Instructions:
1. This is a closed book examination.
2. Attempt all questions.
3. Assume transistor Q1 is different from Q2 (i.e., $\beta_1$ and $\beta_2$ are different. $r_{\pi 1}$ and $r_{\pi 2}$ are different).
4. Assume simplified hybrid $\pi$-model or T-model (i.e., ignore $r_h$, $r_o$ and $r_x$) unless stated otherwise.
5. Useful formulae: $r_{\pi} = \beta/g_m$, $r_{\pi} = (1+\beta) r_e$, $\alpha = \beta/(1+\beta)$, $r_e g_m = \alpha$.
6. Miller’s theorem: $Y_1 = Y (1 - V_2/V_1)$, $Y_2 = Y (1 - V_1/V_2)$.
7. Show all analysis and assumptions you make.

Question 1:

1. Draw the mid-band small-signal AC equivalent circuit. (3 marks)
2. Find the mid-band input resistance $R_{in}$. (1 mark)
3. Find the mid-band output resistance $R_{out}$, including $r_o$ and $r_\mu$ in both transistors with the exception that $r_{\mu 2}$ can be ignored. (1 mark)
4. Find the mid-band gain $v_o/v_s$. (5 marks)
5. Find the low frequency poles associated with $C_1$, $C_2$, $C_B$, and $C_E$. (5 marks)
6. Find the high frequency poles, including $C_{\pi 1}$, $C_{\pi 2}$, $C_{\mu 1}$, and $C_{\mu 2}$. (5 marks)
7. DC analysis: Assume that both transistors are active, $V_{EB1} = V_{BE2} = 0.7V$, $\beta_1 = 100$, and $\beta_2 = 200$. Given $R_1 = 30K\Omega$, $R_2 = 70K\Omega$, $R_{C1} = 4.5K\Omega$, $R_{E1} = 2.3K\Omega$, $R_{C2} = 3K\Omega$, $R_{E2} = 3.4K\Omega$, calculate $I_{E1}$, $I_{E2}$, $V_{C1}$, $V_{C2}$, (5 marks). (To simplify calculation, you can assume that the base current of Q2 is negligible compared to the collector current of Q1. However you must consider the base current of Q1 in the calculation.)
1. \[ \begin{align*}
R_s & \quad v_s \\
R_{E1} & \quad v_{e1} \\
R_{\pi 1} & \quad v_{\pi 1} \\
R_{\pi 1} & \quad v_{\pi 1} \\
g_{m1}v_{\pi 1} & \quad v_{c1} \\
g_{m2}v_{\pi 2} & \quad v_{c2} \\
R_{C1} & \quad v_{b2} \\
R_{C2} & \quad v_{o} \\
R_L & \quad v_{o} \\
\end{align*} \]

2. \[ R_m = R_{E1}\parallel r_{e1} \]

3. \[ R_{out} = R_{C2}\parallel r_{a2} \]

4. \[ \begin{align*}
\frac{v_o}{v_{a2}} &= -g_{m2}(R_{C2}\parallel R_{L}) \\
\frac{v_{e2}}{v_{a1}} &= -g_{m1}(R_{C1}\parallel R_{\pi 2}) \\
\frac{v_{\pi 1}}{v_{e1}} &= -1, \\
\frac{v_{e1}}{v_s} &= R_{E1}\parallel (r_{e1} + R_{S}) \\
\frac{v_o}{v_{a2}} &= v_o = \frac{v_o}{v_{a2}} v_{a1} v_{e1} v_s, \\
\end{align*} \]

5. \[ \omega_{L1} = \frac{1}{(R_s + R_m)C_1}, \text{ where } R_m = R_{E1}\parallel r_{e1} \text{ from Question 2}, \]

\[ \omega_{L2} = \frac{1}{(R_{C2} + R_L)C_2}, \]

\[ \omega_{L3} = \frac{1}{R_{CB}C_{B}}, \text{ where } R_{CB} = R_{e2}\parallel R_{e1}\parallel R_{S}
(1 + \beta_2), \]

\[ \omega_{L4} = \frac{1}{R_{CE}C_E} \text{ where } R_{CE} = R_{E2}\parallel \left(\frac{r_{a2} + R_{CE}}{1 + \beta_2}\right) \]

6. \[ \omega_{H1} = \frac{1}{(R_s\parallel r_{e1})C_{\pi 1}}, \]

\[ \omega_{H2} = \frac{1}{(R_{C1}\parallel R_{\pi 2})C_{\mu 1} + C_{\pi 2} + (1 + g_{m2}R_{L})C_{\mu 2}}, \]

\[ \omega_{H3} = \frac{1}{(R_{C2}\parallel R_L)\left(\frac{1 + \frac{1}{g_{m2}R_{L}}}{1 + \frac{1}{g_{m2}R_{L}}}\right)C_{\mu 2}} \]

7. Thevenin Voltage source on the base of \( Q_1 \) is 7V, Thevenin series resistance is 30k\( \parallel \)70k = 21k.

KVL: \[ 0.7V + I_{B1} \cdot 21k\Omega + (1 + \beta_1)I_{B1} \cdot 2.3k\Omega + 7V = 10V \]

solve for \( I_{B1} \):

\[ I_{B1} = \frac{10 - 7 - 0.7}{21 + 101 \cdot 2.3} = 9.08\mu A \]

\[ I_{E1} = (1 + \beta_1) \cdot 9.08\mu A = 0.917mA, \quad I_{C1} \approx I_{E1} = 0.917mA \]

or more exactly, \( I_{C1} = \frac{\beta_1}{1 + \beta_1} \cdot 0.917mA = 0.908mA \).

\[ V_{C1} = V_{B1} = I_{C1} \cdot 4.5k = 4.086V \text{ where we have assumed that } I_{B2} \text{ is negligible compared to } I_{C1} \text{ as stated in the problem.} \]

\[ I_{E2} = \frac{V_{C1} - 0.7}{3.4k} = \frac{4.127 - 0.7}{3.4k} = 0.996mA \approx 1mA \]

If we assume that \( I_{C2} \approx I_{E2} \) then \( V_{C2} = 10V - 3k \cdot I_{C2} = 10V - 3k \cdot 1mA = 7V \)