

Solutions



Carleton UNIVERSITY

QJZ

FINAL EXAMINATION
DECEMBER 2005

DURATION: 3 HOURS

No. of Students: 205

Department Name & Course Number: ELEC 2501A,B,C
Course Instructor(s) T.G. Ray and Q.J. Zhang

AUTHORIZED MEMORANDA

Self-contained electronic calculator with memory cleared.

No graphical calculators

Students **MUST** count the number of pages in this examination question paper before beginning to write, and report any discrepancy to a proctor. This question paper has twelve pages.

This examination question paper may not be taken from the examination room.

In addition to this question paper, students require: an examination booklet yes no
a Scantron sheet yes no

INSTRUCTIONS and INFORMATION:

1. PRINT YOUR NAME AND STUDENT NUMBER CLEARLY.

NAME:

NUMBER:

marks/1:

2. ATTEMPT ALL QUESTIONS.

3. PUT YOUR ANSWERS ONLY IN THE APPROPRIATE SPACES PROVIDED. Your exam booklet is for rough work and will not be marked.

4. FULL MARKS FOR EACH QUESTION ARE INDICATED. Part marks are possible depending on the question.

5. THE EXAM MARKS TOTAL 100.

Faculty Use Only:

Page	1	2	3	4	5	6	7	8	9	10	11	12
Marks	1											
	1	8	8	14	7	6	4	9	10	11	11	11

(Q1)

Q1. RMS Value and Power

(16 marks)

a) For the periodic waveform in Figure 1:

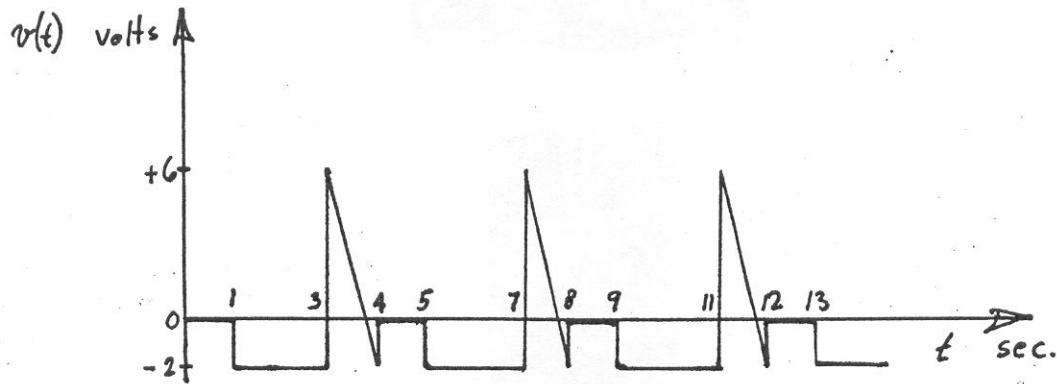


Figure 1.

ai) What are representative equations for the non-zero parts of the waveform?

Answer: $v(t) = \overset{\textcircled{1/2}}{-2}$ for $1 < t < 3$ etc. $\overset{\textcircled{1/2}}{-8t + 30}$ for $3 < t < 4$ etc. marks/3: _____

aii) Using the equations above, what is the equation for calculating the RMS value of the waveform?

Answer: $\text{RMS} = \sqrt{\frac{1}{4} \left[\int_1^3 (-2)^2 dt + \int_3^4 (-8t + 30)^2 dt \right]}$ marks/3: _____

check limits: minus 1 if any wrong

aiii) What is the RMS value of the waveform?

Answer: $\text{RMS} = 2.08$ volts marks/2: _____

$\textcircled{2}$ or 0

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1b) Find the power that is absorbed or supplied by each of the elements in the following circuit:

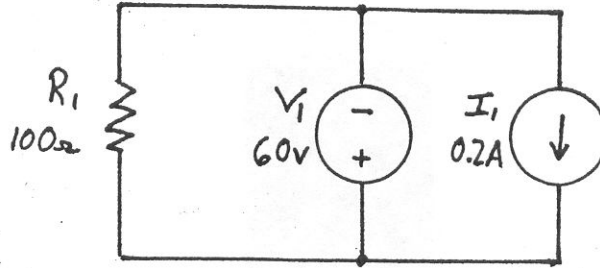


Figure 2.

Signs must be correct, however
 + ≡ "consumed", "dissipated", "lost" etc
 - ≡ "supplied", "sourced" etc.
 No part marks.

