Information and Instructions:

1. Attempt all questions.
2. Show all analysis.
3. The exam marks total 91.
4. Unless otherwise specified, use only the simplified transistor model for the BJT, i.e., take $r_x=0$, $r_o=\infty$, and $r_\mu=\infty$.

Useful Formulas

\[ I_c = I_s e^{V_s}, r_x = \frac{\beta}{g_m}, \quad r_e = (\beta+1)r_x, \quad E = mc^2, \quad \alpha = \frac{\beta}{\beta+1}, \quad g_m = \frac{I_c}{V_T}, \quad V_T = 25mV \atop 20^\circ C \]

\[ g_m = \sqrt{2\mu C_{ox} \left( \frac{W}{L} \right) I_{DS}}, \quad \Gamma = \frac{Z_L - Z_o}{Z_L + Z_o}, \quad \nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t}, \quad i = C \frac{dv}{dt}, \quad V = IR \]

The average of a half sine wave with amplitude of 1 is $1/\pi$.

\[ a = \frac{dy}{dt} = \frac{d^2x}{dt^2}, \quad \text{Money} = \text{Power} = VI = I^2R = \sqrt{\text{Evil}} \]

\[ |\text{forward biased } V_{BE}| = 0.7 \text{ Volts} \]

\[ \omega_\omega = \omega_{\omega_1} + \omega_{\omega_2} + \omega_{\omega_3} + \ldots \quad \text{and} \quad \frac{1}{\omega_{\omega_1}} = \frac{1}{\omega_{\omega_2}} + \frac{1}{\omega_{\omega_3}} + \ldots \]

Miller’s Theorem: \[ Y_1 = Y \left( 1 - \frac{\omega_2}{\omega_1} \right), \quad Y_2 = Y \left( 1 - \frac{\omega_1}{\omega_2} \right) \]

Sensitivity: \[ S_x^2 = \frac{dy}{dx} \cdot \frac{x}{y}, \quad \text{Low Pass Filter: } H(s) = \frac{H_o \omega_o^2}{s^2 + s \frac{\omega_o}{Q} + \omega_o^2} \]
Question 1 (Total 16 Marks)

Answer the following questions.

2 marks (a) Draw the T model for a bipolar transistor including $C_{\pi}$, $C_{\mu}$, $r_o$, and $r_x$.

2 marks (b) A transistor has an $I_s = 9.27 \cdot 10^{-13} \text{A}$ and $V_T = 25 \text{mV}$. If $V_{BE} = 0.69 \text{V}$, what is the collector current?

6 marks (b) Consider the class C amplifier shown below. The transistor collector current is a pulse waveform from 0 to 2A with 5% duty cycle as shown in Figure 1. The LC resonator forms a perfect open circuit at 1GHz and a perfect short circuit at all other frequencies. What DC current and power is drawn from the supply? Now assuming that all of the collector current at the FUNDAMENTAL frequency of 1GHz flows through the load resistor, size $R_{\text{Load}}$ so that the output voltage swing is maximized. Now draw the output voltage waveform. What is the power delivered to the load? What is the efficiency of the amplifier?

![Figure 1](image1.png)

NOTE: A square wave with a duty cycle of x% has a current at the fundamental frequency given by: $i_{\text{fund}} = \frac{4I_{DC}}{\pi} \sin \left( \frac{360 \cdot x}{100} \right)$, where $I_{DC}$ is the average current in the pulse waveform.

2 marks (c) A power transistor is needed that dissipates 100W. The transistor must operate in an environment where the temperature could vary from –10 to 35°C. If the transistor’s junction temperature exceeds 125°C the transistor will melt. What junction to air thermal resistance is needed for this design?

4 marks (d) Set the value of $R_f$ so that the input impedance of this circuit is 50Ω. Note that you can assume that the presence of $R_f$ will not change the gain of the circuit significantly.

![Figure 2](image2.png)
**Question 2** (Total 22 Marks)

When analyzing the amplifier circuit in Figure 2, use appropriate models, to find generalized expressions (i.e., without component values unless specifically asked for).

