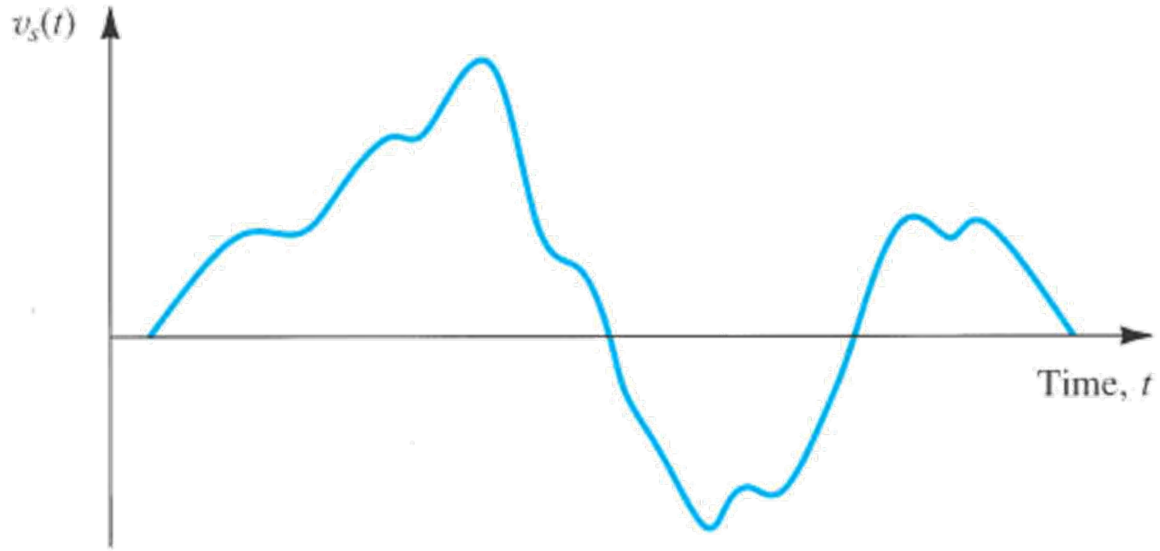
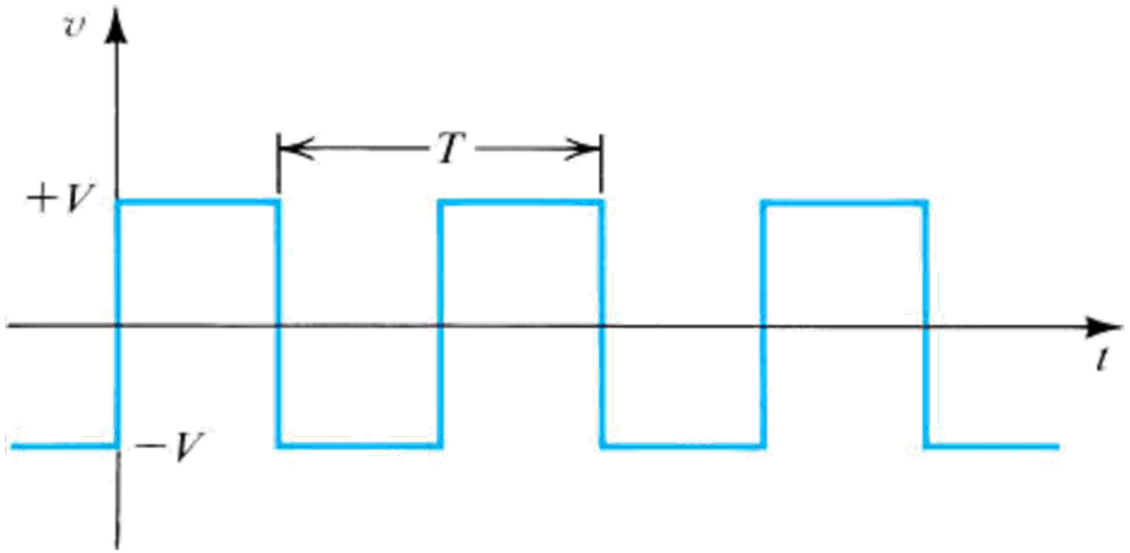
BASIC CONCEPTS

