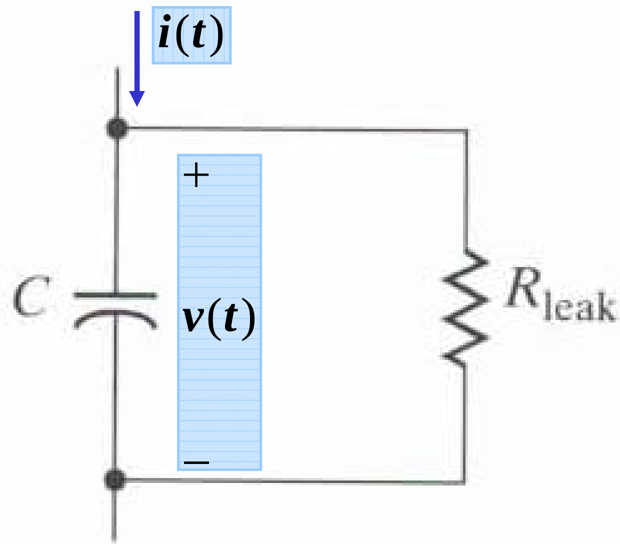
The Dual Relationship for Capacitors and Inductors

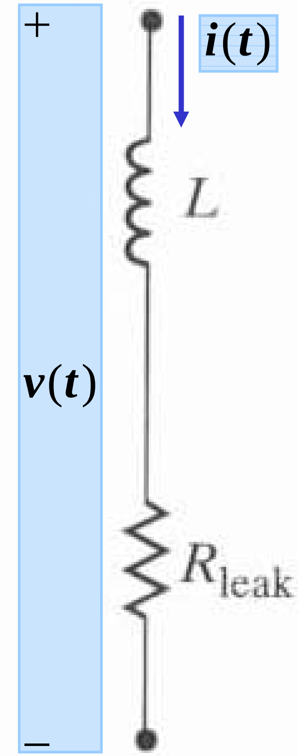
Capacitor		Inductor
$i(t) = C\dfrac{dv(t)}{dt}$		$v(t) = L\dfrac{di(t)}{dt}$
$v(t) = \dfrac{1}{C}\int_{t_0}^{t} i(x)\,dx + v(t_0)$		$i(t) = \dfrac{1}{L}\int_{t_0}^{t} v(x)\,dx + i(t_0)$
$p(t) = Cv(t)\dfrac{dv(t)}{dt}$	$C \to L$	$p(t) = Li(t)\dfrac{di(t)}{dt}$
	$v \to i$	
$w(t) = \frac{1}{2}Cv(t)^2$	$i \to v$	$w(t) = \frac{1}{2}Li^2(t)$

IDEAL AND PRACTICAL ELEMENTS



IDEAL ELEMENTS





CAPACITOR/INDUCTOR MODELS INCLUDING LEAKAGE RESISTANCE

$$i(t) = C\frac{dv}{dt}(t)$$

$$v(t) = L\frac{di}{dt}(t)$$

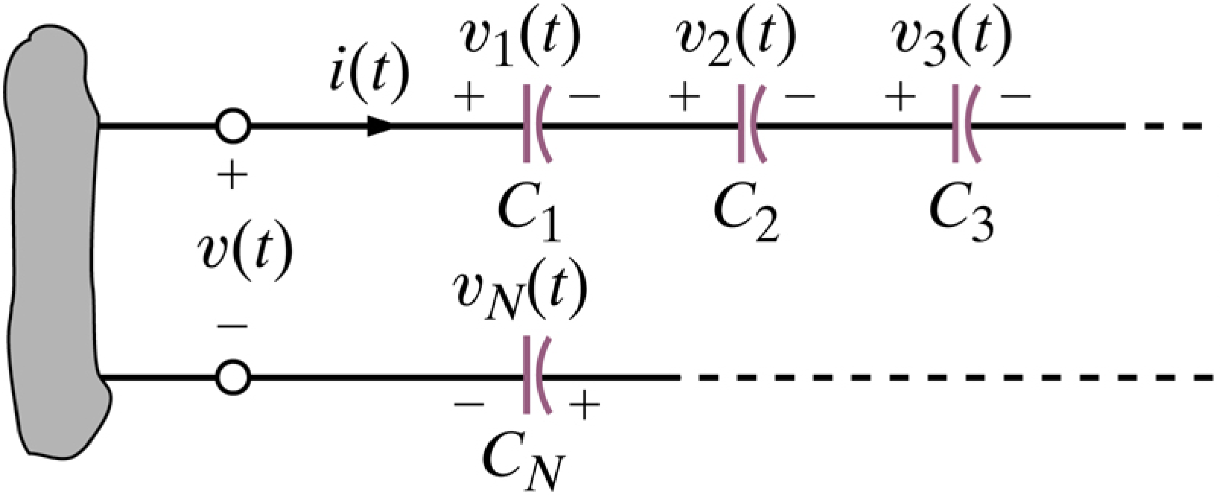
$$i(t) = \frac{v(t)}{R_{leak}} + C\frac{dv}{dt}(t)$$

