EQUATION SHEETS FOR ELEC 4702 **February 2017**

Photons

$$E = h\upsilon = hf = \hbar\omega \quad (J) \qquad \hbar = \frac{h}{2\pi} \qquad \omega = 2\pi\upsilon \quad or \quad 2\pi f \qquad \lambda_g(\mu m) = \frac{1.24}{E_g(eV)}$$

$$p = \hbar k = \left(\frac{h}{2\pi}\right)\left(\frac{2\pi}{\lambda}\right) = \frac{h}{\lambda} \qquad v = \frac{c}{\sqrt{\varepsilon_r}} = \frac{c}{n} = \upsilon\lambda \quad or \quad f\lambda \quad (m/s)$$

Geometric Optics (Use R>0 convex or R<0 concave)

Snell's Law $n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$ $\quad \theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$

Spherical Mirror $f = -\frac{R}{2}$ $\quad \frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{f}$

Spherical boundary (n_1 to n_2, radius R) $\frac{n_1}{z_1} + \frac{n_2}{z_2} \cong \frac{n_2 - n_1}{R}$ $\quad y_2 = -\left(\frac{n_1}{n_2}\right)\left(\frac{z_2}{z_1}\right)y_1$

Spherical lens (R_1 first surface, R_2 second surface) $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ $\quad \theta_2 \cong \theta_1 - \frac{y}{f}$

$$\frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{f} \qquad m = \frac{y_2}{y_1} = -\frac{z_2}{z_1} \qquad f/\# = \frac{f}{\phi} \qquad NA = \sin\theta = \frac{\phi}{2f} \qquad |y_1|\theta_1 = |y_2|\theta_2$$

Optical fiber (core n_1, cladding n_2, radius a)

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right) \qquad NA = n_0 \sin\theta_a = \sqrt{n_1^2 - n_2^2} \qquad \Delta = \frac{n_1 - n_2}{n_1}$$

Gaussian Beams

Parameter	Before Lens	After Lens
Waist radius	W_0	$W_0^{'} = MW_0$
Waist location	z	z' where $(z' - f) = M^2 (z - f)$
Waist at lens	$W(z)$	$W'(z') = W(z)$
Beam curvature at lens	$R(z)$ note $R > 0$ is diverging	R' where $\frac{1}{R'} = \frac{1}{R} - \frac{1}{f}$
Depth of focus	$2z_0$	$2z_0^{'} = M^2 (2z_0)$
Beam Divergence	$2\theta_0$	$2\theta_0^{'} = 2\theta_0 / M$
Magnification		$M = \frac{M_r}{\sqrt{1+r^2}} \qquad M_r = \left\lvert\frac{f}{z-f}\right\rvert$
		$r = \frac{z_0}{z - f}$

$$W_0 = \sqrt{\frac{\lambda z_0}{\pi}} \qquad W(z) = W_0\sqrt{1+\left(\frac{z}{z_0}\right)^2} \qquad \theta_0 = \frac{W_0}{z_0} = \frac{\lambda}{\pi W_0} \qquad 2z_0 = \frac{2\pi W_0^2}{\lambda}$$

$$I(r,z) = I_0\left[\frac{W_0}{W(z)}\right]^2 \exp\left[\frac{-2r^2}{W^2(z)}\right] \qquad I(r,z) = \frac{2P}{\pi W^2(z)}\exp\left[\frac{-2r^2}{W^2(z)}\right] \qquad P = \frac{1}{2}I_0\pi W_0^2$$

$$R(z) = z\left[1+\left(\frac{z_0}{z}\right)^2\right] \qquad \phi(r,z) = kz - \xi(z) + \frac{kr^2}{2R(z)} \qquad \xi(z) = \tan^{-1}\left(\frac{z}{z_0}\right)$$

For incident well-collimated beam $W_0' \approx \frac{f}{z_0}W_0 = \theta_0 f = \frac{\lambda f}{\pi W_0}$

Wave Optics

$$U(\vec{r},t) = U(\vec{r})\exp(j2\pi f t) \qquad U(\vec{r}) = a(\vec{r})e^{j\phi(\vec{r})} \qquad I(\vec{r}) = |U(\vec{r})|^2$$