Answer: Power in R_1 : +36 ^① watts $P = \frac{V^2}{R}$
 Power in V_1 : -24 ^② watts $\sum P = 0$, or KCL
 Power in I_1 : -12 ^① watts $P = VI$ marks/4: _____

1c) Given the circuit of Figure 3:

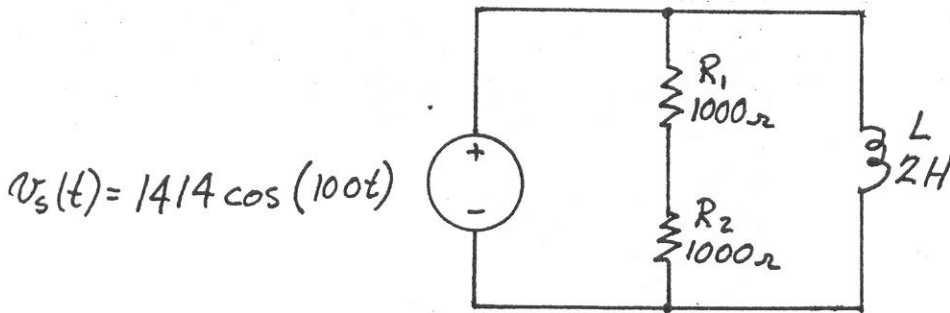


Figure 3.

(missed R_2 completely) OR
 1000 worth 0.5 (missed $\sqrt{2}$, missed $-P_{R2}$)
 500 worth 1.5 (missed $-P_{R2}$ or missed $\sqrt{2}$)

What is the average power dissipated in each of R_1 and L ?

Answer: Average power in R_1 is 250 ^③ watts marks/3: _____
 Average power in L is 0 ^① watts marks/1: _____

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Q2. Thevenin, Norton and Superposition

(21 marks)

a) For the circuit shown in Figure 4, find the **Thevenin** equivalent source between the terminals A and B.

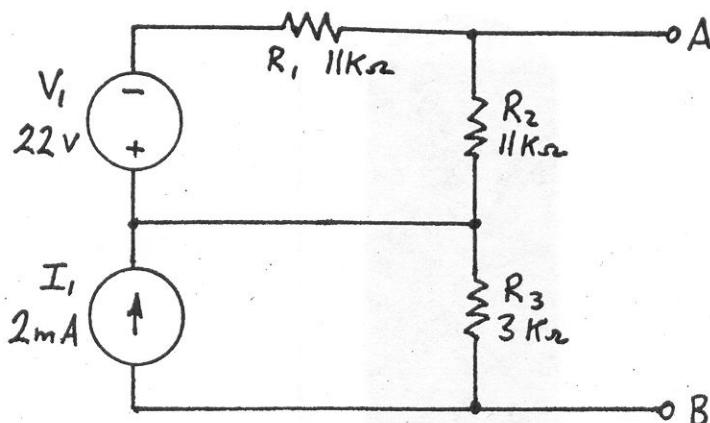
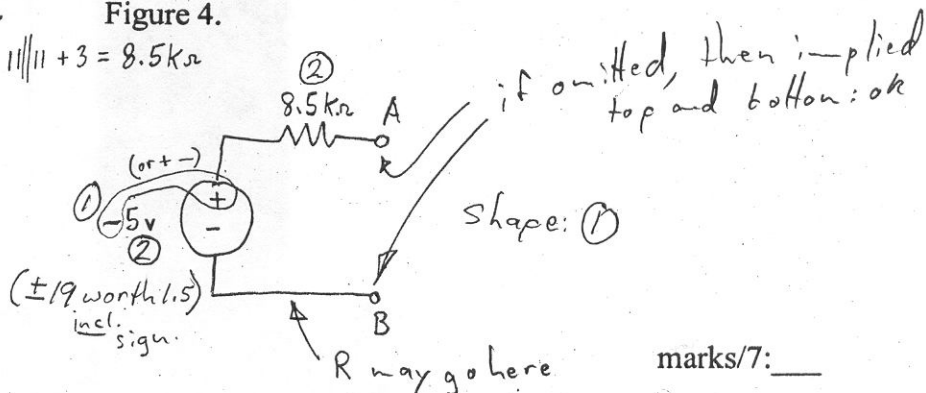


Figure 4.

Answer: Thevenin equivalent source: $11 \parallel 11 + 3 = 8.5k\Omega$



marks/7: _____

b) For the circuit shown in Figure 5, find the **Norton** equivalent source between the terminals A and B.