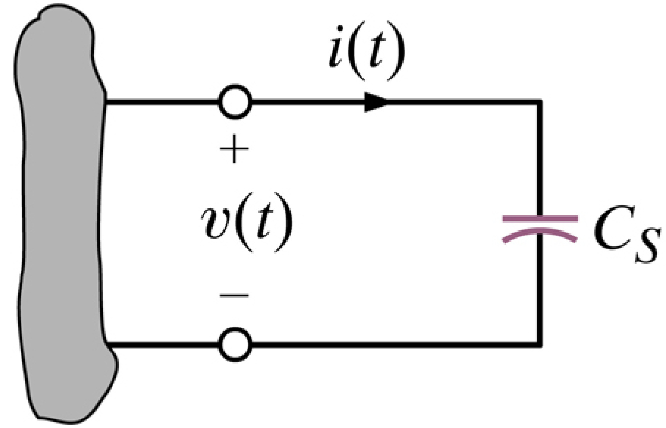
MODEL FOR "LEAKY" CAPACITOR

$$v(t) = R_{leak}\,i(t) + L\frac{di}{dt}(t)$$

MODEL FOR "LEAKY" INDUCTORS

SERIES CAPACITORS





$$v(t) = v_1(t) + v_2(t) + v_3(t) + \cdots + v_N(t)$$

$$v_i(t) = \frac{1}{C_i}\int_{t_0}^{t} i(t)\,dt + v_i(t_0)$$

$$v(t) = \left(\sum_{i=1}^{N}\frac{1}{C_i}\right)\int_{t_0}^{t} i(t)\,dt + \sum_{i=1}^{N} v_i(t_0)$$

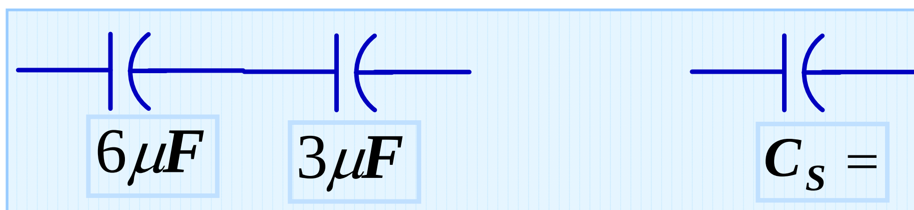
$$\frac{1}{C_S} = \sum_{i=1}^{N}\frac{1}{C_i} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_N}$$