Plane wave $U(\vec{r}) = A\exp(-j\vec{k}\bullet\vec{r})$ $\quad k = |\vec{k}| = \frac{\omega}{v} = \frac{2\pi}{\lambda} = nk_0$ $\quad I(\vec{r}) = |U(\vec{r})|^2 = |A|^2$

Spherical wave $U(\vec{r}) = \frac{A}{r}\exp(-jkr)$ $\quad k = \frac{\omega}{v}$ $\quad I(\vec{r}) = \frac{|A|^2}{r^2}$

Diffraction from rectangular slit of width D_x by D_y in far field where $\frac{D_x^2}{\lambda d} << 1$ and $\frac{D_y^2}{\lambda d} << 1$

$$I(x,y) = I_0\,\mathrm{sinc}^2\left(\frac{xD_x}{\lambda d}\right)\mathrm{sinc}^2\left(\frac{yD_y}{\lambda d}\right)$$

Diffraction from circular aperture of diameter D in far field where $\frac{D^2}{\lambda d} << 1$

$$I(r) = I_0\left[\frac{2J_1\left(\frac{\pi Dr}{\lambda d}\right)}{\left(\frac{\pi Dr}{\lambda d}\right)}\right]^2 \qquad r_{Airy} = 1.22\frac{\lambda d}{D}$$

Diffraction grating (spacing a) $\quad AB - CD = a(\sin\theta_m - \sin\theta_i) = m\lambda$

Bragg grating (periodicity Λ) $\quad \sin\theta_i = \sin\theta_d = \frac{\lambda}{2\Lambda}$

Electromagnetic Optics (charge free region)

$$\vec{E}'(\vec{r},t) = \mathrm{Re}\left\{\vec{E}(\vec{r})e^{j\omega t}\right\} \qquad \vec{H}'(\vec{r},t) = \mathrm{Re}\left\{\vec{H}(\vec{r})e^{j\omega t}\right\}$$

$$\vec{\nabla}\times\vec{H} = j\omega\vec{D} = j\omega\varepsilon\,\vec{E} \qquad \vec{\nabla}\times\vec{E} = -j\omega\vec{B} = -j\omega\mu\,\vec{H} \qquad \vec{\nabla}\bullet\vec{D} = 0 \qquad \vec{\nabla}\bullet\vec{B} = 0$$

$$\nabla^2\vec{E} + k^2\vec{E} = 0 \qquad \nabla^2\vec{H} + k^2\vec{H} = 0$$

$k = nk_o = \omega/v = 2\pi/\lambda$ $\quad k_o = \omega/c$

$\vec{S} = \frac{1}{2}\mathrm{Re}\{\vec{E} \times \vec{H}^*\}$ W/m^2 $I = |\vec{S}|$ $D_{2n}=D_{1n}$ $B_{2n}=B_{1n}$ $E_{2t}=E_{1t}$ $H_{2t}=H_{1t}$

Plane wave $\vec{E}(\vec{r}) = \vec{E}_0 \exp(-j\vec{k} \bullet \vec{r})$ $\vec{H}(\vec{r}) = \vec{H}_0 \exp(-j\vec{k} \bullet \vec{r})$

$\vec{k} = k_x\hat{x} + k_y\hat{y} + k_z\hat{z}$ $k = |\vec{k}| = \frac{\omega}{v} = \frac{2\pi}{\lambda} = nk_0$ $\vec{k} \times \vec{E}_0 = \omega\mu_0\vec{H}_0$ $\vec{k} \times \vec{H}_0 = -\omega\varepsilon_0\vec{E}_0$

$\frac{|\vec{E}_0|}{|\vec{H}_0|} = \frac{E_0}{H_0} = \eta = \frac{\eta_0}{n}$ $\eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0}{\varepsilon}}$ $\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi = 377\Omega$

Fresnel reflection and transmission (n_1 to n_2) $\cos\theta_2 = \sqrt{1-\sin^2\theta_2} = \sqrt{1-\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_1}$

$R = \frac{Power\ reflected}{Power\ incident} = |r_{TE}|^2\ or\ |r_{TM}|^2$ $T = \frac{Power\ transmitted}{Power\ incident} = 1 - R$

TE $r_\perp = r_{TE} = \left.\frac{E_3}{E_1}\right|_{boundary} = \frac{n_1\cos\theta_1 - n_2\cos\theta_2}{n_1\cos\theta_1 + n_2\cos\theta_2}$ $t_\perp = t_{TE} = \left.\frac{E_2}{E_1}\right|_{boundary} = 1 + r_{TE}$

