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Question 1

Let $\phi = [\phi_1 \quad \phi_2]^T$ be a vector of design variables. Error functions for design optimization are

$$f_1(\phi) = \phi_2 - 4$$
$$f_2(\phi) = 2 - \phi_2$$
$$f_3(\phi) = 1 - \phi_1$$
$$f_4(\phi) = \phi_1 - 7$$

Let ϕ^0 be the nominal point (design center) and $\mathbf{x}$ be a 2-dimensional random variable. We consider 5 random outcomes of $\mathbf{x}$:

$$\mathbf{x}^{(1)} = [0 \quad 2]^T, \quad \mathbf{x}^{(2)} = [-2 \quad 0]^T, \quad \mathbf{x}^{(3)} = [-1 \quad -2]^T,$$
$$\mathbf{x}^{(4)} = [1 \quad -2]^T, \quad \mathbf{x}^{(5)} = [2 \quad 0]^T.$$

The corresponding 5 circuit outcomes are

$$\phi^{(k)} = \phi^0 + \mathbf{x}^{(k)}, \quad k = 1, 2, \ldots, 5.$$

The objective function for generalized l_p centering is

$$U(\phi^0) = H_p(\mathbf{u})$$

where the kth component of vector $\mathbf{u}$ is defined as

$$u_k = \alpha_k H_q(\mathbf{f}(\phi^{(k)})), \quad k = 1, 2, \ldots, 5$$

and the weighting factor $\alpha_k = 1$ for $k = 1, 2, \ldots, 5$.

Suppose the starting point for optimization is $\phi^0 = [\phi^0{}_1 \quad \phi^0{}_2]^T = [4.0 \quad 4.2]^T$. Suppose $\phi^0{}_1$ is fixed at 4.0 throughout optimization.

Case 1: p = q = 1

Requirements:

(1) Derive the expressions for the objective function U vs. variable $\phi^0{}_2$ for $0 \le \phi^0{}_2 < \infty$. Draw graphically U vs. $\phi^0{}_2$ for $0 \le \phi^0{}_2 < \infty$.

(2) Find the optimal design center ϕ^0, i.e., the minimum of $U(\phi^0)$.

(3) Draw the 2-dimensional acceptance region and the locations of the 5 circuit outcomes before optimization.

(4) Draw the 2-dimensional acceptance region and the locations of the 5 circuit outcomes after optimization.

Case 2: $p = q = \infty$

Requirements (1) - (4): same as in Case 1.

Compare the yield and worst-case performances between the solutions of the two cases.

Solution

Case 1

(1) Since ϕ_1^0 is fixed, $U(\underset{\sim}{\phi}^0)$ is a function of only ϕ_2^0.

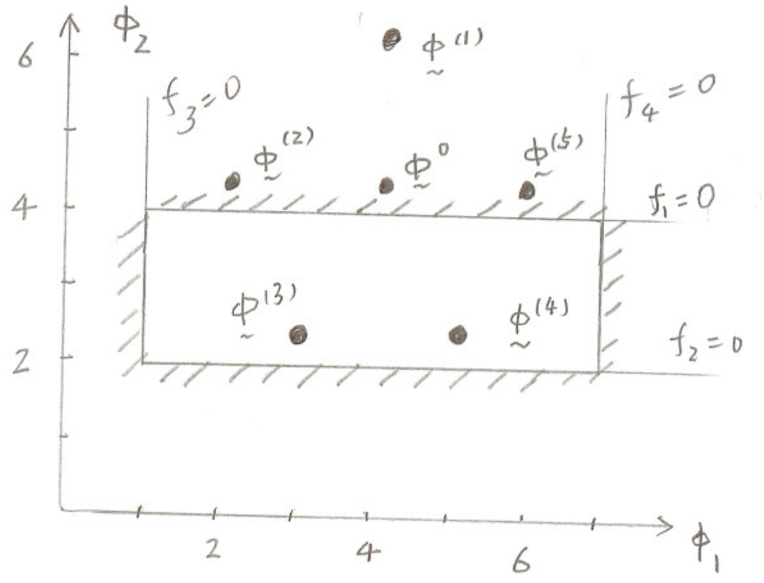

Figure 1

$$\underset{\sim}{\phi}^{(1)} = \underset{\sim}{\phi}^0 + \underset{\sim}{\chi}^{(1)} = \begin{bmatrix} 4 \\ \phi_2^0 + 2 \end{bmatrix}$$