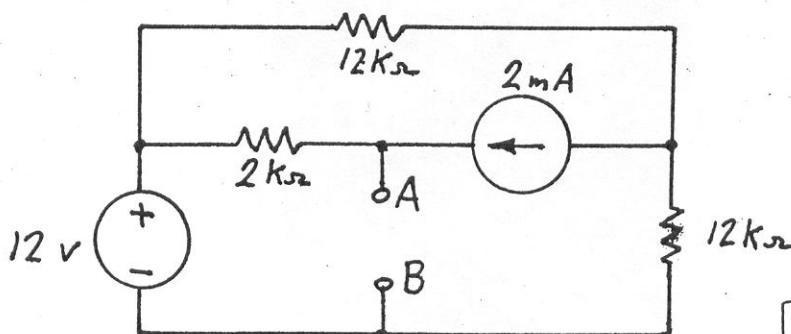
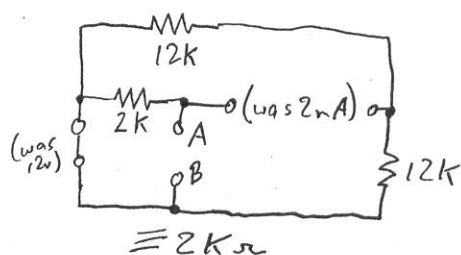
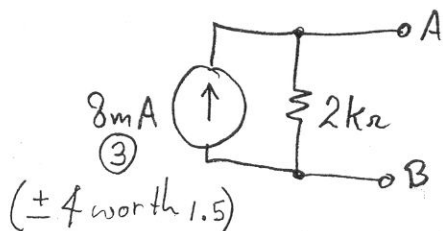


Figure 5.

$$+ 2mA + \frac{12V}{2K} = 8mA$$

Answer: Norton equivalent source:



(Corrected version)

marks/7: _____

(Q32)

2c) For the circuit shown in Figure 6:

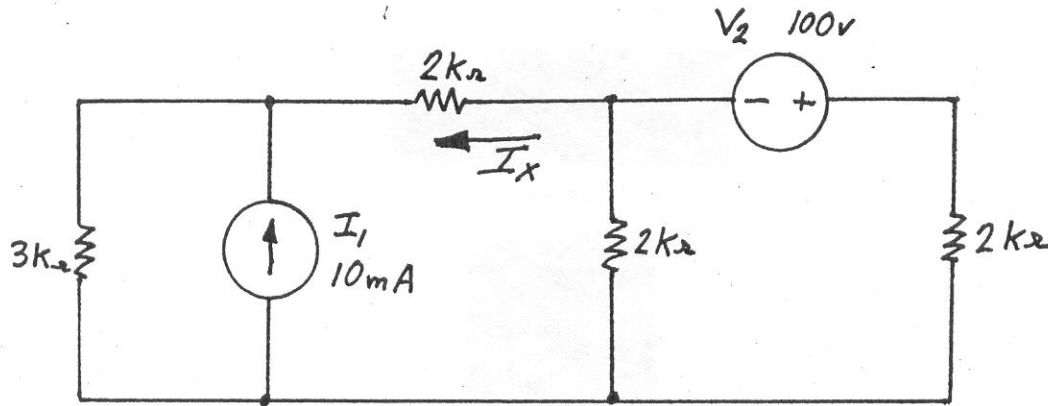


Figure 6.

Use superposition to find the contribution of each source to the current I_x , and then find the total current I_x .

$$I_x(I_1) = -10\text{mA} \cdot \frac{3\text{k}}{3\text{k} + (2\text{k} + 2\text{k} \parallel 2\text{k})} = -5\text{mA}$$

Answer: Contribution to I_x made by the source I_1 : ① ②
- 5mA

marks/3:

$$I_x(V_2) = \left(\frac{-100\text{V}}{2\text{k} + 2\text{k} \parallel 5\text{k}} \right) \left(\frac{2\text{k}}{2\text{k} + 5\text{k}} \right) = -8.33\text{mA}$$

Contribution to I_x made by the source V_2 : ① ②
- 8.33mA

marks/3:

Total I_x using superposition theorem: - 13.33mA

marks/1:

① Accept any correct sum only if at least one of the components is completely correct.