- **Signal Waveforms**

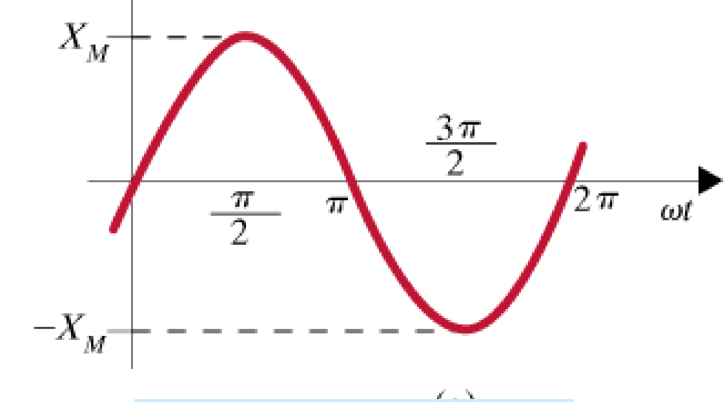
Continuous / Discontinuous



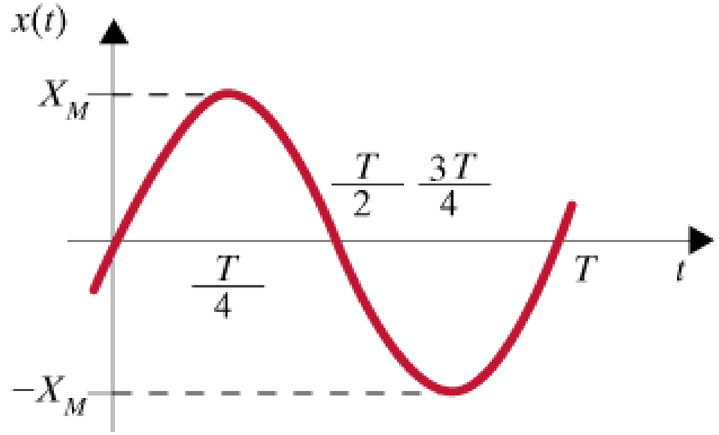


SINUSOIDS

$$x(t) = X_M \sin \omega t$$



Adimensional plot



As function of time

X_M = amplitude or maximum value

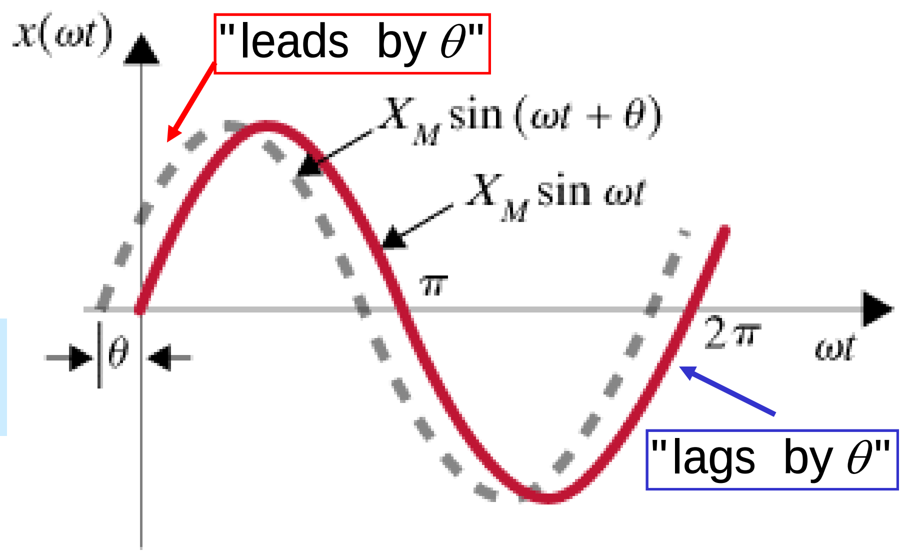
ω = angular frequency (rads/sec)

