97.472 Problem Analysis

Week 0 (January 3-5)

1) Prove Snell's Law at a planar boundary using Fermat's Principle.

Week 1 (January 8-12)

2) A prism having refractive index n has a small apex angle α. Show that paraxial rays entering one face of the prism at angle of incidence θ ,with respect to the normal, exit the opposite face at an angle approximately given by $\theta_d = (n-1)\alpha$.

3) Prove Equation (2.5) of the notes for paraxial rays.

4) Determine the NA and acceptance angle from air of a silica glass fiber if the refractive index of the core is 1.475 and
   (a) the cladding has index 1.46
   (b) the cladding is stripped so that the core is surrounded by air.
   (c) Repeat if the light enters the fiber through index matching gel having index 1.4.

5) Tiny glass balls are often used as lenses to couple light into and out of optical fiber. The fiber end face is located at distance f from the sphere. For a sphere of radius a=1mm and refractive index 1.48, determine the distance f such that a ray parallel to the optical axis at height y=0.7mm is focused into the fiber core.

6) An optical fiber has core diameter 100μm and NA=0.36 . The output light from the fiber is collimated by a lens with focal length 25 mm. What is the diameter of the collimated beam and what is the beam divergence?

Week 2 (January 15-19)

7) A plano-concave lens with f= -50 mm and a plano-convex lens with f=250mm are separated by 200 mm with the plane surfaces facing each other. A laser beam with diameter 0.66 mm and beam divergence 1.4 mrad is incident on the concave surface. What is the diameter and divergence of the output beam from the convex surface? What does this lens combination accomplish?

8) The light from a Nd:YAG laser at wavelength 1.06 μm is a Gaussian beam of 1W optical power and beam divergence $2\theta_o$ = 1mrad. Determine the beam waist radius, the depth of focus, the maximum intensity, and the intensity on the beam axis at distance z=100 cm from the beam waist.

9) An argon-ion laser produces a Gaussian beam of wavelength λ=488nm and waist radius W_o=0.5mm. Choose a single lens to focus the light to a spot of diameter 100 μm. What is the shortest focal-length lens that may be used?

Week 3 (January 22-26)

10) A 25mm diameter lens with focal length 5 cm is stopped down to aperture diameter 50 μm. A collimated laser beam which has been expanded to 10 mm is incident on the lens. If the wavelength is 635 nm, calculate and plot (use MathCAD or MATLAB) the diffraction pattern at the focal plane. What is the diameter of the focused spot? What is the F/# of the lens?

11) Determine the electric field magnitude at the center of a Gaussian beam in air (a point on the beam axis at the beam wais) if the beam is 1W and the beam waist radius is W_o=0.1mm .

12) TM polarized light in a medium with index n_1 is incident at angle θ_1 on a plane boundary to a medium having index n_2 where $n_1<n_2$. Show that the Brewster angle θ_B where no reflection occurs for a plane wave is given by $\tan \theta_B=n_2/n_1$.

13) A plane wave crosses a plane boundary between GaAs (n=3.6) and air. Calculate and plot (use MathCAD or MATLAB) the reflection coefficient and reflectance for angle of incidence θ_1 from 0-90 degrees for
    (a) TE polarization
    (b) TM polarization.

Week 4 (January 29- February 2)

14) A fiber has core radius 25 μm, core index n_1=1.48 and Δ=0.01.
    (a) If λ=1320 nm, what is the V-number?
    (b) How many modes propagate in the fiber?
    (c) What percentage of the power flows in the cladding?
    (d) Repeat (a)-(c) if the index difference is reduced to Δ=0.003.

15) Find the core radius necessary for single-mode operation at 1320 nm of a step-index fiber with n1=1.480, and n2=1.478. What are the NA and maximum acceptance angle for this fiber?

16) Use MATLAB or MathCAD to plot $E(r)/E_o$ for $0<r/a<3$ for for a fiber having V=1.0, 1.4, 1.8, 2.2, 2.6, and 3.0.

Week 5 (February 5- February 9)

17) Plot the refractive index profiles from n_1 to n_2 as a function of radial distance $r \leq a$ for GRIN fibers having a=1,2,4,8,and ∞. Assume the core radius is 25 μm , n_1=1.48 and Δ=0.01

18) Calculate the number of modes at 820 nm and 1300 nm in a GRIN fiber having parabolic index profile, 25 μm core radius n_1=1.48 and Δ=0.02. How does this compare to step-index fiber?