[Handwritten signature] (Q3)

Q3. Nodal Analysis and Loop Analysis

(10 marks)

a) Given the circuit in Figure 7:

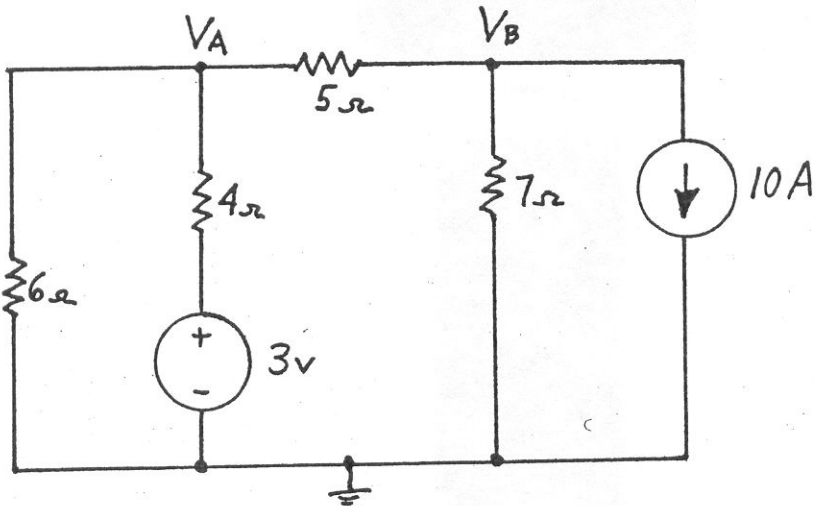


Figure 7.

Write the nodal equations at nodes A and B, using the convention that currents entering a node are positive.

Answer: At node A: $\frac{0 - V_A}{6\Omega} + \frac{3V - V_A}{4\Omega} + \frac{V_B - V_A}{5\Omega} = 0$ marks/3: _____

At node B: $\frac{V_A - V_B}{5\Omega} + \frac{0 - V_B}{7\Omega} - 10A = 0$ marks/3: _____

May be combined to mitigate penalty } Any term must be completely correct.
 If convention is opposite, -1 in each equation where it is opposite.

If = 0 is missed, -1/4 in each equation where it is missed

Equivalent forms of these equations are acceptable.

(Q52)

b) Given the circuit of Figure 8:

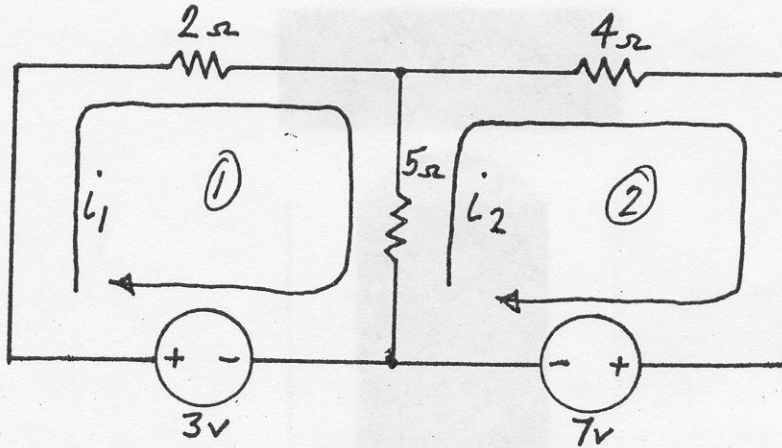


Figure 8.

Write the loop equations for the two loops.

Answer: Around loop1:
$$\overset{\textcircled{1/2}}{+2i_1} + \overset{\textcircled{1}}{5(i_1 - i_2)} - \overset{\textcircled{1/2}}{3V} = 0$$
 marks/2: _____

Around loop2:
$$\overset{\textcircled{1}}{5(i_2 - i_1)} + \overset{\textcircled{1/2}}{4i_2} + \overset{\textcircled{1/2}}{7V} = 0$$
 marks/2: _____

same comments as a)

Yan (EX) L

Q4. Phasor Analysis

(19 marks)

Note: Answers may be in either polar or Cartesian form, but still must be simplified, e.g. $15 \angle 30^\circ$ volts, or e.g. $30-j27$ ohms.

a) For the circuit given in Figure 9:

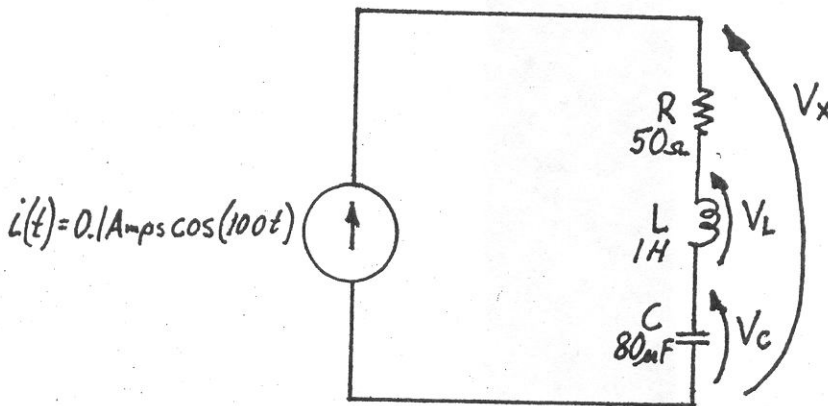


Figure 9.

a i) Find the impedance of the inductor and the impedance of the capacitor.

Answer: $Z_L = +j100 \Omega$ ^① or $100 \angle +90^\circ$ marks /1:

$\frac{1}{j\omega C} = \frac{1}{j100 \cdot 80 \mu F} = -j125 \Omega$
 Answer: $Z_C = -j125 \Omega$ ^① or $125 \angle -90^\circ$ ^① marks /2:

a ii) Find the inductor and capacitor voltages as phasors.

Answer: $V_L = +j10$ ^② or $10 \angle +90^\circ$ marks /2:

Answer: $V_C = -j12.5$ ^② or $12.5 \angle -90^\circ$ marks /2:

a iii) Find the voltage V_x as a phasor.

Answer: $V_x = \frac{5 \angle 0^\circ + j10 - j12.5}{1} = 5 - j2.5$ or $5.59 \angle -26.6^\circ$ marks /2:

① → if one of V_L or V_C is wrong (but $\pm 90^\circ$) then check $V_L + V_C$ for ①

(252)

b) Given the KCL equation $I_x = I_1 + I_2$ in which $I_1 = 20 \angle +30^\circ$ amps and $I_2 = 50 \angle -90^\circ$ amps:

$$(17.32 + j10) + (-j50)$$

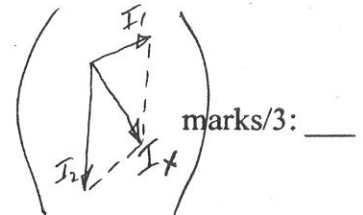
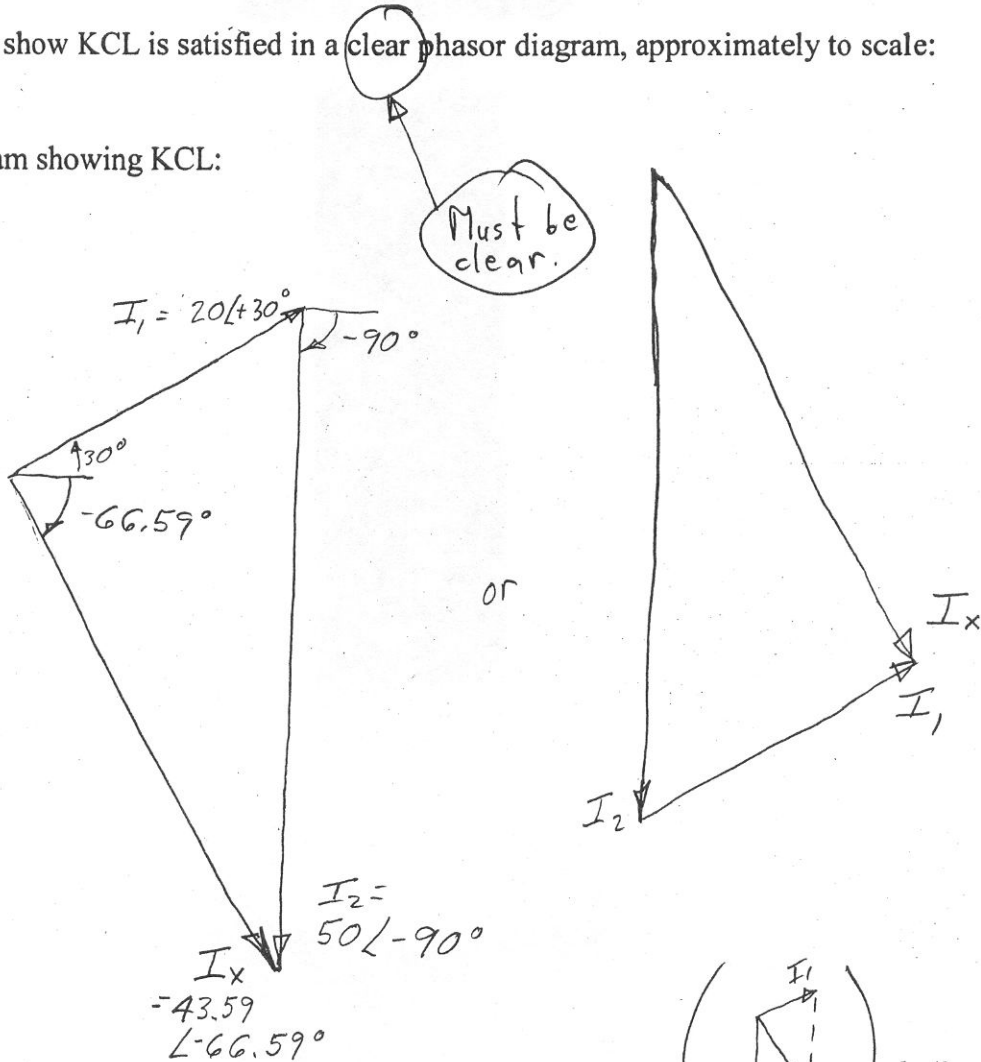
b i) Calculate I_x as a phasor:

Answer: $I_x = \frac{17.32 - j40}{\text{①} \quad \text{①}}$ or $43.59 \angle -66.59^\circ$ amps

marks /2: _____

b ii) For $I_x = I_1 + I_2$, show KCL is satisfied in a clear phasor diagram, approximately to scale:

Answer: Phasor diagram showing KCL:



c) A series RLC resonant circuit is to have a resonant frequency of 1 MHz and is to use $L = 0.1$ mH.

ci) What value of C is required?

$$\omega_0 = 2\pi f_0 = 6.28 \text{ Mr/s} \quad \omega_0 = \frac{1}{\sqrt{LC}} \quad \therefore \frac{1}{\omega_0^2} = LC$$

Answer: $C = 253 \text{ pf}$ (②) 10^x error: give (①) 10 nF give (①) marks /2: _____

cii) If the circuit is to have a bandwidth (BW) of 12 kHz, what is the value of Q? $Q = \frac{\omega_0}{\text{BW}}$

Answer: $Q = 83.3$ marks /2: _____

ciii) At resonance, what is the combined impedance of the inductor plus capacitor?

Answer: $Z = 0$ marks /1: _____

(Q12)

Q5. Frequency Response and Bode Plots

(11 marks)

Given the transfer function $H(j\omega) = V_{OUT} / V_{IN} = j10\omega / (1+j0.025\omega)$:

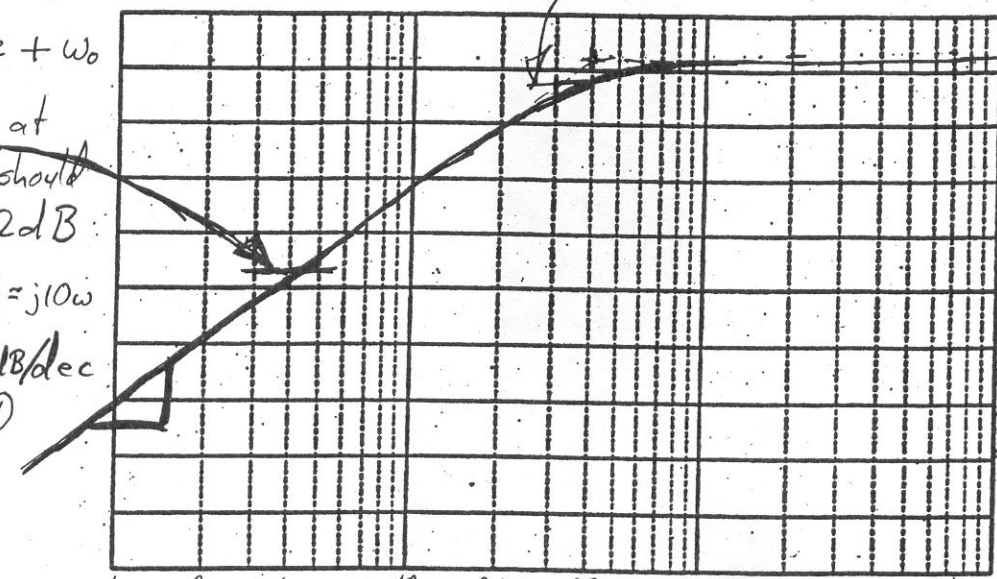
a) What is the value of the corner frequency in radians per second?