ωt = argument (radians)

$$T = \frac{2\pi}{\omega} = \text{Period} \Rightarrow x(t) = x(t+T), \forall t$$

$$f = \frac{1}{T} = \frac{\omega}{2\pi} = \text{frequency in Hertz (cycle/sec)}$$

$$\omega = 2\pi f$$



BASIC TRIGONOMETRY

ESSENTIAL IDENTITIES

$$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$$

$$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$$

$$\sin(-\alpha)=-\sin\alpha$$

$$\cos(-\alpha)=\cos\alpha$$

SOME DERIVED IDENTITIES

$$\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$$

$$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$$

$$\sin\alpha\cos\beta=\frac{1}{2}\sin(\alpha+\beta)+\frac{1}{2}\sin(\alpha-\beta)$$

$$\cos\alpha\cos\beta=\frac{1}{2}\cos(\alpha+\beta)+\frac{1}{2}\cos(\alpha-\beta)$$

APPLICATIONS

$$\cos\omega\boldsymbol{t}=\sin(\omega\boldsymbol{t}+\frac{\pi}{2})$$

$$\sin\omega\boldsymbol{t}=\cos(\omega\boldsymbol{t}-\frac{\pi}{2})$$

$$\cos\omega\boldsymbol{t}=-\cos(\omega\boldsymbol{t}\pm\pi)$$

$$\sin\omega\boldsymbol{t}=-\sin(\omega\boldsymbol{t}\pm\pi)$$

RADIANS AND DEGREES

2π radians = 360 degrees

$$\theta(\text{rads})=\frac{180}{\pi}\theta\ (\text{degrees})$$

ACCEPTED EE CONVENTION

$$\sin(\omega\boldsymbol{t}+\frac{\pi}{2})=\sin(\omega\boldsymbol{t}+90°)$$

Derivative and Integral

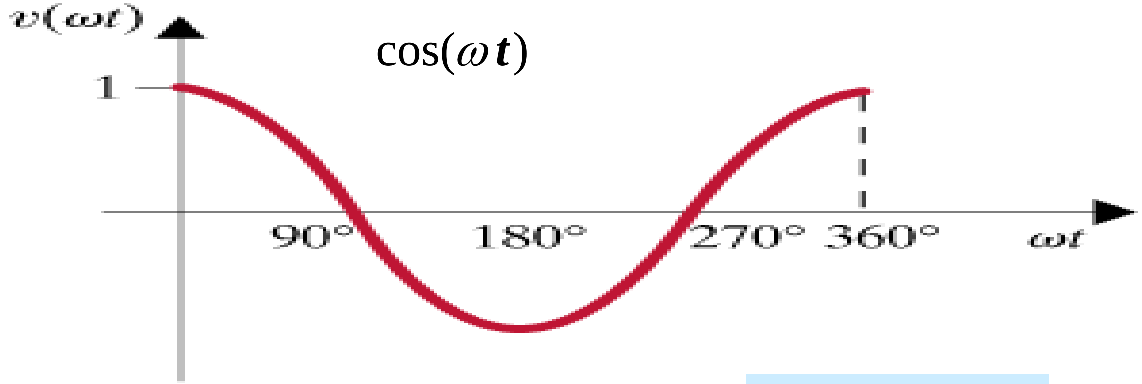
$$a=A\sin(\omega t)$$

$$\frac{da}{dt}=\omega A\cos(\omega t)=\omega A\sin(\omega t+\frac{\pi}{2})$$

$$\int A\sin(\omega t)dt=-\frac{A}{\omega}\cos(\omega t)+k=-\frac{A}{\omega}\sin(\omega t+\frac{\pi}{2})+k$$