TM $r_\parallel = r_{TM} = \left.\frac{E_3}{E_1}\right|_{boundary} = \frac{n_2\cos\theta_1 - n_1\cos\theta_2}{n_2\cos\theta_1 + n_1\cos\theta_2}$ $t_\parallel = t_{TM} = \left.\frac{E_2}{E_1}\right|_{boundary} = \frac{n_1}{n_2}(1 + r_{TM})$

Fresnel reflectance (normal incidence) $R = \left(\frac{n_2 - n_1}{n_2 + n_1}\right)^2$

Brewster angle for TM ($n_1<n_2$) $\theta_B = \tan^{-1}\left(\frac{n_2}{n_1}\right)$

Slab Waveguide (core n_1, both claddings n_2, thickness d=2a)

$K_1 = K_0 n_1 = \frac{2\pi n_1}{\lambda_0}\ or\ \frac{2\pi n_1}{\lambda}$ $V = \frac{2\pi a}{\lambda}\sqrt{n_1^2 - n_2^2}$ $m \le \frac{2V - \phi_m}{\pi}$ $M = Int\left(\frac{2V}{\pi}\right) + 1$

Single mode $V_{max}=\pi/2$ $\alpha_{cladding} = \frac{2\pi n_2}{\lambda}\sqrt{\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_{inc} - 1}$

$MFD = 2W_0 \approx 2a + 2\frac{a}{V} = 2a\left(\frac{V+1}{V}\right)$ $NA = n_0 \sin\theta_{max} = \sqrt{n_1^2 - n_2^2}$

TE ($\sin\theta_m > \sin\theta_c$) $2aK_1\ \cos\theta_m - m\pi = \phi_m$ $\tan\left(\frac{\phi_m}{2}\right) = \frac{\sqrt{\sin^2\theta_m - \left(\frac{n_2}{n_1}\right)^2}}{\cos\theta_m}$

TM (sin θ_m > sin θ_c) $2aK_1 \ \cos\theta_m - m\pi = \phi_m$ $\tan\left(\frac{\phi_m}{2}\right) = \frac{\sqrt{\sin^2\theta_m - \left(\frac{n_2}{n_1}\right)^2}}{\cos\theta_m \left(\frac{n_2}{n_1}\right)^2}$

Stepped Index Optical Fiber (core n_1, cladding n_2, core radius a)

$\Delta = \frac{n_1 - n_2}{n_1}$ $NA = n_0 \sin\theta_{\max} = \sqrt{n_1^2 - n_2^2}$ $V = \frac{2\pi a}{\lambda}\sqrt{n_1^2 - n_2^2}$

Single mode V_{max}=2.405 $M \approx \frac{V^2}{2}$ $\frac{P_{cladding}}{P_{total}} \approx \frac{4}{3\sqrt{M}}$

$b = \frac{\left(\frac{\beta}{k}\right) - n_2}{n_1 - n_2}$ MFD=2W_0 single mode $W_0 = a\left(0.65 + 1.619V^{-1.5} + 2.879V^{-6}\right)$

$B_F = n_y - n_x$ $L_p = \frac{2\pi}{K_0\left(n_y - n_x\right)} = \frac{2\pi}{K_0 B_F}$

Graded Index Optical Fiber (core n(r), cladding n_2, core radius a)

$$n(r) = \begin{cases} n_1\sqrt{1 - 2\Delta\left(\frac{r}{a}\right)^{\alpha}}, & 0 \le r \le a, \ core \\ n_1\sqrt{1-2\Delta} \approx n_1\left(1-\Delta\right) = n_2, & r \ge a, \ cladding \end{cases}$$

$$\Delta = \frac{n_1^2 - n_2^2}{2n_1^2} \approx \frac{n_1 - n_2}{n_1}$$

$axial \ NA(0) = \sqrt{n^2(0) - n_2^2} = \sqrt{n_1^2 - n_2^2}$ $M \approx \left(\frac{\alpha}{\alpha+2}\right) a^2 K_0^2 n_1^2 \Delta$