Answer: The corner frequency is 40 r/s (2) marks/2: _____

b) Draw the magnitude and phase Bode plots, clearly indicating the *important values* on each of them.

Answer: Magnitude Bode Plot:

① shape + ω_0
 ① value at $\frac{1}{10}\omega_0 = 4 \text{ r/s}$ should be $\approx 32 \text{ dB}$
 $\omega \rightarrow 0: H(j\omega) = j10\omega$
 $\rightarrow +20 \text{ dB/dec}$
 ①

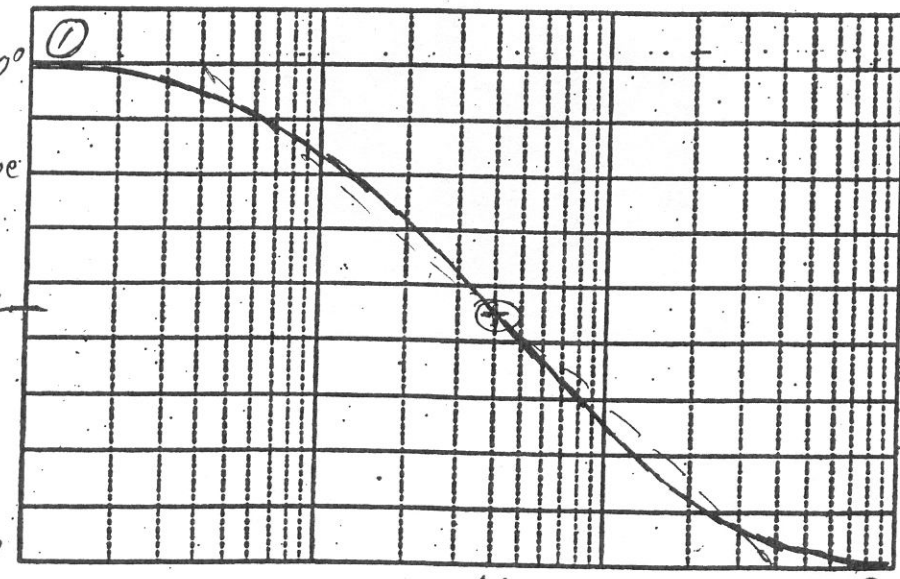


$\omega \rightarrow \infty: H(j\omega) = 400$
 \downarrow
 52 dB ①

marks/5: _____

Answer: Phase Bode plot:

$\omega \rightarrow 0: \rightarrow +90^\circ$ ①
 ① smooth shape
 $\omega = \omega_0: \rightarrow 45^\circ$ ①
 $\omega \rightarrow \infty: \rightarrow 0^\circ$



marks/4: _____

(Handwritten signature)

Q6. Transient Analysis

(22 marks)

a) In the circuit of Figure 10, switch S is open for all $t < 0$. At $t = 0$ the switch is closed.

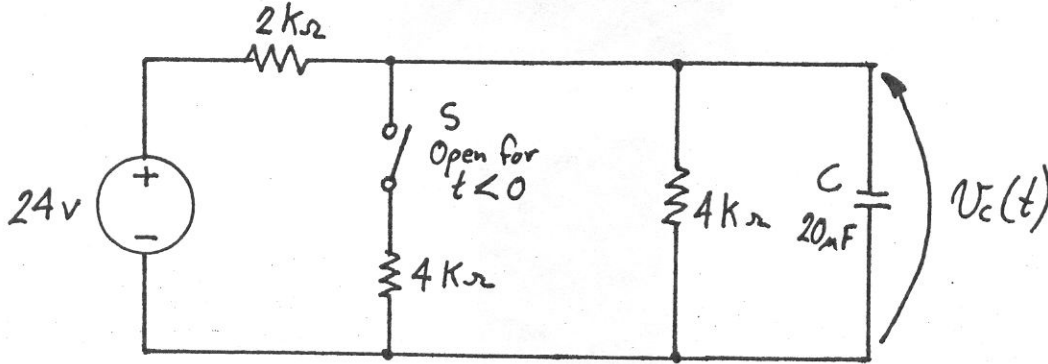


Figure 10.

a i) Find the voltage across the capacitor, v_C , for $t < 0$.

Answer: For $t < 0$, $v_C = +16\text{V}$ marks/1: 1

a ii) Derive an expression for the voltage across the capacitor, $v_C(t)$, for $t > 0$.

