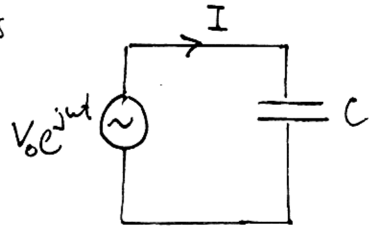
LCR Circuits with Phasors

Represent sinusoidal power supply voltage as phasor $V(t) = V_0 e^{j\omega t}$

$\mathrm{Re}\{V(t)\} = V_0 \cos \omega t$ has physical meaning

1. Capacitors

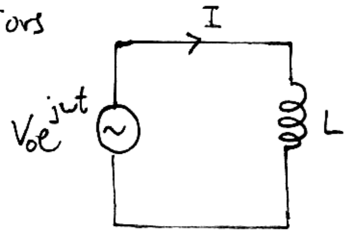


$$I = \frac{dQ}{dt} = \frac{C\,dV}{dt} = j\omega C V_0 e^{j\omega t}$$

$\therefore$ impedance $Z_{cap} = \dfrac{V}{I} = \dfrac{1}{j\omega C}$ admittance $Y_{cap} = \dfrac{1}{Z_{cap}} = j\omega C$

NB current and voltage are 90° out of phase in capacitor; I leads V

2. Inductors

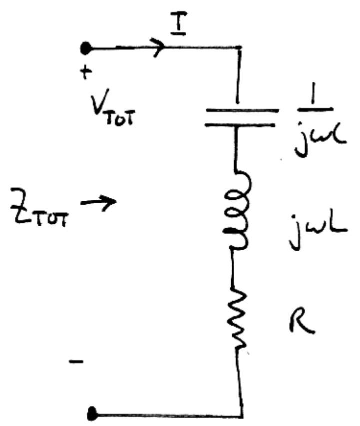


$V = L\dfrac{dI}{dt} \longrightarrow$ integrate over t to get $I = \dfrac{V_0}{j\omega L} e^{j\omega t}$

$\therefore$ impedance $Z_{ind} = \dfrac{V}{I} = j\omega L$ admittance $Y_{ind} = \dfrac{1}{j\omega L}$

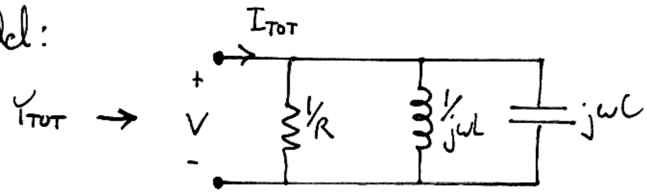
I lags V by 90°

3. Impedances add in series:



$$Z_{TOT} = \frac{V_{TOT}}{I} = \frac{1}{j\omega C} + R + j\omega L$$

4. Admittances add in parallel:



$$Y_{TOT} = \frac{I_{TOT}}{V} = \frac{1}{R} + \frac{1}{j\omega L} + j\omega C$$

5. Resonance: suppose $\omega L = \frac{1}{\omega C}$ in either the series or par^{ll} circuit above;

then the inductive and capacitive terms cancel out and we are left with only the resistance!

This is called a resonance condition NB $\omega L = \frac{1}{\omega C} \Rightarrow \omega = \frac{1}{\sqrt{LC}}$