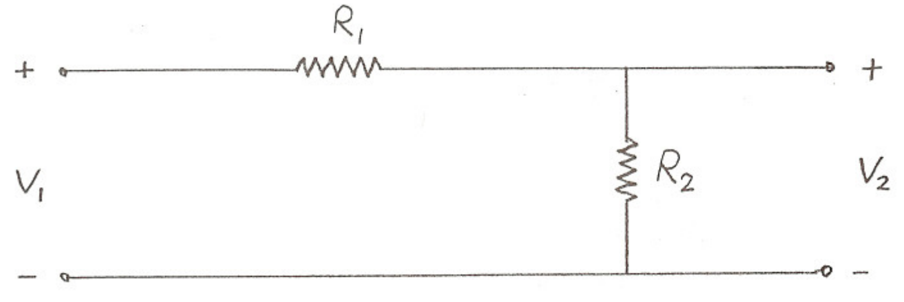
Question 3

Consider the voltage divider shown



The circuit is linear. Given $R_1 = 2\Omega$ and $R_2 = 1.5\Omega$. Let $\phi = [R_1 \quad R_2]^T$ and $T = V_2/V_1$.

(1) For the given values of R_1 and R_2, calculate $\partial T/\partial R_1$ and $\partial T/\partial R_2$ using direct differentiation of T.

(2) Suppose the tolerance on R_1 is ±5% and on R_2 is ±10%. Suppose the design specification on T is $0.39 \leq T \leq 0.46$. Find the values of the worst-case vertex of ϕ and the corresponding worst case performance T using sensitivity information. Verify your predicted worst-case by calculating T at all vertices.

(3) List all optimization variables and constraints for integrated design centering, tolerancing and tuning of this circuit.

(4) In your list of constraints, are any of the constraints inactive throughout optimization? If so, provide another list (reduced list) of variables and constraints that would result in the same solution of optimization as in (3).

Solution

(1) $$T = \frac{R_2}{R_1+R_2} = \frac{1.5}{3.5} = 0.42857$$

$$\frac{\partial T}{\partial R_1} = \frac{-R_2}{(R_1+R_2)^2} = \frac{-1.5}{(1.5+2)^2} = \frac{-1.5}{(3.5)^2} = -0.12244898$$

$$\frac{\partial T}{\partial R_2} = \frac{-R_2}{(R_1+R_2)^2} + \frac{1}{R_1+R_2} = 0.1632653$$

(2) Error functions:

$$f_1(\underset{\sim}{\phi}) = T - 0.46$$

$$f_2(\underset{\sim}{\phi}) = 0.39 - T, \quad \text{where} \quad \underset{\sim}{\phi} = \begin{bmatrix} R_1 \\ R_2 \end{bmatrix}$$

worst-case vertex for f_1:

$$\underset{\sim}{u} = \text{sign}\left[\frac{\partial f_1}{\partial \underset{\sim}{\phi}}\right] = \text{sign}\left[\frac{\partial T}{\partial \underset{\sim}{\phi}}\right] = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$\underset{\sim}{\phi} = \underset{\sim}{\phi}^0 + \begin{bmatrix} \varepsilon_1 & \\ & \varepsilon_2 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 - 2\times 0.05 \\ 1.5 + 1.5\times 0.1 \end{bmatrix} = \begin{bmatrix} 1.9 \\ 1.65 \end{bmatrix}$$

at this vertex:

$$T = \frac{R_2}{R_1+R_2} = \frac{1.65}{1.65+1.9} = 0.464788$$

$$\underset{\sim}{f} = \begin{bmatrix} T - .46 \\ .39 - T \end{bmatrix} = \begin{bmatrix} .004788 \\ -0.074788 \end{bmatrix}$$

worst case for f_2:

$$\underset{\sim}{u} = \text{Sign}\left[\frac{\partial f_2}{\partial \underset{\sim}{\phi}}\right] = -\text{sign}\left[\frac{\partial T}{\partial \underset{\sim}{\phi}}\right] = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\underset{\sim}{\phi} = \underset{\sim}{\phi}^{\circ} + \begin{bmatrix} \varepsilon_1 & \\ & \varepsilon_2 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 2 + 2 \cdot 0.05 \\ 1.5 - 1.5 \cdot 0.1 \end{bmatrix} = \begin{bmatrix} 2.1 \\ 1.35 \end{bmatrix}$$

at this vertex,

$$T = \frac{R_2}{R_1 + R_2} = \frac{1.35}{2.1 + 1.35} = 0.3913.$$

$$\underset{\sim}{f} = \begin{bmatrix} T - .46 \\ .39 - T \end{bmatrix} = \begin{bmatrix} -0.0687 \\ -0.0013 \end{bmatrix}$$

Worst case for overall circuit:

$$\underset{\sim}{\phi}^{wc} = \begin{bmatrix} 1.9 \\ 1.65 \end{bmatrix},$$

$$T = 0.464788$$

$$f^{wc} = 0.004788$$

Other vertices:

$$\phi^{\circ} + \begin{bmatrix} \varepsilon_1 & \\ & \varepsilon_2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2.1 \\ 1.65 \end{bmatrix}$$

$T = 0.44$, within specification

$$\phi^{\circ} + \begin{bmatrix} \varepsilon_1 & \\ & \varepsilon_2 \end{bmatrix} \begin{bmatrix} -1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1.9 \\ 1.35 \end{bmatrix}$$

$T = 0.4153846$ within specification.