$$v(t_0) = \sum_{i=1}^{N} v_i(t_0)$$

NOTICE SIMILARITY WITH RESITORS IN PARALLEL

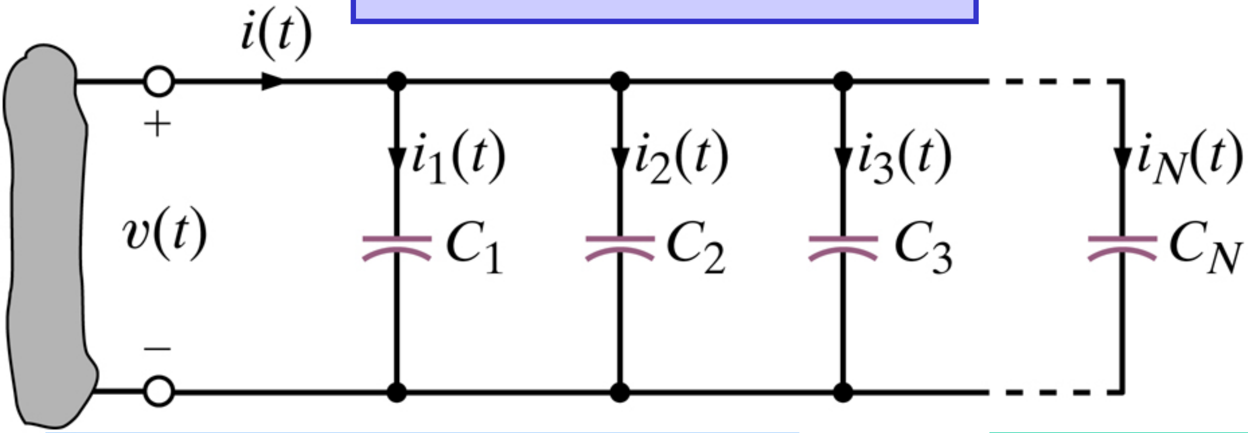
$$C_s = \frac{C_1 C_2}{C_1 + C_2}$$

Series Combination of two capacitors



$2\mu F$

PARALLEL CAPACITORS



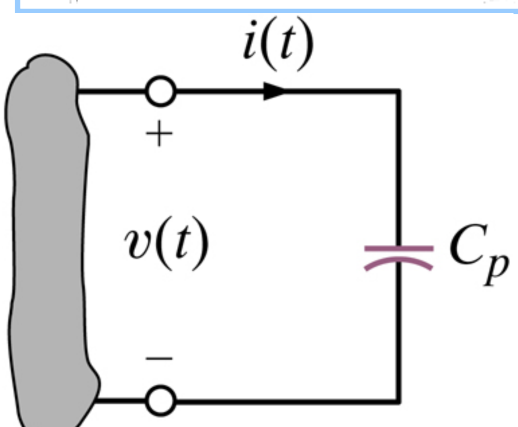
$$i(t) = i_1(t) + i_2(t) + i_3(t) + \cdots + i_N(t)$$

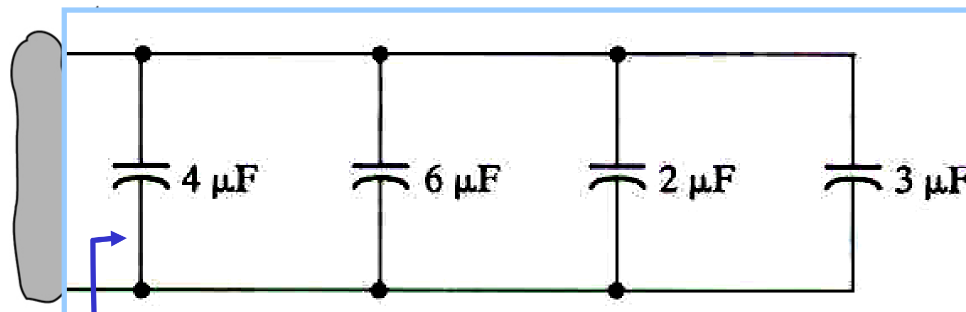
$$i_k(t) = C_k \frac{dv}{dt}(t)$$

$$i(t) = C_1 \frac{dv(t)}{dt} + C_2 \frac{dv(t)}{dt} + C_3 \frac{dv(t)}{dt} + \cdots + C_N \frac{dv(t)}{dt}$$