19) Sunlight impinges on a transmission grating that is formed with 5000 lines per cm. Does the third-order spectrum overlap the second-order spectrum? Take red to be 780 nm wavelength and violet to be 390 nm wavelength.

Week 6 and 7 (February 12- March 2)

20) Derive the expression for the group index N_g and show that $v_g=c_o/N_g$.

21) It is desired to use multimode fiber with core diameter 50μm. together with a source that has very narrow linewidth at 1300nm. What is the BL product for the following:
   (a) stepped index with $n_1=1.480$, $\Delta=0.0135$
   (b) GRIN with $n_1=1.480$, $\Delta=0.0135$, $\alpha=2$.

22) A single-mode silica fiber has $D_{mat}=22$ ps/(nm-km) and $D_{wg}= -6$ ps/(nm-km) at 1550 nm. Compare the bit rate possible over 10 km of fiber for
   (a) an LED operating at 1550 nm with linewidth 45nm
   (b) a laser diode operating at 1550 nm with linewidth 5 nm.

23) Determine the core radius of multimode SI fiber with NA=0.1 if the number of modes is 5000 when the wavelength is 0.87 μm. If the core refractive index is $n_1=1.445$, the group index $N_1=1.456$ and Δ is approximately independent of wavelength, determine the pulse spread for a 2 km fiber.

Week 8 (March 5 – March 9)

24) A Fabry-Perot optical cavity has identical mirrors with reflectance 0.9 spaced 100 μm apart in air. Calculate the cavity mode nearest to 900 μm. Calculate the mode separation, the finesse, and the spectral width of each mode in terms of frequency and wavelength.

25) A silicon pn-junction diode has $N_d=N_a=10^{18}$ cm^{-3} and $n_i=1.45\text{x}10^{10}$ cm^{-3} and area 1 mm^2. For silicon $\varepsilon_r=11.9$. When $N_d=10^{18}$ cm^{-3} , $\mu_p=130$ cm^{2}V^{-1}s^{-1} and when $N_a=10^{18}$ cm^{-3} , $\mu_n=250$ cm^{2}V^{-1}s^{-1} . Also the recombination times $\tau_n=\tau_p=25$ ns. Given that a forward bias 0.6V is applied, calculate the diffusion and recombination contributions to the diode current. What is your conclusion?

26) GaAs has an effective density of states $N_C=4.7\text{x}10^{17}$ cm^{-3} and $N_V=7\text{x}10^{18}$ cm^{-3}. Given a bandgap $E_g=1.42$ eV, calculate the intrinsic carrier concentration and location of the Fermi level at 27^{o}C. Assuming that N_C and N_V scale with temperature as $T^{3/2}$, what is the intrinsic carrier concentration at 127^{o}C?

Week 9 (March 12 – March 16)

27) Derive an expression for the temperature sensitivity of the peak wavelength of an LED spectrum. For GaAs at 300K, $E_g=1.42$ eV, $dE_g/dT= -5\text{x}10^{-4}$ eV/^{o}C , what is the shift in wavelength for 10^{o}C temperature increase?

28) An infrared LED is made using $In_{1-x}Ga_xAs_yP_{1-y}$ alloy. What is the alloy composition for peak emission at 1300nm at 27^{o}C? What is the expected linewidth?

29) An AlGaAs LED emitter for use in a local optical fiber network is designed for peak emission at 820 nm at 25^oC. The linewidth at 25^oC is 40 nm.

(a) What is the bandgap of AlGaAs in this LED at 25^oC?
(b) Given that the bandgap changes with temperature as –4.5x10^{-4} eV/^oC, what is the peak emission wavelength at temperatures –40^oC and at 85 ^oC?
(c) What is the linewidth at temperatures –40^oC and at 85 ^oC?
(d) What is the composition of the AlGaAs in this LED?

Week 9 (March 12 – March 16)

30) The LED in problem 29 has responsivity 30μW/mA.When the forward current is 50 mA at 25^oC, the voltage across the LED is 1.5V.
(a) What is the LED output power?
(b) What is the external efficiency?
(c) If the radiative lifetime is 50ns and the non-radiative lifetime is 100ns, what is the internal efficiency?
(d) What is the internal optical power generated?
(e) If the optical power coupled into a multimode fiber through a small spherical glass lens is 500 μW, what is the overall efficiency?
(f) What is the modulation bandwidth?