Fiber Dispersion and Loss

$P(z) = P_0 e^{-\alpha_p z}$ $\alpha_p = \frac{1}{z}\ln\frac{P(0)}{P(z)} \quad Np/m$ $\alpha = 4.343\alpha_p \quad dB/m$ $\alpha(\lambda) = \alpha_0\left(\frac{\lambda_0}{\lambda}\right)^4$

GRIN (parameter α) $N_{eff} = N_\infty\left\{1 - \frac{\alpha+2}{2\alpha\,\Delta}\left[\frac{2a}{R} + \left(\frac{3}{2n_2 kR}\right)^{2/3}\right]\right\}$ $N_\infty = \frac{\alpha}{\alpha+2}\left(n_1 ka\right)^2 \Delta$

$I_{\max} = I_{in} e^{-\alpha L}$ $\tau_g = \frac{L}{v_g}$ $\Delta\tau_{rms} = 2\sigma$

Gaussian $I = I_{\max} e^{-\frac{t^2}{2\sigma^2}}$ $\sigma = 0.425\Delta\tau_{1/2}$ $\Delta\tau_{1/2} = 2.35\sigma$

Rectangular $\sigma = 0.29\Delta\tau_{1/2}$

$B = \frac{1}{T_{\min}}$ RZ format $T_{\min} = 2\Delta\tau_{1/2}$ *or* 4σ NRZ format $T_{\min} = \Delta\tau_{1/2}$ *or* 2σ

$$f_{electrical} = \frac{f_{optical}}{\sqrt{2}}$$

Multimode SI fiber $\Delta\tau_{1/2} \approx \frac{L}{v_{g,\min}} - \frac{L}{v_{g,\max}} \approx \frac{L}{c}\frac{n_1}{n_2}(n_1 - n_2) = \frac{Ln_1^2\Delta}{cn_2}$ $\sigma_{intermode} = 0.29\Delta\tau_{1/2}$

GRIN fiber $\Delta\tau_{1/2} = \frac{Ln_1\Delta^2}{8c}$ $\Delta\tau_{rms} = \sigma_{intermode} \approx \frac{Ln_1\Delta^2}{20\sqrt{3}\,c}$

$$D_{mat} = \frac{1}{L}\left(\frac{d\tau_g}{d\lambda}\right) \qquad \Delta\tau = \left|\frac{d\tau_g}{d\lambda}\right|\Delta\lambda = L\left|D_{mat}\right|\Delta\lambda$$

$D_{wg} = \frac{1}{L}\left(\frac{d\tau_g}{d\lambda}\right) = \frac{d}{d\lambda}\left(\frac{1}{v_g}\right) = -\frac{2\pi c}{\lambda^2}\left(\frac{d^2\beta}{d\omega^2}\right)$ $\frac{d^2\beta}{d\omega^2}$ = GVD $\Delta\tau = \left|\frac{d\tau_g}{d\lambda}\right|\Delta\lambda = L\left|D_{wg}\right|\Delta\lambda$

$$D_{ch} = D_{mat} + D_{wg} \qquad \sigma_{total}^2 = \sigma_{intermodal}^2 + \sigma_{chromatic}^2$$

$$t_{SYS}^2 = t_{TX}^2 + \sigma_{TOT}^2 + t_{RX}^2 \qquad t_{RX} = 2.2RC \qquad BW = \frac{1}{2\pi RC} \qquad L_C = \frac{v_g}{\Delta f}$$

$$n = \frac{c}{v_p} \qquad n_g = \frac{c}{v_g} = c\frac{\partial\beta}{\partial\omega} = \frac{\partial}{\partial\omega}\left[\omega n(\omega)\right] = n(\omega) + \omega\frac{\partial n}{\partial\omega} = n(\lambda) - \lambda\frac{dn}{d\lambda}$$

Fiber Coupling Loss (lateral d, angular θ, longitudinal s)

$$\eta_F = \left[1 - \left(\frac{n_1 - n_0}{n_1 + n_0}\right)^2\right]^2 = \frac{16\left(\frac{n_1}{n_0}\right)^2}{\left(1 + \frac{n_1}{n_0}\right)^4}$$

Lateral: SI multimode fiber $A_C = 2a^2\cos^{-1}\left(\frac{d}{2a}\right) - d\sqrt{a^2 - \left(\frac{d}{2}\right)^2}$ $\eta = \frac{A_C}{\pi a^2}$