$$f_1(\underset{\sim}{\phi}^{(1)}) = \phi_2^0 - 2 = \begin{cases} > 0 & \text{if } \phi_2^0 > 2 \\ \leq 0 & \text{if } \phi_2^0 \leq 2 \end{cases}$$

$$f_2(\underset{\sim}{\phi}^{(1)}) = -\phi_2^0 = \begin{cases} > 0 & \text{if } \phi_2^0 < 0 \\ \leq 0 & \text{if } \phi_2^0 \geq 0 \end{cases}$$

$$f_3(\underset{\sim}{\phi}^{(1)}) = -3 \leq 0$$

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$$u_1 = \begin{cases} \sum_{j \in J} f_j(\underset{\sim}{\phi}^{(1)}) = f_2(\underset{\sim}{\phi}^{(1)}) = -\phi_2^0, & \text{if } \phi_2^0 \leq 0 \quad (J = \{2\}) \\ \sum_{j \in J} f_j(\underset{\sim}{\phi}^{(1)}) = f_1(\underset{\sim}{\phi}^{(1)}) = \phi_2^0 - 2, & \text{if } \phi_2^0 \geq 2 \quad (J = \{1\}) \\ -\left(\sum_{j=1}^{4} \left(-f_j(\underset{\sim}{\phi}^{(1)})\right)^{-1} \right)^{-1}, & \text{if } 0 < \phi_2^0 < 2 \end{cases}$$

Notice that $u_1 = \begin{cases} > 0 & \text{if } \phi_2^0 < 0 \text{ or } \phi_2^0 > 2 \\ \leq 0 & \text{otherwise} \end{cases}$

Similarly:

$$u_2 = H_1(\underset{\sim}{f}(\underset{\sim}{\phi}^{(2)})) = \begin{cases} f_1(\underset{\sim}{\phi}^{(2)}) = \phi_2^0 - 4 & \text{if } \phi_2^0 \geq 4 \\ f_2(\underset{\sim}{\phi}^{(2)}) = 2 - \phi_2^0 & \text{if } \phi_2^0 \leq 2 \\ -\left(\sum_{j=1}^{4} (-f_j(\underset{\sim}{\phi}^{(2)}))^{-1}\right)^{-1} & \text{if } 2 < \phi_2^0 < 4 \end{cases}$$

Notice $u_2 = \begin{cases} > 0 & \text{if } \phi_2^0 < 2 \quad \text{or} \quad \phi_2^0 > 4 \\ \leq 0 & \text{otherwise} \end{cases}$

$$u_3 = H_1(\underset{\sim}{f}(\underset{\sim}{\phi}^{(3)})) = \begin{cases} f_2(\underset{\sim}{\phi}^{(3)}) = 4 - \phi_2^0 & \text{if } \phi_2^0 \leq 4 \\ f_1(\underset{\sim}{\phi}^{(1)}) = \phi_2^0 - 6 & \text{if } \phi_2^0 \geq 6 \\ -\left(\sum_{j=1}^{4} (-f_j(\underset{\sim}{\phi}^{(3)}))^{-1}\right)^{-1} & \text{if } 4 < \phi_2^0 < 6 \end{cases}$$

$$u_3 = \begin{cases} > 0 & \text{if } \phi_2^0 < 4 \quad \text{or} \quad \phi_2^0 > 6 \\ \leq 0 & \text{otherwise} \end{cases}$$