31) What wavelengths will silicon (E_g=1.12eV) absorb? What wavelengths will GaAs (E_g=1.42eV) absorb? Design a AR coating for a silicon (n=3.6) photodetector at 635 nm.

Week 10-11 (March 19 – March 30)

32) A semiconductor Fabry-Perot optical cavity has cavity length 200 μm with end mirrors that have reflectance 0.8 each. The refractive index is 3.7 .
(a) What is the cavity mode nearest to the free space wavelength of 1300 nm?
(b) Calculate the separation of the modes and the spectral width.

33) Consider an InGaAsP-InP laser diode which has an optical cavity of length 250 μm. The peak radiation is at 1550 nm and the refractive index is 3.4 . The optical gain bandwidth (FWHM) is 8 nm at the operating DC diode current.
(a) What is the mode integer m of the peak radiation?
(b) What is the mode spacing in wavelength?
(c) How many modes are there in the cavity?
(d) What is the reflectance at the cleaved mirror surfaces?
(e) What would the cavity length need to be to operate single mode?

34) A Bragg grating is added to the laser diode in problem 33 to make a DFB laser. What is the required period for a first-order grating?

35) Consider a double heterostructure InGaAsP laser diode operating at 1310 nm. The cavity has length 60 μm , active layer width 10 μm and active layer thickness 0.25 μm. The refractive index is n=3.5 . The loss coefficient for the cavity is 10 cm^{-1}.
   (a) Find the total loss and gain at threshold and the photon lifetime.
   (b) If the threshold current density is 500 A/cm^2 and τ_{sp}=10 ps, what is the threshold electron concentration?
   (c) What is the lasing optical power and intensity when the diode current is 5 mA?

Week 12 (April2 –April6)

36) Consider an ideal photodiode with 100% quantum efficiency and no dark current. What is the expression for the NEP for the quantum noise limit? If the device operates at 1300nm with a bandwidth of 500MHz, calculate the NEP and the corresponding photocurrent.
37) A silicon photodiode has an intrinsic layer which is 20 μm wide. The P+ layer on the illumination side is very thin (0.1μm). The diode is reverse biased at 100V. A very short pulse of light at wavelength 900nm illuminates the active area. What is the duration of the photocurrent if absorption occurs over the entire I-layer? (use absorption coefficient $3x10^4$ m^{-1} at 900nm, $v_e \sim v_{esat}$, $v_h=7x10^4$ m/s)
38) Consider a Ge pn junction photodiode which has responsivity 0.25A/W, 0.55A/W, and 0.72A/W at 850nm, 1300nm, and 1550nm, respectively. Its photosensitive area is .01 mm^2. It is used at reverse bias 10V when the dark current is 0.3 μA and the junction capacitance is 4 pF. The rise time of the diode is 0.5ns.
   (a) Calculate its quantum efficiency at 850nm, 1300nm, and 1550nm.
   (b) What is the intensity of light at 1550nm that gives a photocurrent equal to the dark current?
   (c) What would be the effect of lowering the temperature on the responsivity?
   (d) Given that the dark current is in the range of microamperes, what would be the advantage in reducing the temperature?
   (e) Suppose that the diode is used with a 100Ω resistance to sample the photocurrent, what limits the speed of the response?
39) Consider an InGaAs APD with x~0.7 which is biased to operate at M=10 at 27^oC. At small bias the responsivity is 0.8 A/W and the dark current is 10nA. The bandwidth is 700MHz when used with a load resistor of 50Ω.
   (a) What is the APD noise current per square root of bandwidth?
   (b) What is the APD noise current for the operating bandwidth?
   (c) What is the thermal noise power?
   (d) What is the minimum optical power for a SNR of 10dB?
   (e) What is the approximate capacitance of the device?
40) A fiber optic link is constructed at 1300 nm using a AlGaAs laser diode, single mode fiber, and a InGaAs pin photodiode. The photodiode has responsivity 0.8A/W at 1300nm.The laser diode can launch 2mW of power into the fiber which has attenuation 0.35dB/km. For an acceptable SNR in the receiver, the photodiode must have at least 5nA photocurrent. What is the maximum length of the link without using repeaters or optical amplifiers?