Lateral: GRIN (parabolic) multimode fiber $\eta_{LAT} = 1 - \frac{8}{3\pi}\left(\frac{d}{a}\right) = 1 - 0.85\left(\frac{d}{a}\right)$

Lateral: single mode fiber $\eta_{SM,LAT} = \exp\left[-\left(\frac{d}{w}\right)^2\right]$

Angular: SI multimode fiber $\eta_A = 1 - \frac{n_0\theta}{\pi n_1\sqrt{2\Delta}}$

Angular: single mode fiber $\eta_{SM,ANG} = \exp\left[-\left(\frac{\pi n_2 w\theta}{\lambda}\right)^2\right]$

Longitudinal: SI multimode fiber $\eta_S = \left(\frac{a}{a + s\tan(\theta_C)} \right)^2$

Longitudinal: single mode fiber $z = \frac{s\lambda}{2\pi n_2 w^2}$ $\eta_{SM,S} = \frac{1}{z^2 + 1}$

Optical Resonator

$$\frac{I_t}{I_i} = \frac{|T|^2}{(1-R)^2 + 4R\sin^2(KL)} \qquad \left.\frac{I_t}{I_i}\right|_{\max} = \frac{I_{\max}}{I_i} = \frac{|T|^2}{(1-R)^2} \qquad F = \frac{\pi\sqrt{R}}{1-R}$$

$$\nu_m = m\left(\frac{c}{2Ln}\right) \qquad \Delta\nu_m = \nu_{m+1} - \nu_m = \nu_f \qquad \nu_f = \frac{c}{2Ln} \qquad F = \frac{\nu_f}{\delta\nu_m}$$

$$\alpha_r = \alpha_s + \frac{1}{2L}\ln\left(\frac{1}{R_1R_2}\right)$$

PN Junctions

$$V_0 = \frac{KT}{e}\ln\left(\frac{N_A N_D}{n_i^2}\right) \qquad W_0 = \sqrt{\frac{2\varepsilon}{q}V_0\left(\frac{1}{N_A} + \frac{1}{N_D}\right)} \qquad C_0 = \frac{\varepsilon A}{W_0}$$

$$W = W_0\sqrt{1 - \frac{V}{V_0}} \qquad C = \frac{\varepsilon A}{W} = \frac{C_0}{\sqrt{1 - \frac{V}{V_0}}}$$

$Al_xGa_{1-x}As$. E_g(eV)=1.424+1.266x+0.266x^2 n~3.59-0.71x with 0<x<0.4

$In_{1-x}Ga_xAs_yP_{1-y}$ E_g(eV)=1.35-0.72y+0.12y^2 with 0<x<0.47 and y=2.15x

$$L_p = \sqrt{D_p\tau_p} \qquad L_n = \sqrt{D_n\tau_n} \qquad D_p = \frac{\mu_p KT}{e} \qquad D_n = \frac{\mu_n KT}{e}$$

$$\tau_p \approx \frac{1}{B(n_{no} + p_{no})} \approx \frac{1}{Bn_{no}} = \frac{1}{BN_D} \qquad \tau_n \approx \frac{1}{B(n_{po} + p_{po})} \approx \frac{1}{Bp_{po}} = \frac{1}{BN_A}$$

$$I_d = I_s\left[\exp\left(\frac{eV}{KT}\right) - 1\right] \approx I_s\exp\left(\frac{eV}{KT}\right) \qquad I_s = A\left(\frac{D_n}{L_n N_A} + \frac{D_p}{L_p N_D}\right)en_i^2$$

$$I_{recomb} = I_r\left[\exp\left(\frac{eV}{2KT}\right) - 1\right] \approx I_r\exp\left(\frac{eV}{2KT}\right) \qquad I_r = \frac{A}{2}\left(\frac{W_N}{\tau_p} + \frac{W_P}{\tau_n}\right)en_i$$

$$I = I_d + I_{recomb} = I_o\left[\exp\left(\frac{eV}{\eta KT}\right) - 1\right]$$