Example

$\cos(\omega t)$

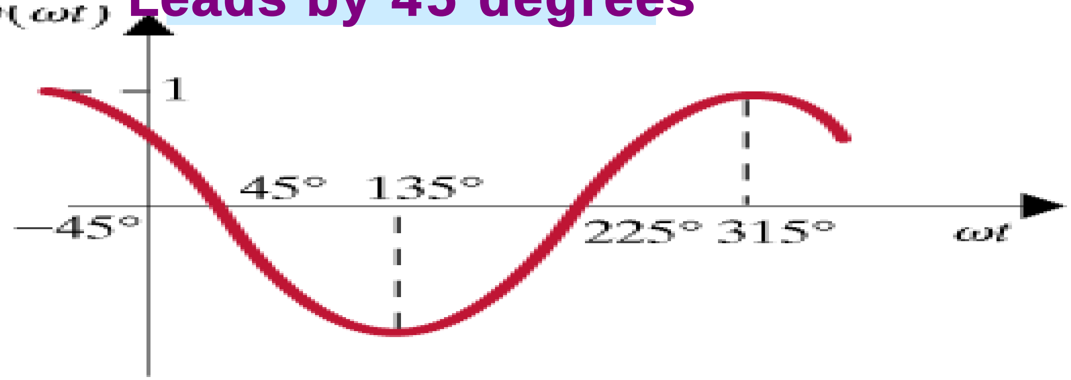


$\cos(\omega t + 45°)$

$\cos(\omega t + 45 - 360)$

Leads by 45 degrees

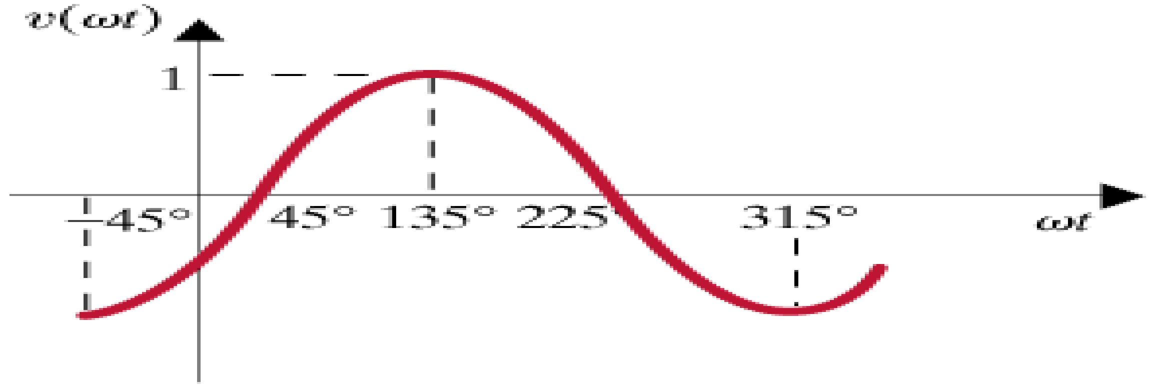
Lags by 315



$-\cos(\omega t + 45°)$

$\cos(\omega t + 45 \pm 180)$

Leads by 225 or lags by 135



(c)

Example

$v_1(t) = 12\sin(1000t + 60°),\ v_2(t) = -6\cos(1000t + 30°)$

FIND FREQUENCY AND PHASE ANGLE BETWEEN VOLTAGES

Frequency in radians per second is the factor of the time variable

$$f\,(Hz) = \frac{\omega}{2\pi} = 159.2\,Hz$$

$\omega = 1000\sec^{-1}$

To find phase angle we must express both sinusoids using the same trigonometric function; either sine or cosine with positive amplitude