(3): variables:

$R_1 \quad R_2$

$\varepsilon_1 \quad \varepsilon_2$

$t_1 \quad t_2$

$\rho_1(1) \quad \rho_2(1)$

$\rho_1(2) \quad \rho_2(2)$

$\rho_1(3) \quad \rho_2(3)$

$\rho_1(4) \quad \rho_2(4)$

Constr.

$f_1(\phi^0 + \underset{\sim}{E} \begin{bmatrix} 1 \\ -1 \end{bmatrix}) \le 0$ (← $\underset{\sim}{I} \cdot \rho$)

$f_2(\phi^0 + \underset{\sim}{E} \begin{bmatrix} 1 \\ -1 \end{bmatrix}) \le 0$ } vertex #1

$\vdots$

$f_1(\underset{\sim}{\phi}^0 + \underset{\sim}{E} \begin{bmatrix} -1 \\ -1 \end{bmatrix}) \le 0$

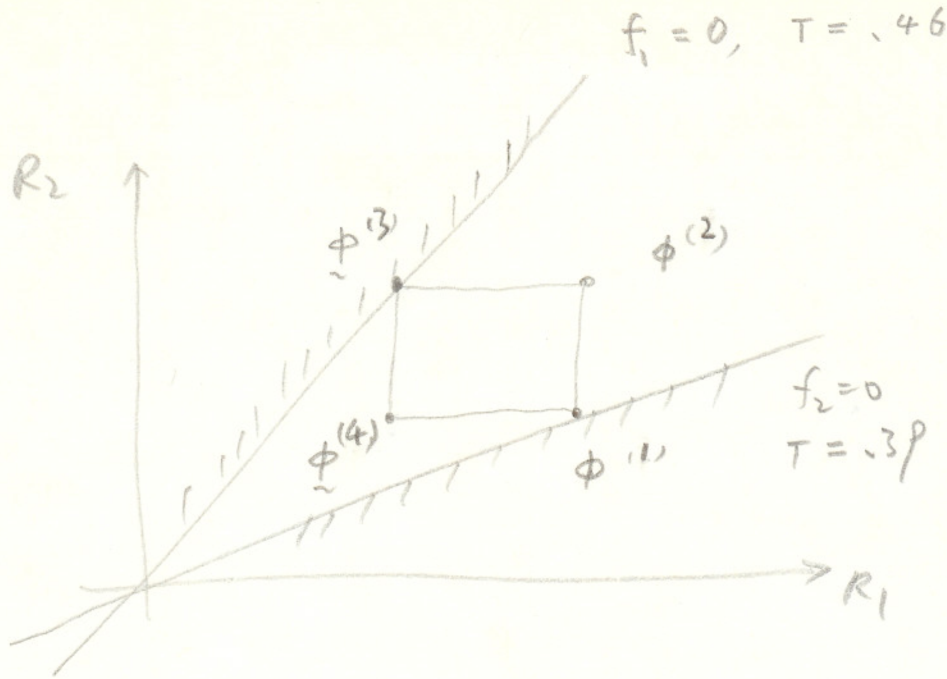
$f_2(\underset{\sim}{\phi}^0 + \underset{\sim}{E} \begin{bmatrix} -1 \\ -1 \end{bmatrix}) \le 0$ } vertex #4

$\varepsilon_i \ge 0 \quad , \quad i = 1, 2$

$t_i \ge 0 \quad , \quad i = 1, 2$

$-1 \le \rho_i(j) \le 1 \quad i = 1, 2 \quad j = 1, 2, 3, 4.$

(4)



$$T = \frac{R_2}{R_1 + R_2} \Longrightarrow R_2 = \frac{T}{1-T} R_1$$

vertices 2 & 4 automatically are within acceptable region as long as vertices 3 & 1 are acceptable

Variables:

$R_1 \quad R_2$

$\varepsilon_1 \quad \varepsilon_2$

$t_1 \quad t_2$

$\rho_1(1), \quad \rho_2(1)$

$\rho_1(3), \quad \rho_2(3)$

Constr.

Same as in (3) but delete ρ (circled: $T \rho$)

$f_1(\phi^0 + E \begin{bmatrix} -1 \\ -1 \end{bmatrix}) \le 0$

$f_2(\phi^0 + E \begin{bmatrix} -1 \\ -1 \end{bmatrix}) \le 0$

$f_1(\phi^0 + E \begin{bmatrix} 1 \\ 1 \end{bmatrix}) \le 0$

$f_2(\phi^0 + E \begin{bmatrix} 1 \\ 1 \end{bmatrix}) \le 0$.