LED

$$\eta_{\text{int}} = \frac{\Phi_{\text{int}}}{I/e} = \frac{\text{internal photon flux}}{\text{electron flux}} \qquad \eta_{ext} = \frac{\Phi_0}{I/e} = \frac{\text{output photon flux}}{\text{electron flux}}$$

$$\eta_{\text{int}} = \frac{\frac{1}{\tau_r}}{\frac{1}{\tau_r} + \frac{1}{\tau_{nr}}} \qquad \eta_{loss} \approx \frac{1}{n(n+1)^2} \qquad \eta_{ext} = \eta_{\text{int}}\eta_{loss} \qquad |\Delta\lambda| = \frac{\lambda^2}{hc}|\Delta E_{ph}|$$

$$P_o = h f \Phi_o = \eta_{ext} \frac{h f I}{e} \qquad \eta = \frac{\textit{Optical Power Out}}{\textit{DC Electrical Power In}} = \frac{P_o}{IV}$$

$$\Re = \frac{\textit{Optical Power Out}}{\textit{DC Current In}} = \frac{P_o}{I} \qquad \Re = \frac{h f \Phi_o}{I} = \eta_{ext}\frac{hf}{e} \qquad \Re = \eta_{ext}\frac{1.24}{\lambda(\mu m)}$$

SLED $I(\theta) = I_{\max}\cos\theta$ $\quad P_{in} \simeq P_0 (NA)^2$ $\quad$ ELED $I(\theta) = I_{\max}\cos^n(\theta)$

$$\frac{p_1}{i_1} \approx \frac{1}{\sqrt{1+(\omega\tau)^2}} \qquad B = \frac{1}{2\pi\tau} \qquad t_r = \frac{0.35}{B}$$

Laser Principles

$$R_{12} = B_{12}N_1\rho(hf) \qquad R_{21} = A_{21}N_2 + B_{21}N_2\rho(hf) \qquad B_{12} = B_{21} \qquad \frac{A_{21}}{B_{21}} = \frac{8\pi hf^3}{c^3}$$

$$\rho_{eq}(hf) = \left(\frac{8\pi f^2}{c^3}\right)(hf)\left(\frac{1}{\exp\left(\frac{hf}{KT}\right)-1}\right)$$

$$P = P_0\exp(gz) \qquad g = \frac{n}{cN_{ph}}(N_2 - N_1)B_{21}\rho(hf) \qquad \rho(hf_0) \approx \frac{N_{ph}hf_0}{\Delta f} \qquad g = (N_2 - N_1)\frac{B_{21}nhf_0}{c\Delta f}$$

$$g_{th} = \alpha_{total} = \gamma + \frac{1}{2L}\ln\left(\frac{1}{R_1R_2}\right) \qquad (N_2 - N_1)_{th} \approx g_{th}\frac{c\Delta f}{B_{21}nhf_0}$$

Laser Diodes

$$L = m\left(\frac{\lambda_0}{2n}\right) \qquad \Delta\lambda_0 = \frac{2nL}{m} - \frac{2nL}{m+1} \approx \frac{2nL}{m^2} = \frac{\lambda_0^2}{2nL} \qquad \Delta f = \frac{c}{2nL} \qquad g(\lambda) = g(0)\exp\left[-\frac{(\lambda-\lambda_c)^2}{2\sigma^2}\right]$$

$$\Lambda = \ell\left(\frac{\lambda_B}{2n_e}\right) \qquad \lambda_B = \frac{2n_c\Lambda}{\ell} \qquad \lambda = \lambda_B \pm \frac{\lambda_B^2}{2n_eL_e}\left(m+\frac{1}{2}\right)$$

$$\eta_{slope} = \frac{\Delta P}{\Delta I} = \frac{P_{out}}{I - I_{th}} \qquad n_{th} = \frac{1}{C\tau_{ph}} \qquad \frac{J_{th}}{ed} = \frac{n_{th}}{\tau_{sp}} \qquad I_{th} = J_{th}A = \frac{n_{th}edWL}{\tau_{sp}}$$

$$P_{out} = \left[\frac{hc^2\tau_{ph}(1-R)}{2en\lambda L}\right](I - I_{th}) \qquad \tau_{ph} = \frac{n}{c\alpha_{total}} = \frac{n}{cg_{th}} \qquad f_{\max} = \left(\frac{1}{2\pi}\right)\frac{1}{\sqrt{\tau_{sp}\tau_{ph}}}\sqrt{\frac{I}{I_{th}} - 1}$$