take care of minus sign with $\cos(\alpha) = -\cos(\alpha \pm 180°)$

$$-6\cos(1000t + 30°) = 6\cos(1000t + 30° + 180°)$$

Change sine into cosine with $\cos(\alpha) = \sin(\alpha + 90°)$

$$6\cos(1000t + 210°) = 6\sin(1000t + 210° + 90°)$$

We like to have the phase shifts less than 180 in absolute value

$$6\sin(1000t + 300°) = 6\sin(1000t - 60°)$$

$v_1(t) = 12\sin(1000t + 60°)$

$v_2(t) = 6\sin(1000t - 60°)$

$(1000t + 60°) - (1000t - 60°) = 120°$

$(1000t - 60°) - (1000t + 60°) = -120°$

v_1 leads v_2 by $120°$

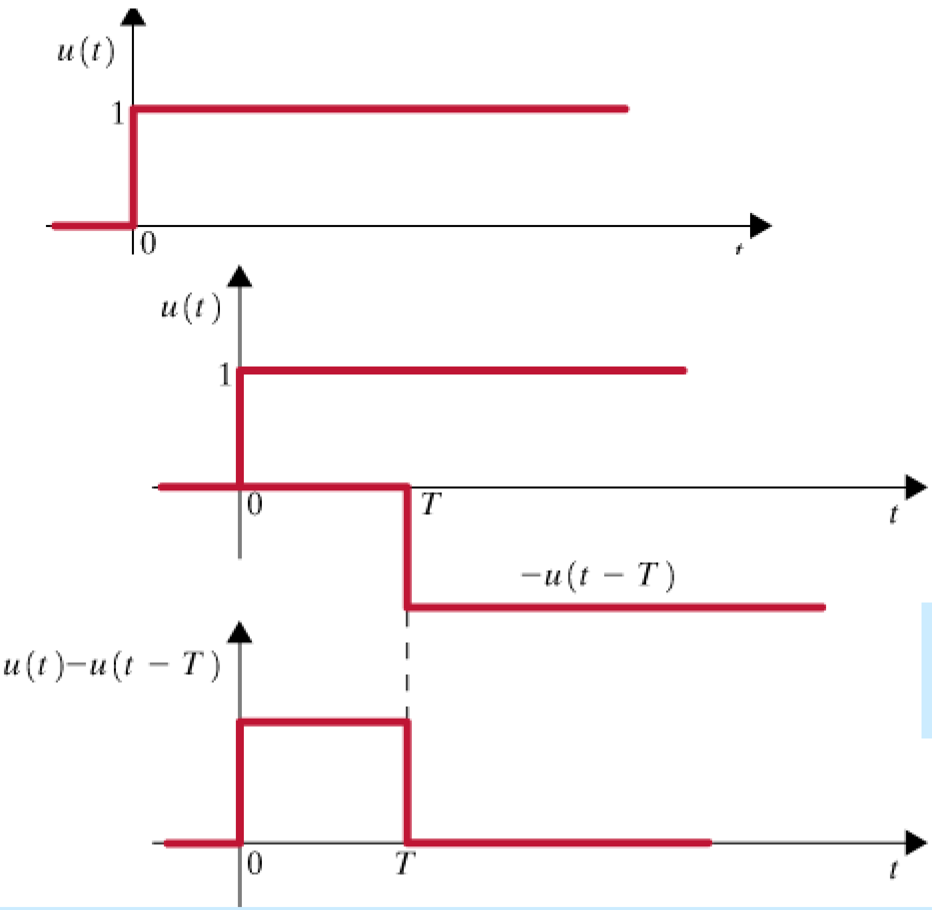
v_2 lags v_1 by $120°$

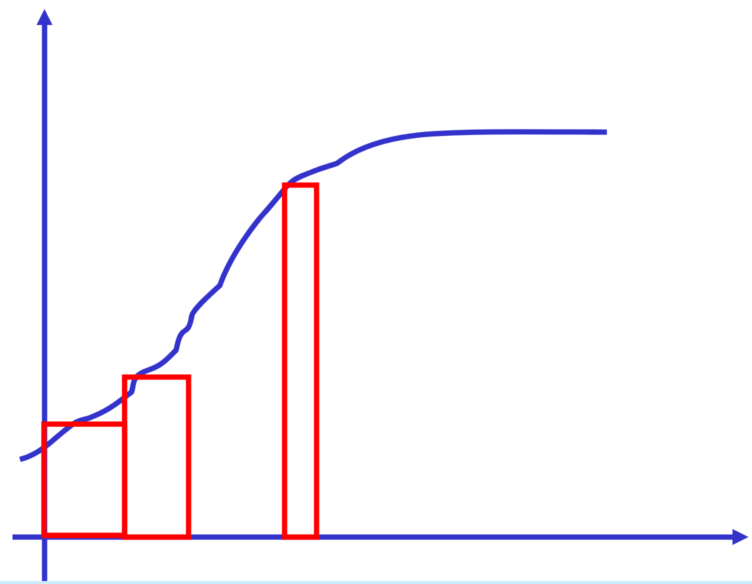
Unit step

$$u(t) = \begin{cases} 0 & t < 0 \\ 1 & t > 0 \end{cases}$$