$$= \left(\sum_{i=1}^{N} C_i \right) \frac{dv(t)}{dt}$$

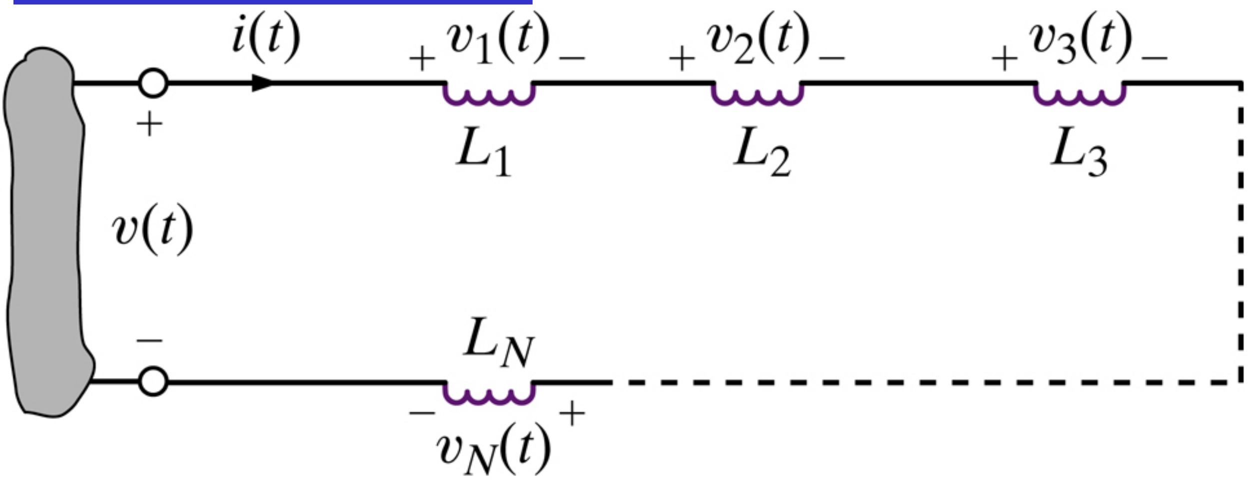
$$C_p = C_1 + C_2 + C_3 + \cdots + C_N$$

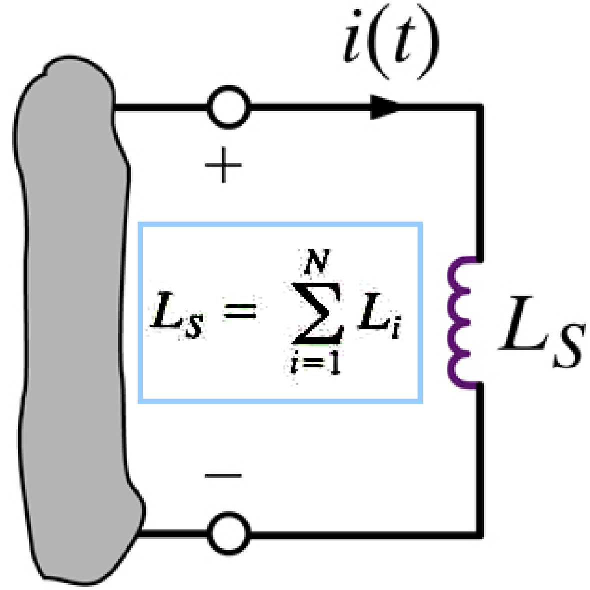




$$C_P = 4 + 6 + 2 + 3 = 15 \mu F$$

SERIES INDUCTORS





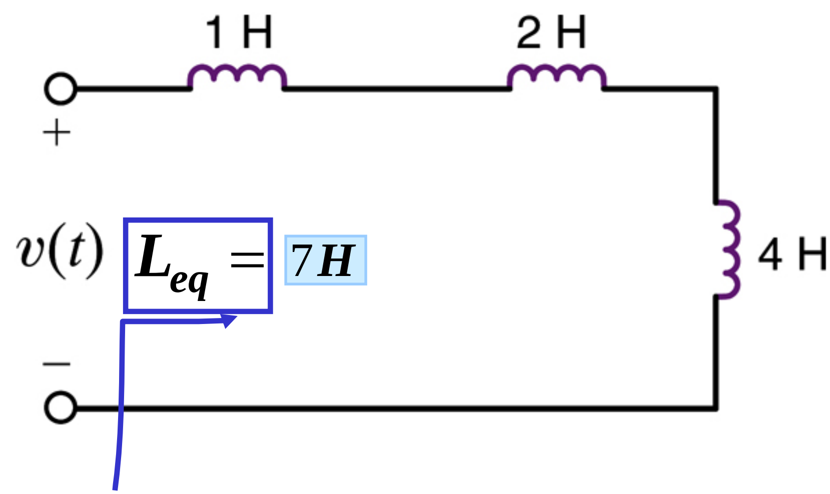
$$v(t) = v_1(t) + v_2(t) + v_3(t) + \cdots + v_N(t)$$