$$u_4 = \begin{cases} u_3 & \text{if } \phi_2^0 \leq 4 \quad \text{or} \quad \phi_2^0 \geq 6 \\ -\left(\sum_{j=1}^{4} (-f_j(\underset{\sim}{\phi}^{(4)}))^{-1}\right)^{-1}, & \text{otherwise} \end{cases}$$

$$u_5 = \begin{cases} u_2 & \text{if } \phi_0^2 \leq 2 \quad \text{or} \quad \phi_2^0 \geq 4 \\ -\left(\sum_{j=1}^{4} (-f_j(\underset{\sim}{\phi}^{(5)}))^{-1}\right)^{-1} & \text{otherwise} \end{cases}$$

$$U(\phi_2^\circ) = \begin{cases} \sum_{k\in K} u_k & \text{if } K \neq \text{Null} \\ -\left(\sum_{k=1}^{5} (-u_k)^{-1}\right)^{-1} & \text{if } K = \text{null} \end{cases}$$

where $K = \{ k \mid \text{if } u_k > 0,\ k = 1, 2, 3, 4, 5 \}$.

It can be found that for any value of ϕ_2°, there is always at least one u_k, $k = 1, 2, 3, 4, 5$, which is positive. Therefore index set K is never Null.

$$U(\phi_2^\circ) = \sum_{k\in K} u_k = \begin{cases} u_1+u_2+u_3+u_4+u_5 & \text{if } \phi_2^\circ < 0, \ (K=\{1,2,3,4,5\}) \\ u_2+u_3+u_4+u_5 & \text{if } 0 \leq \phi_2^\circ < 2, \ (K=\{2,3,4,5\}) \\ u_1+u_3+u_4 & \text{if } 2 \leq \phi_2^\circ < 4, \ (K=\{1,3,4\}) \\ u_1+u_2+u_5 & \text{if } 4 \leq \phi_2^\circ < 6, \ (K=\{1,2,5\}) \\ u_1+u_2+u_3+u_4+u_5, & \text{if } 6 \leq \phi_2^\circ, \ (K=\{1,2,3,4,5\}) \end{cases}$$

i.e.,

$$U(\phi_2^\circ) = \begin{cases} 12-5\phi^\circ & \text{if } \phi_2^\circ < 0 \\ 12-4\phi_2^\circ & \text{if } 0 \leq \phi_2^\circ < 2 \\ 6-\phi_2^\circ & \text{if } 2 \leq \phi_2^\circ < 4 \\ 3\phi_2^\circ - 10 & \text{if } 4 \leq \phi_2^\circ < 6 \\ 5\phi_2^\circ - 22 & \text{if } 6 \leq \phi_2^\circ \end{cases}$$