For positive time functions

$$f(t) = f(t)u(t)$$





Using square pulses to approximate an arbitrary function

Pulse width = T, Pulse height = 1

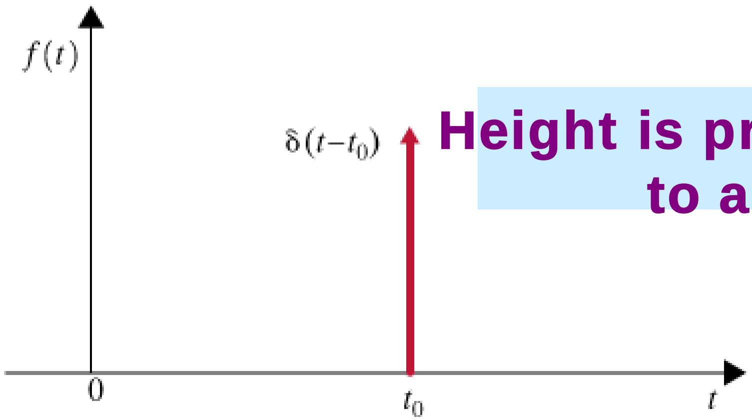
Using the unit step to build a pulse function

THE IMPULSE FUNCTION

(Good model for impact, lightning, and other well known phenomena)

$$\delta(t - t_0) = 0 \qquad t \neq t_0$$

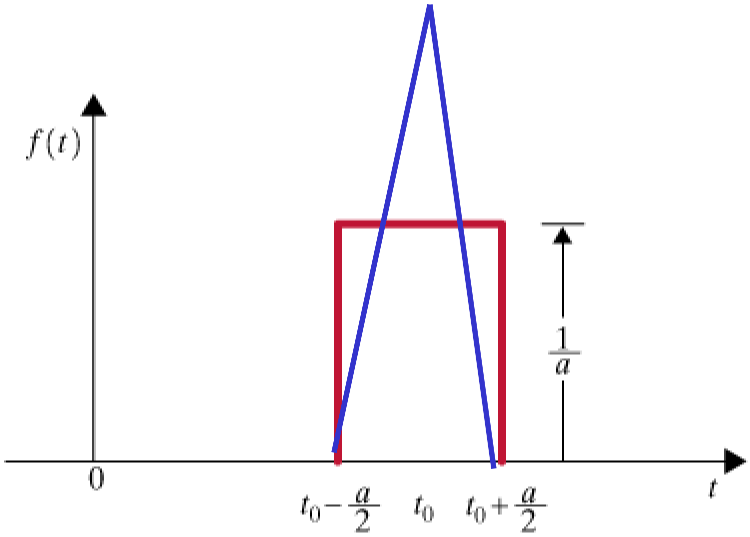
$$\int_{t_0-\varepsilon}^{t_0+\varepsilon} \delta(t - t_0)\, dt = 1 \qquad \varepsilon > 0$$



Height is proportional to area

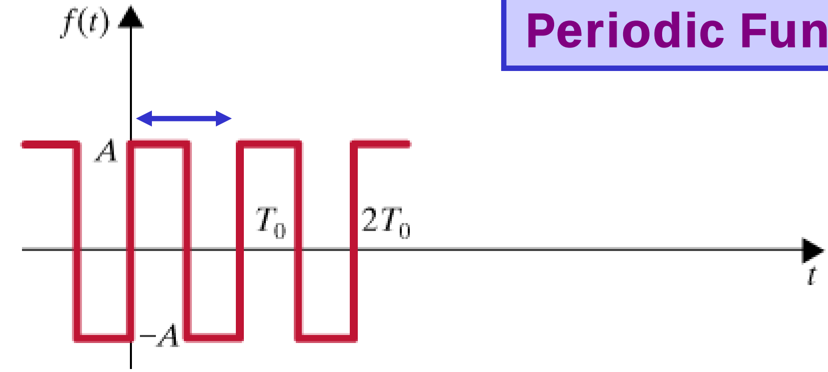
Representation of the impulse

Approximations to the impulse



(a)