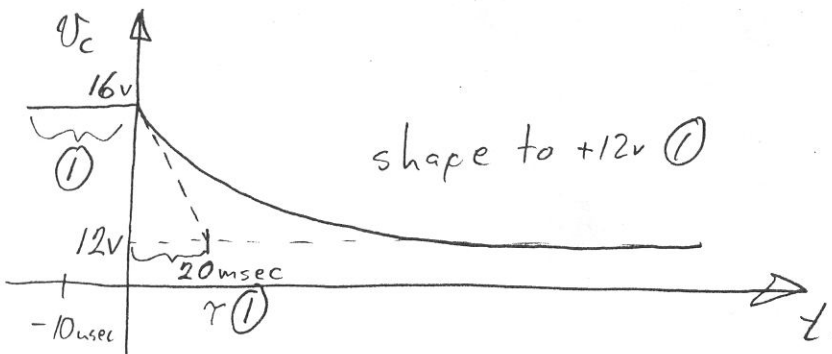
$V_I = 16\text{V}$
 $V_F = 12\text{V}$
 $\tau = 20\text{msec}$

Answer: For $t > 0$, $v_C(t) = \frac{V_F}{V_I - V_F} + \frac{V_I - V_F}{V_I - V_F} e^{-t/20\text{msec}}$ marks/7: 2
 eqn ①

Handwritten notes: e^{-50t} is ok.
 $e^{-t/20}$ is -1
 $e^{-t/20 \times 10^6}$ is -1, n=3, etc.

a iii) Sketch $v_C(t)$ from $t = -10\text{ msec.}$ to $t = +60\text{ msec.}$ clearly showing the time constant and its relationship to the waveform.

Answer: Graph of $v_C(t)$:



Handwritten notes: One of V_I , V_F or τ is correct; follow through. Two wrong: follow through -1. Three wrong: -2.

marks/3:

232

a iv) If the response above is interrupted at $t = +10$ msec. by the switch being opened, derive a new equation for $v_C(t)$ for $t > +10$ msec.

Answer: For $t > +10$ msec., $v_C(t) = 16 - 1.574 e^{-(t-10\text{msec})/26.7\text{msec}}$ marks/5: _____

$V_{I_{\text{new}}} = 12 + 4e^{-10/20} = 14.426$ $V_{F_{\text{new}}} = 16 \text{ v}$ $\tau_{\text{new}} = 26.7 \text{ msec}$

b) In the circuit shown below in Figure 11, switch S is open for all $t < 0$. At $t = 0$ the switch is closed.

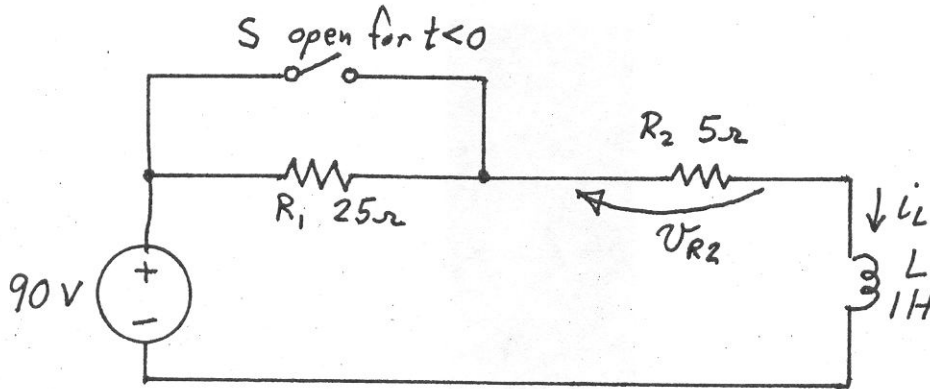


Figure 11.

b i) Find the inductor current, i_L , just before the switch is closed.

Answer: $i_L = \frac{90\text{V}}{30\Omega} + 3\text{A}$ (2) marks/2: _____

b ii) Find v_{R2} at the instant just after the switch is closed. $+(3\text{A})(5\Omega)$

Answer: $v_{R2} = +15\text{V}$ (2) marks/2: _____

b iii) Find the final value of v_{R2} .

Answer: $v_{R2} = +90\text{V}$ (1) marks/1: _____

b iv) Find the time constant for the response of v_{R2} for $t > 0$.

Answer: Time Constant = $\frac{L}{R} = \frac{1\text{H}}{5\Omega} = 0.2\text{sec}$ (1) marks/1: _____

Best Wishes for a Safe and Happy Holiday!