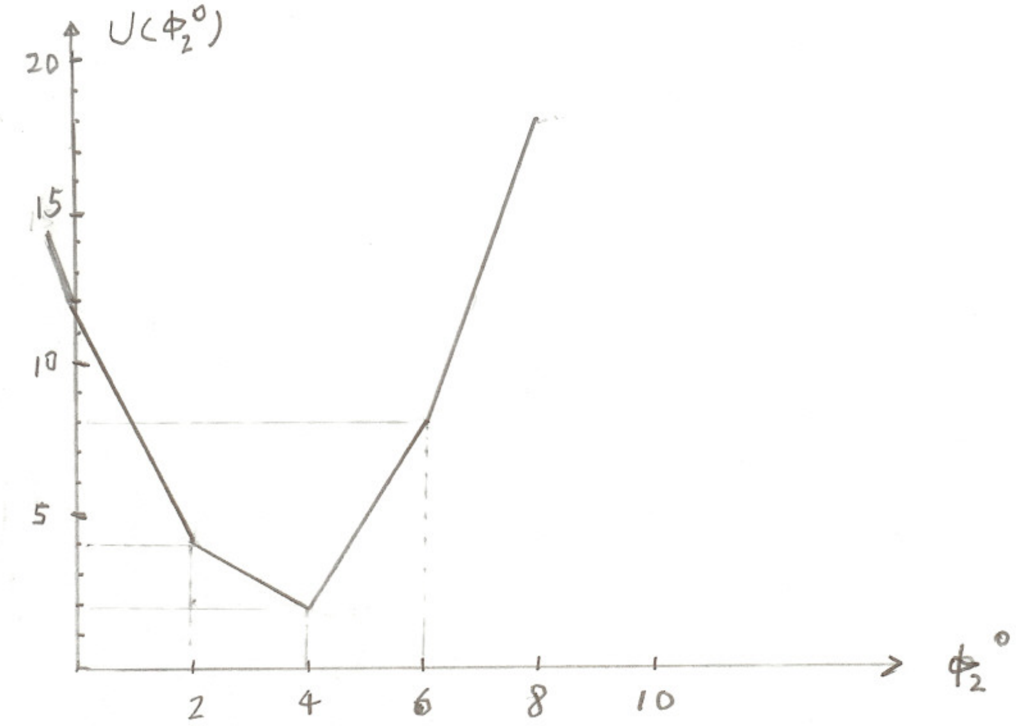

Figure 2

(2) From above figure, optimal value $\phi_2^0 = 4$

& $\min U(\phi_2^0) = 2$.

(3) See Figure 1

(4)

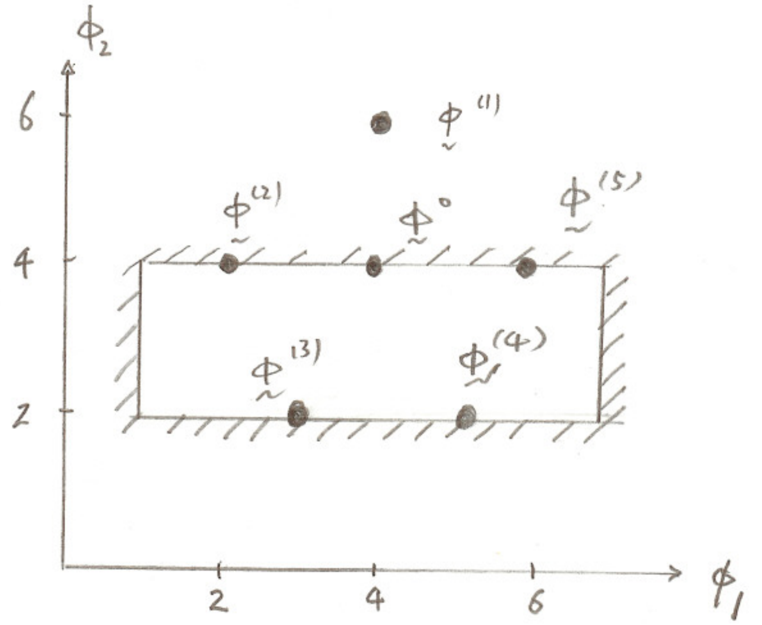

Figure 3

Case 2

(1). Case 2 can be solved in a way similar to case 1, i.e, first find

$$U_k = \max_j \{ f_j (\underset{\sim}{\Phi}^{(k)}) \}$$

for $k=1, 2, 3, 4, 5$ and then find

$$U(\Phi_2^0) = \max_k \{ U_k \}.$$

Here we present a different, simpler, (but not general) approach. (Graphic inspection approach)

$$U(\Phi_2^0) = \max_k \{ U_k \} = \max_k \left\{ \max_j \{ f_j (\underset{\sim}{\Phi}^{(k)}), \} \right\}$$

$$= \max_{j, k,} \{ f_j (\underset{\sim}{\Phi}^{(k)}) \}.$$

Notice that for this particular example,

$f_j (\underset{\sim}{\Phi}^{k)}) =$ distance from $\underset{\sim}{\Phi}^{(k)}$ to the boundary defined by $f_j = 0$, see Figure 1

By inspecting Figure 1, either

$$f_1 (\Phi^{(1)})$$

or

$$f_2 (\Phi^{(3)})$$

Will represent the most vilation of specifications.