Periodic Functions

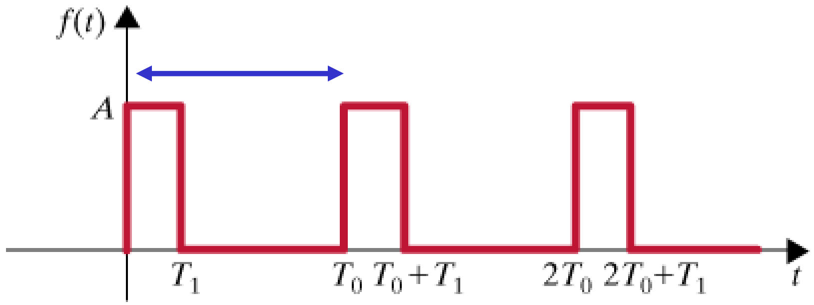


$$v = A \qquad \text{for } 0 < t < \frac{To}{2}$$

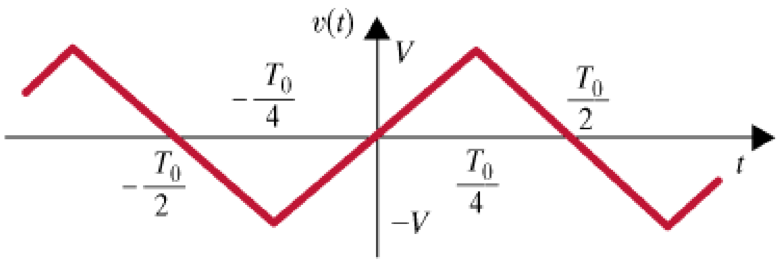
$$v = -A \qquad \text{for } \frac{To}{2} < t < To$$

$$v = A \qquad \text{for } To < t < \frac{3To}{2}$$

etc...

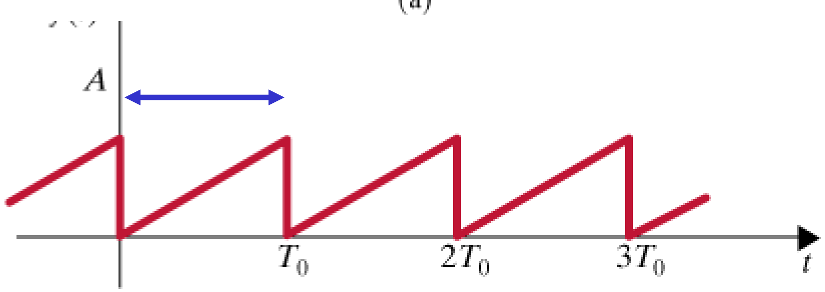


Square Waveforms



Triangle Wavefrom

(a)



Sawtooth Waveform

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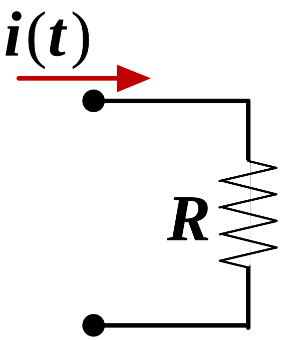
EFFECTIVE OR RMS VALUES



Instantaneous power

$$p(t) = i^2(t)R$$

If the current is sinusoidal the average power is known to be

$$P_{av} = \frac{1}{2} I_M^2 R$$

$$\therefore I_{eff}^2 = \frac{1}{2} I_M^2$$

For a sinusoidal signal

$$x(t) = X_M \cos(\omega t + \theta)$$

the effective value is

$$X_{eff} = \frac{X_M}{\sqrt{2}}$$

The effective value is the equivalent DC value that supplies the same average power

If current is periodic with period T

$$P_{av} = \frac{1}{T}\int_{t_0}^{t_0+T} p(t)dt = R\left(\frac{1}{T}\int_{t_0}^{t_0+T} i^2(t)dt\right)$$