$$v(t) = L_S \frac{di}{dt}(t)$$

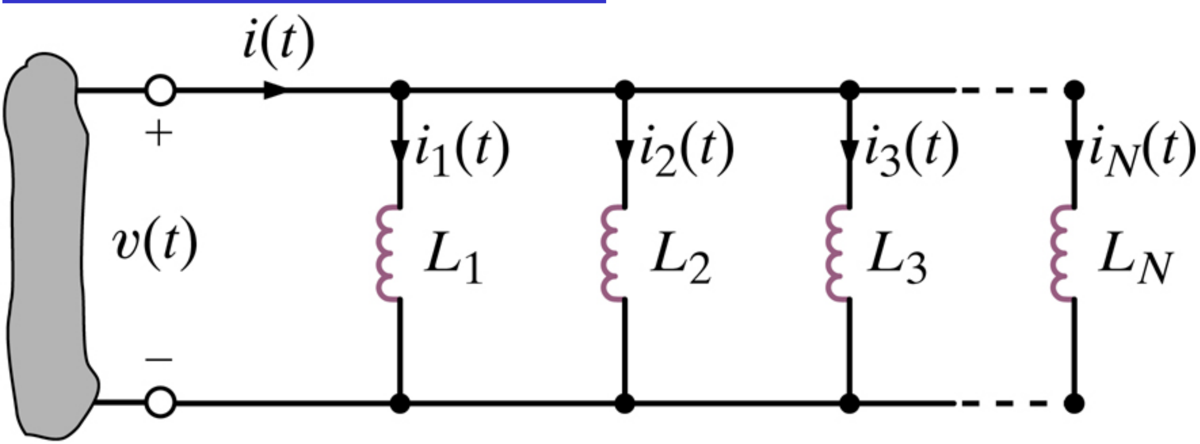
$$v_k(t) = L_k \frac{di}{dt}(t)$$

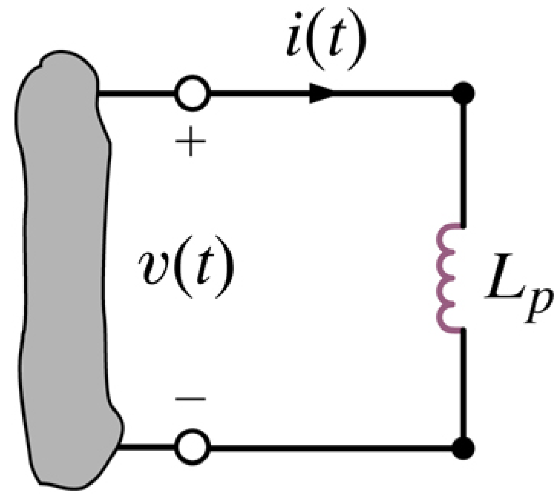
$$v(t) = L_1 \frac{di(t)}{dt} + L_2 \frac{di(t)}{dt} + L_3 \frac{di(t)}{dt} + \cdots + L_N \frac{di(t)}{dt}$$
$$= \left(\sum_{i=1}^{N} L_i \right) \frac{di(t)}{dt}$$