To be more specific,

$$U(\phi_2^0) = \begin{cases} f_1(\underset{\sim}{\phi}^{(1)}) = \phi_2^0 - 2 & \text{if } \phi_2^0 > 3 \\ f_2(\phi^{(3)}) = 4 - \phi_2^0 & \text{if } \phi_2^0 < 3 \end{cases}$$

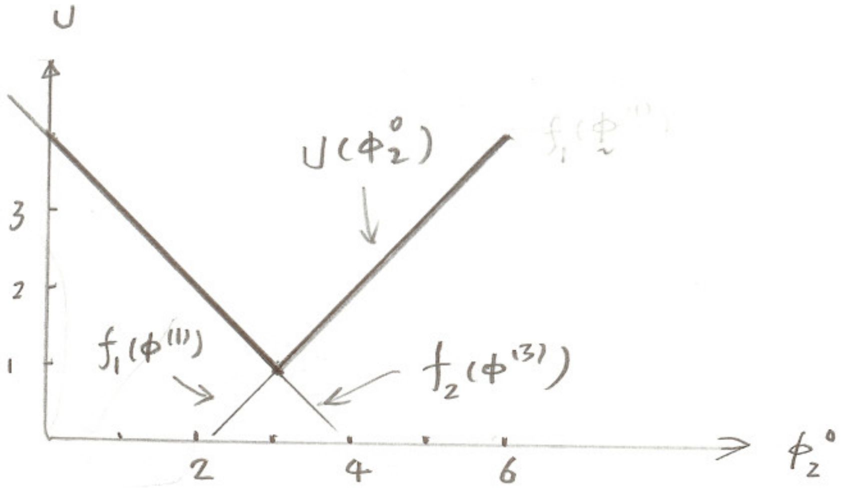

Figure 4

(2) From Figure 4, optimal solution is $\phi_2^0 = 3$.
and the minimum of $U(\phi_2^0)$ is 1

(3) See Figure 1.

(4)

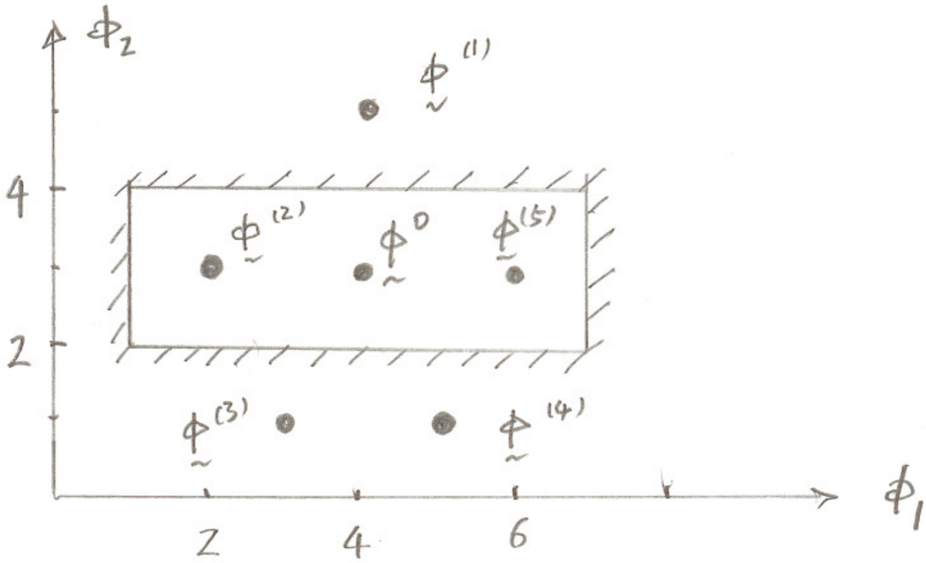

Figure 5

Comparison :

	Solution of Case 1	Solution of Case 2
Yield (not counting ϕ^0)	80%	40%
Worst Case Performance	largest violation of all specs by all the 5 circuits is = 2	= 1

Therefore case 2 is suitable for Worst case design. Case 1 is suitable for yield maximization