- **4 marks** (a) Draw the small-signal equivalent circuit.
- **3 marks** (b) Find the mid-band gain $A_v$.
- **2 marks** (c) Find the mid-band $R_{in}$.
- **2 marks** (d) Find the mid band $R_{out}$ (include $r_{o1}$, $r_{o2}$ and $r_{µ2}$).
- **3 marks** (e) Find the low frequency poles ($\omega_L$’s). Do not solve for the pole associated with $C_3$.
- **4 marks** (f) Find the high frequency poles ($\omega_H$’s) for the circuit. You may ignore $C_{π2}$.
- **4 marks** (g) With $\beta_1 = \beta_2 = 100$, $V_{CC} = 15V$, set $R_1$ so that $I_{C1} \approx I_{C2} \approx 1mA$. Assuming that $V_{C2} = 7V$, set $R_2$, $R_3$, and $R_5$ so that $I_{R2} \approx 10 I_{B2}$ and the output swing is roughly maximized.

![Figure 3](image-url)
**Question 3** (Total 22 Marks)

In this question, all transistors are assumed to be matched. Perform the following analysis on the op amp circuit of Figure 4. Assume that $r_{\mu}=\infty$, $V_A = 100V$, $C_\pi = 15pF$, and $C_\mu = 5pF$ for all transistors. **NOTE: Include $r_o$s in your calculations where needed!**

12 marks (a) Draw the small signal model and use it to find:

(i) The differential-mode gain $A_{DM}$ with output at $v_x$.
(ii) The common-mode gain $A_{CM}$ with output at $v_x$.
(iii) The differential mode input impedance $R_{DM}$.
(iv) The common mode input impedance $R_{CM}$.
(v) Assuming that the amplifier labeled $A_v$ has an input impedance of $R_i$, what is the load impedance at node $v_x$?

10 marks (b) With $V_{cc} = 15V$, and $V_{EE} = -15V$

(i) Set $R_{bias}$ so that $Q_1$ and $Q_2$ each draw 10µA.
(ii) The “N” associated with $Q_3$ means that it is actually $N$ transistors in parallel. What value of $N$ will make sure that $Q_4$ is biased correctly?
(iii) At this current what is the required value of $A_v$ needed to make the gain of the op-amp 10,000 assuming that $R_i = 250k\Omega$, where $R_i$ is the input impedance of the second stage amplifier.
(iv) What value of $C_{comp}$ is needed to have a corner frequency of 20Hz?
(v) What is the slew rate of this op-amp?
(vi) At what frequency will the common mode gain start to increase?
Question 4 (Total 20 Marks)

7 marks  (a) Derive the transfer function $v_o(s)/v_i(s)$ of the circuit shown in Figure 4. Note that there are two resistors labeled $R_1$ and five resistors labeled $R_2$.

![Figure 5](image)

4 marks  (b) For the following filter transfer function:

$$\frac{v_o(s)}{v_i(s)} = \frac{R_2 R_3}{R_1 s^2 R_2 R_3 R_4 C_1 C_2 + s R_4 C_2 + R_4} \frac{1}{R_1}$$

find expressions for $H_0$, $\omega_o$ and $Q$.

3 marks  (c) For the filter in (b), given that $C_1 = C_2 = 0.5 \mu F$, and $R_1 = 1 \text{k}\Omega$, find values for the remaining components (that is for $R_2$, $R_3$, and $R_4$) to set $H_0 = 2$, $Q = 5$, and $\omega_o = 1000 \text{ rad/s}$.

4 marks  (d) For a second-order low-pass filter with $H_0 = 2$, $Q = 5$, and $\omega_o = 1000 \text{ rad/s}$, as in (b) and (c), sketch the transfer function approximately to scale and clearly identify $H_0$, $Q$, $\omega_o$, and the high-frequency rolloff.

2 marks  (e) For the filter in (b), derive an expression, (showing your steps) for the sensitivity for $\omega_o$ as a function of $R_1$, that is determine $S_{R_1}^{\omega_o}$.
**Question 5** (Total 11 Marks)

7 marks  (a) For the oscillator shown in Figure 6, apply Barkhausen criteria to find the oscillating frequency showing any necessary derivation. Then find out what is the necessary value of $K$ to start the oscillator.

![Figure 6](image)

4 marks  (b) What is the minimum number of stages for the circuit above to oscillate? If the gain of the amplifier is made positive instead of negative (that is, it is $+K$ instead of $-K$), how many stages are required?