Question

Suppose ϕ is a n-vector containing circuit design variables. Suppose $f(\phi)$ is a m-vector containing error functions in the form

$$W_U(F(\phi, \omega) - S_U(\omega))$$

and

$$-W_L(F(\phi, \omega) - S_L(\omega))$$

where S_U and S_L are upper and lower specifications on the circuit response F, respectively. W_U and W_L are weighting factors.

Suppose the quality of the circuit is measured by the generalized l_p function:

$$v(\phi) = H_p(f) = H_p(f(\phi)).$$

Suppose the circuit can be classified into 4 categories according to the following table.

category	criteria	retail price per circuit
high quality	If $v(\phi) \leq -1$	\$500
standard	If $-1 < v(\phi) \leq 0$	\$100
discounted	If $0 < v(\phi) \leq 1$	\$20
discarded	If $1 < v(\phi)$	-\$0.0

Consider the mass production of the circuit. Due to process variations and machine tolerances during production, some circuits maybe of high-quality while others may be standard or non-acceptable. Formulate a generalized l_p optimization to maximize the total income from the sales of the circuits.

Solution

Let $A = \$500.00$, $B = \$100.00$, $C = \$20.00$, $D = \$0.00$.

Let $\underset{\sim}{\phi}^0$ be the nominal point. Let $\underset{\sim}{x}$ be a n-vector of random variables representing the random variation in $\underset{\sim}{\phi}$. Consider K random samples $\underset{\sim}{x}^{(k)}$, $k=1,2,\cdots,K$, and K circuits represented by

$$\phi^{(k)} = \underset{\sim}{\phi}^0 + \underset{\sim}{x}^{(k)}, \quad k=1,2,\cdots,K \tag{1}$$

Consider generalized l_p function

$$H_p (\underset{\sim}{u}_A (\underset{\sim}{\phi}^0)) \tag{2}$$

where

$$\underset{\sim}{u}_A = \begin{bmatrix} \alpha_A^{(1)} (v(\underset{\sim}{\phi}^{(1)}) + 1) \\ \alpha_A^{(2)} (v(\underset{\sim}{\phi}^{(2)}) + 1) \\ \vdots \\ \alpha_A^{(K)} (v(\underset{\sim}{\phi}^{(K)}) + 1) \end{bmatrix} \tag{3}$$

(hint = "$v(\phi)+1$" is derived from high-quality criteria $v(\underset{\sim}{\phi}) \leq -1$, i.e., $v(\underset{\sim}{\phi}) + 1 \leq 0$)

If we choose the weighting factors $\alpha_A^{(k)}$, $k=1,2,\cdots,K$ to be constant which equals $\alpha_A^{(k)} = \dfrac{1}{v(\underset{\sim}{\phi}^{(k)})+1}$ at the starting point of optimization, then

$$H_p^+ (\underset{\sim}{\mu}_A (\underset{\sim}{\Phi}^0)) \tag{4}$$

equals the number of non-high-quality circuits at starting point of optimization. If not all the K circuits are of high-quality, then

$$H_p (\underset{\sim}{\mu}_A (\underset{\sim}{\Phi}^0)) = H_p^+ (\underset{\sim}{\mu}_A (\underset{\sim}{\Phi}^0)) \tag{5}$$

is an approximation of the number of non-high-quality circuits during optimization.

Similarly, we define K-vectors $\underset{\sim}{\mu}_B$ and $\underset{\sim}{\mu}_C$ whose kth element, $k = 1, 2, \cdots K$, are $\alpha_B^{(k)} \nu (\underset{\sim}{\Phi}^{(k)})$ and $\alpha_C^{(k)} (\nu (\underset{\sim}{\Phi}^{(k)}) - 1)$, respectively.

$$\alpha_B^{(k)} = \frac{1}{\nu (\underset{\sim}{\Phi}^{(k)})} \tag{6}$$

$$\alpha_C^{(k)} = \frac{1}{\nu (\underset{\sim}{\Phi}^{(k)} - 1)} \tag{7}$$

are constant weighting factors calculated at starting point of optimization and kept unchanged during optimization.

$H_p(\underset{\sim}{\mu}_B)$ approximates the number of discounted and failed circuits. $H_p(\underset{\sim}{\mu}_c)$ approximates the number of failed circuits.

Total income from the sales of K circuits:

$$\begin{aligned}\text{income} &= A[K - H_p(\underset{\sim}{\mu}_A)] + B[H_p(\underset{\sim}{\mu}_A) - H_p(\underset{\sim}{\mu}_B)] \\ &\quad + C[H_p(\underset{\sim}{\mu}_B) - H_p(\underset{\sim}{\mu}_c)] + D H_p(\underset{\sim}{\mu}_c) \\ &= -(A-B) H_p(\underset{\sim}{\mu}_A) - (B-C) H_p(\underset{\sim}{\mu}_B) \\ &\quad - (C-D) H_p(\underset{\sim}{\mu}_c) + A \cdot K \qquad (8)\end{aligned}$$

The optimization problem for maximizing income is

$$\min_{\underset{\sim}{\Phi}^{\circ}} U(\underset{\sim}{\Phi}^{\circ}) \qquad (9)$$

where

$$\begin{aligned}U(\underset{\sim}{\Phi}^{\circ}) &= (A-B) H_p(\underset{\sim}{\mu}_A(\underset{\sim}{\Phi}^{\circ})) + (B-C) H_p(\underset{\sim}{\mu}_B(\underset{\sim}{\Phi}^{\circ})) \\ &\quad + (C-D) H_p(\underset{\sim}{\mu}_c(\underset{\sim}{\Phi}^{\circ})) \\ &= 400 H_p(\underset{\sim}{\mu}_A(\underset{\sim}{\Phi}^{\circ})) + 180 \cdot H_p(\underset{\sim}{\mu}_B(\underset{\sim}{\Phi}^{\circ})) \\ &\quad + 20 \cdot H_p(\underset{\sim}{\mu}_c(\underset{\sim}{\Phi}^{\circ})). \qquad (10)\end{aligned}$$

(Notice "A·K" in (8) is a constant and not included in (10))