$$L_S = \sum_{i=1}^{N} L_i$$



PARALLEL INDUCTORS





$$i(t) = i_1(t) + i_2(t) + i_3(t) + \cdots + i_N(t)$$

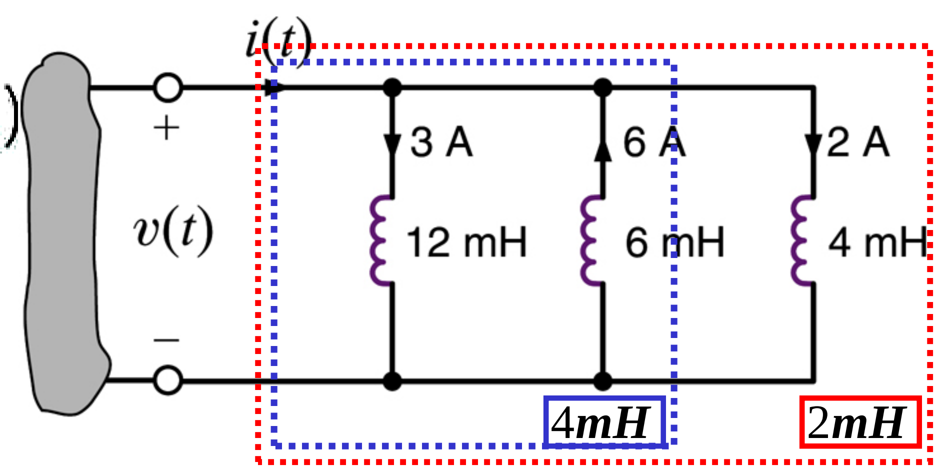
$$i_j(t) = \frac{1}{L_j}\int_{t_0}^{t} v(x)\,dx + i_j(t_0)$$

$$i(t) = \frac{1}{L_P}\int_{t_0}^{t} v(x)\,dx + i(t_0)$$

$$i(t) = \left(\sum_{j=1}^{N}\frac{1}{L_j}\right)\int_{t_0}^{t} v(x)\,dx + \sum_{j=1}^{N} i_j(t_0)$$

$$\frac{1}{L_p} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots + \frac{1}{L_N}$$

$$i(t_0) = \sum_{j=1}^{N} i_j(t_0)$$



$$i(t_0) = 3A - 6A + 2A = -1A$$

INDUCTORS COMBINE LIKE RESISTORS
CAPACITORS COMBINE LIKE CONDUCTANCES

SUMMARY

- The important (dual) relationships for capacitors and inductors are as follows:

$$q = Cv$$

$$i(t) = C\frac{dv(t)}{dt} \qquad v(t) = L\frac{di(t)}{dt}$$

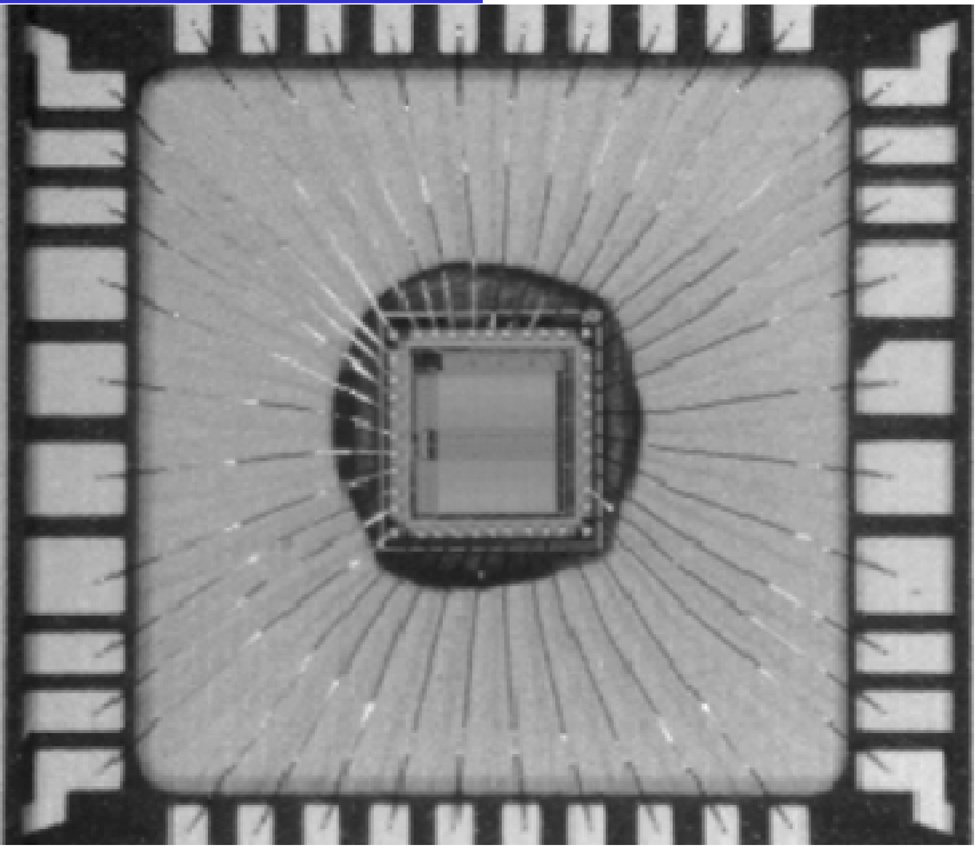
$$v(t) = \frac{1}{C}\int_{-\infty}^{t} i(x)\,dx \qquad i(t) = \frac{1}{L}\int_{-\infty}^{t} v(x)\,dx$$