For sinusoidal case $P_{av} = \frac{1}{2} V_M I_M \cos(\theta_v - \theta_i)$

If current is DC ($i(t) = I_{dc}$) then

$$P_{dc} = RI_{dc}^2$$

$$I_{eff} : P_{av} = P_{dc}$$

$$P_{av} = V_{eff} I_{eff} \cos(\theta_v - \theta_i)$$

effective $\approx$ rms (root mean square)

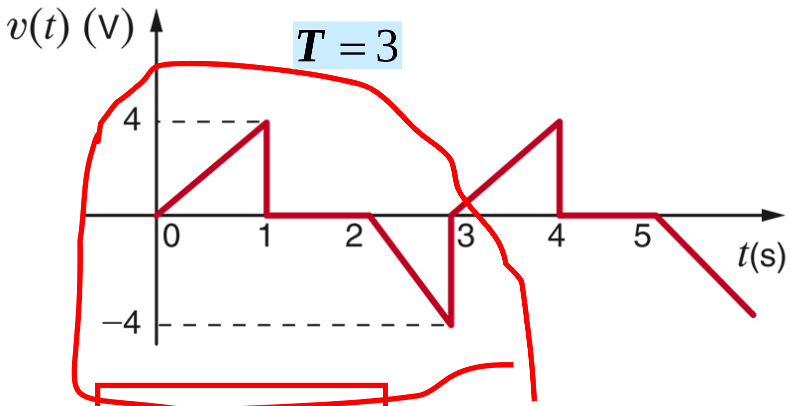
$$I_{eff}^2 = \frac{1}{T}\int_{t_0}^{t_0+T} i^2(t)dt$$

$$I_{eff} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} i^2(t)dt}$$

Definition is valid for ANY periodic signal with period T

EXAMPLE

Compute the rms value of the voltage waveform



One period

$$X_{rms} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} x^2(t)dt}$$

$$v(t) = \begin{cases} 4t & 0 < t \le 1 \\ 0 & 1 < t \le 2 \\ -4(t-2) & 2 < t \le 3 \end{cases}$$

The two integrals have the same value

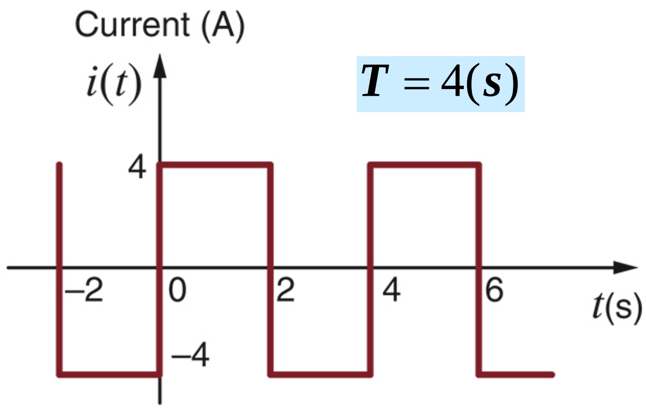
$$\int_0^T v^2(t)dt = \int_0^1 (4t)^2 dt + \int_2^3 (4(t-2))^2 dt$$

$$\int_0^3 v^2(t)dt = 2 \times \left[\frac{16}{3}t^3\right]_0^1 = \frac{32}{3}$$

$$V_{rms} = \sqrt{\frac{1}{3} \times \frac{32}{3}} = 1.89(V)$$

EXAMPLE

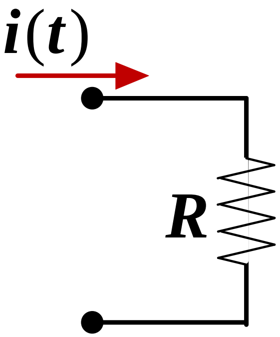
Compute the rms value of the voltage waveform and use it todetermine the average power supplied to the resistor



$T = 4(s)$

$i^2(t) = 16;\ 0 \le t < 4$

$I_{rms} = 4(A)$



$R = 2\Omega$

$$X_{rms} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} x^2(t)dt}$$

$$P_{av} = RI_{rms}^2 = 32(W)$$