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Question 1

Consider the simple low-pass LC filter circuit



The output voltage can be expressed as

$$V_0(\omega) = \frac{1}{CL_1L_2S^3 + C(L_1 + L_2)S^2 + (L_1 + L_2 + C)S + 2} \quad \text{where } S = j\omega$$

The insertion loss of the circuit is defined as

$$L = 20 \log_{10} \left(\frac{R_s}{R_s + R_L} \frac{|V_i|}{|V_0|} \right)$$

In this example, $V_i = 1$ (corresponding to a 1 ampere source with $R_s = 1$).

The design specifications imposed on the filter insertion loss are

 $S_1(\omega) = 25DB$, if $\omega \ge 2.5$ rad/second

 $S_u(\omega) = 1.5DB$, if $\omega \le 1$ rad/second

Design variables for the circuit are

$$\Phi = \begin{bmatrix} L_1 \\ L_2 \\ C \end{bmatrix}$$

Suppose 10 uniformly spaced frequency samples in the passband, (i.e., from 0.1 to 1 rad/second), and 1 frequency sample at $\omega = 2.5$ rad/second are chosen.

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Part 1:

(1) Write explicitly the vector of error functions $f(\Phi)$ in terms of $V_0(\omega)$ and the specific values of S_u , S_1 and ω .

Part 2 (parts 2 and 3 are NOT part of the assignment. They are an example of how to do a possible project).

Carry out simulation and optimization using the ADS program according the following requirements.

- (2) Simulate the circuit before optimization to obtain insertion loss plot
- (3) Perform minimax optimization. Record the number of iterations, objective function values in the first iterations, and the solution of design variables.
- (4) Simulate the circuit after optimization to obtain insertion loss plot.

Part 3:

Carry out statistical design using ADS. Assume the following statistics:

All resistors: uniform distribution with a tolerance of $\pm 6\%$ Capacitors: normal distribution with standard deviation of 5% Inductors: normal distribution with standard derivation of 5% Correlation coefficient between the two inductors is 0.6

- (5) Perform Monte-Carlo analysis with 500 outcomes with design variables having the same value as in (2) to obtain the yield, and plot of insertion loss vs. frequency
- (6) Perform Monte-Carlo analysis with 500 outcomes with design variables having the same value as (4) to obtain the yield, and plot of insertion loss vs. frequency
- (7) Perform yield optimization. Record the number of iterations, objective function at first and last iteration, the optimal solutions of design variables.
- (8) Perform Monte-Carlo analysis with 500 outcomes after yield optimization to obtain the yield, and plot of insertion loss vs. frequency.

Notice

Notice that (2) and (5) can be achieved using one input file, so can (4) and (6). Attach hardcopies of the files:

- Three datafiles, i.e., at starting points of Φ , after nominal optimization, after yield optimization.
- The log file which records the procedure of program execution.

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Question 2

Suppose the design variable is ϕ and the error functions are $\mathbf{f} = [f_1 \ f_2]^T$, where

$$f_1(\phi) = 97.5 - \phi$$

 $f_2(\phi) = 5\phi - 502.5$

The design objective function is

 $U(\phi) = \max \{f_1 \ f_2\}$

- What is the solution of φ so that U(φ) is minimized? What is the acceptance region of φ for this design example?
- (2) Suppose φ varies randomly around its nominal value according to a uniform distribution within the tolerance range of 1.6. What is the optimal design center φ⁰ such that yield is maximized?
- (3) What is the yield if we use the optimal solution of (1) as the nominal value of φ?. What is the yield if we use the optimal design center in (2) as the nominal value for φ?.

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Question 3

Let $\varphi = \begin{bmatrix} \varphi_1 & \varphi_2 \end{bmatrix}^T$ be a vector of design variables. Error functions for design optimization are

$$\begin{array}{l} f_1 \left(\pmb{\varphi} \right) = \varphi_2 - 4 \\ f_2 \left(\pmb{\varphi} \right) = 2 - \varphi_2 \\ f_3 \left(\pmb{\varphi} \right) = 1 - \varphi_1 \\ f_4 \left(\pmb{\varphi} \right) = \varphi_1 - 7 \end{array}$$

Let φ^0 be the nominal point (design center) and **x** be a 2-dimensional random variable. We consider 5 random outcomes of **x**:

$$\mathbf{x}^{(1)} = \begin{bmatrix} 0 & 2 \end{bmatrix}^{\mathrm{T}}, \quad \mathbf{x}^{(2)} = \begin{bmatrix} -2 & 0 \end{bmatrix}^{\mathrm{T}}, \quad \mathbf{x}^{(3)} = \begin{bmatrix} -1 & -2 \end{bmatrix}^{\mathrm{T}}, \quad \mathbf{x}^{(4)} = \begin{bmatrix} 1 & -2 \end{bmatrix}^{\mathrm{T}}, \quad \mathbf{x}^{(5)} = \begin{bmatrix} 2 & 0 \end{bmatrix}^{\mathrm{T}}.$$

The corresponding 5 circuit outcomes are

$$\boldsymbol{\varphi}^{(k)} = \varphi^0 + \mathbf{x}^{(k)}, \quad k = 1, 2, \dots, 5.$$

The objective function for generalized l_p centering is

$$U(\boldsymbol{\varphi}^0) = H_p(\mathbf{u})$$

where the kth component of vector **u** is defined as

$$u_k = \alpha_k H_q (f(\boldsymbol{\varphi}^{(k)})), k = 1, 2, ..., 5$$

and the weighting factor $\alpha_k = 1$ for $k = 1, 2, \ldots, 5$.

Suppose the starting point for optimization is $\varphi^0 = [\varphi_1^0 \ \varphi_2^0]^T = [4.0 \ 4.2]^T$. Suppose φ_1^0 is fixed at 4.0 throughout optimization. For this exercise, we use p=q=1.

- 1) Derive the expressions for the objective function U vs. variable φ_2^0 for $0 \le \varphi_2^0 < \infty$. ∞ . Draw graphically U vs. φ_2^0 for $0 \le \varphi_2^0 < \infty$.
- 2) Find the optimal design center φ^0 , i.e., the minimum of U (φ^0).
- 3) Draw the 2-dimensional acceptance region and the locations of the 5 circuit outcomes before optimization.
- 4) Draw the 2-dimensional acceptance region and the locations of the 5 circuit outcomes after optimization.
- 5) What is the yield before and after optimization.