$$p(t) = Cv(t)\frac{dv(t)}{dt} \qquad p(t) = Li(t)\frac{di(t)}{dt}$$

$$W_C(t) = 1/2Cv^2(t) \qquad W_L(t) = 1/2Li^2(t)$$

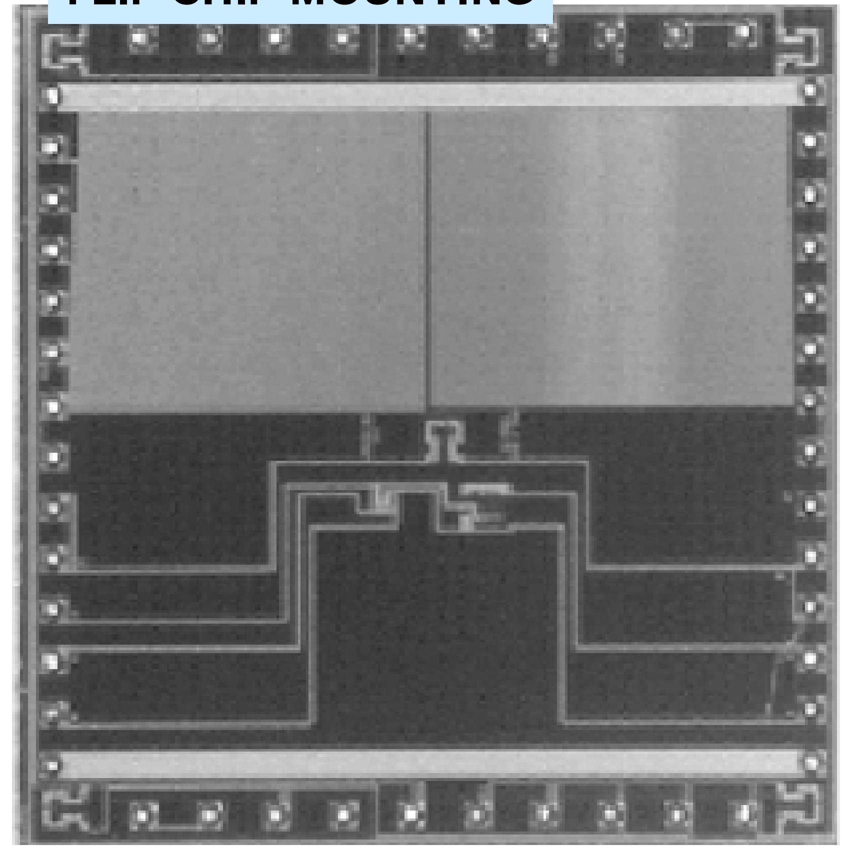
- The passive sign convention is used with capacitors and inductors.
- In dc steady state a capacitor looks like an open circuit and an inductor looks like a short circuit.
- Leakage resistance is present in practical capacitors and inductors.
- When capacitors are interconnected, their equivalent capacitance is determined as follows: Capacitors in series combine like resistors in parallel and capacitors in parallel combine like resistors in series.
- When inductors are interconnected, their equivalent inductance is determined as follows: Inductors in series combine like resistors in series and inductors in parallel combine like resistors in parallel.
- *RC* operational amplifier circuits can be used to differentiate or integrate an electrical signal.

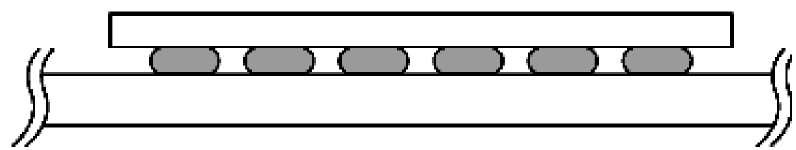
LEARNING EXAMPLE



IC WITH WIREBONDS TO THE OUTSIDE

FLIP CHIP MOUNTING

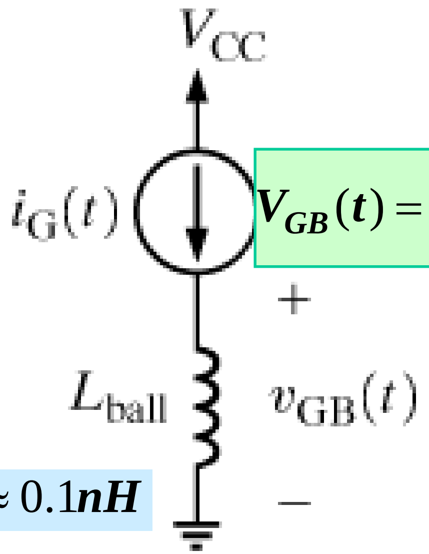




GOAL: REDUCE INDUCTANCE IN THE WIRING AND REDUCE THE "GROUND BOUNCE" EFFECT

A SIMPLE MODEL CAN BE USED TO DESCRIBE GROUND BOUNCE

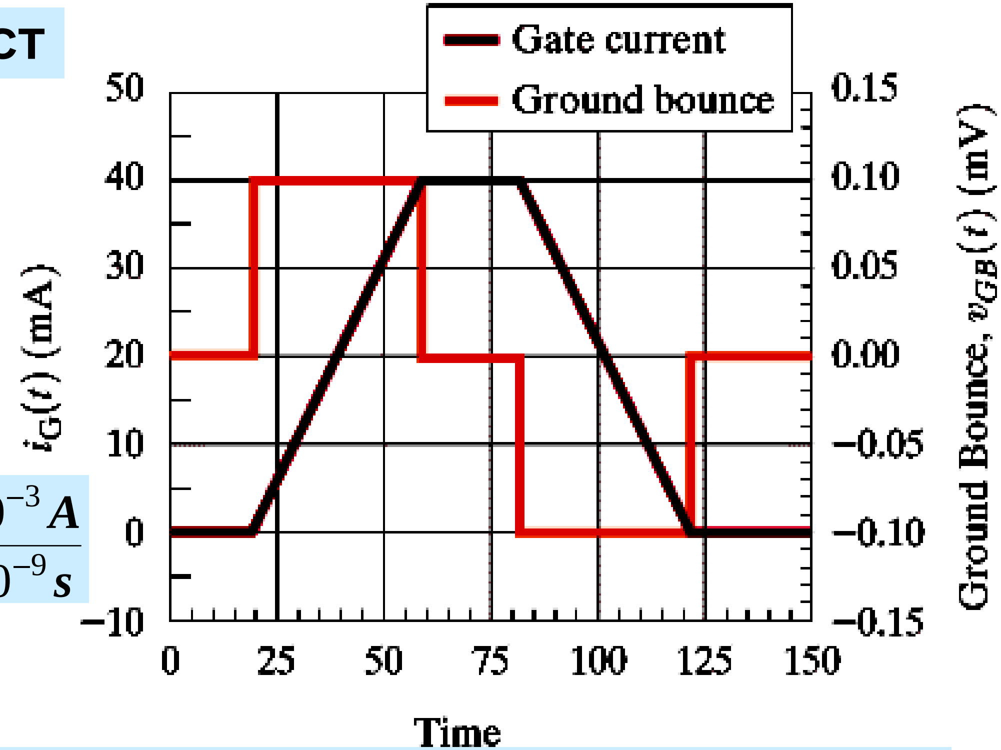
MODELING THE GROUND BOUNCE EFFECT



$$V_{GB}(t) = L_{ball}\frac{di_G}{dt}(t)$$

$$L_{ball} \approx 0.1nH$$

$$m = \frac{40\times10^{-3}A}{40\times10^{-9}s}$$



IF ALL GATES IN A CHIP ARE CONNECTED TO A SINGLE GROUND THE CURRENT CAN BE QUITE HIGH AND THE BOUNCE MAY BECOME UNACCEPTABLE

USE SEVERAL GROUND CONNECTIONS (BALLS) AND ALLOCATE A FRACTION OF THE GATES TO EACH BALL