Photodetectors

$$\lambda_g(\mu m) = \frac{1.24}{E_g(eV)} \qquad I(x) = I_0\exp(-\alpha x) \qquad \delta = 1/\alpha$$

$$\eta = \frac{I_{ph}/e}{P_0/hf} = \frac{hfI_{ph}}{eP_0} \qquad \Re = \frac{I_{ph}}{P_0} \qquad \Re = \frac{\eta e}{hf} = \eta\frac{e\lambda}{hc} \qquad V_{out} = \frac{R_L I_{ph}}{1 + j\omega R_L C_L}$$

$$t_d = \frac{W}{v_d} \qquad t_{diffusion} = \frac{L^2}{2D_n} \qquad t_r = \sqrt{t_{diffusion}^2 + t_{drift}^2 + \left(R_L C_L\right)^2}$$

$$R_j = \frac{dV_d}{dI_d} = \left[\frac{eI_s}{\eta KT}\exp\left(\frac{eV_d}{\eta KT}\right)\right]^{-1} \approx \frac{\eta KT}{eI_d} \qquad M = \frac{I_{ph}}{I_{ph0}} \qquad M = \frac{1}{1-\left(\frac{V_d}{V_{BR}}\right)^n}$$

Noise and Receiver Design

$$P_i = P_0\left[1 + mf(t)\right] \qquad I_S = MI_{s0} = M\Re P_0 \qquad i_s^2 = M^2\Re^2 P_0^2 m^2\left\langle f^2(t)\right\rangle = I_s^2 m^2\left\langle f^2(t)\right\rangle$$

$$i_n^2 = 2e\left(I_{d0} + I_{s0}\right)M^2 FB \qquad F = M^x \qquad i_{th}^2 = \frac{4KTB}{R_L} \qquad \{i_{nA}^2\} = 2eI_{in} \qquad B_{RC} = \frac{1}{2\pi RC}$$

$$RIN = \frac{\left\{P_{nL}^2\right\}}{P_0^2} \qquad i_{nL}^2 = M^2\Re^2\left\{P_{nL}^2\right\}B = \left(RIN\right)M^2\Re^2 BP_0^2 \qquad SNR = \frac{i_s^2 R_L}{i_n^2 R_L + i_{th}^2 R_L + i_{nL}^2 R_L} = \frac{I_s^2 m^2\left\langle f^2(t)\right\rangle}{i_n^2 + i_{th}^2 + i_{nL}^2}$$

Without equalization
$$SNR = \frac{I_{s0}^2 m^2\left\langle f^2(t)\right\rangle}{\left[2e\left(I_{s0} + I_{d0}\right)F + \frac{4KT}{M^2 R} + \frac{\left\{i_{nA}^2\right\}}{M^2}\right]\left(\frac{\pi B_{RC}}{4}\right) + \frac{\left\{v_{nA}^2\right\}B_{RC}}{M^2 R^2}}$$

With equalization
$$SNR = \frac{I_{s0}^2 m^2\left\langle f^2(t)\right\rangle}{\left[2e\left(I_{s0} + I_{d0}\right)F + \frac{4KT}{M^2 R} + \frac{\left\{i_{nA}^2\right\}}{M^2} + \frac{\left\{v_{nA}^2\right\}}{M^2}\left(\frac{1}{R^2} + \frac{(2\pi BC)^2}{3}\right)\right]B}$$

$$\omega_E = \frac{1}{R_E C_E} \qquad \omega_2 = \frac{1}{\left(R_{A,out} + R_E + R_0\right)C_E}$$

Transimpedance amplifier

$$SNR = \frac{I_{s0}^2 m^2\left\langle f^2(t)\right\rangle}{\left\{2e\left(I_{s0} + I_{d0}\right)F + \frac{4KT}{M^2}\left(\frac{1}{R} + \frac{1}{R_F}\right) + \frac{\left\{i_{nA}^2\right\}}{M^2} + \frac{\left\{v_{nA}^2\right\}}{M^2}\left[\left(\frac{1}{R} + \frac{1}{R_F}\right)^2 + \frac{(2\pi BC)^2}{3}\right]\right\}B}$$

Optical Components

$$n = n_1 + \Delta n = n_1 + pE \qquad strain = \frac{\Delta L}{L} = kE$$

MZI
$$P_{out} = \frac{1}{2}P_{in}\left(1 + \cos(\Delta\phi)\right) \qquad \Delta\phi = \pi\frac{V(t)}{V_\pi}$$

EDFA
$$NF = \frac{SNR_{in}}{SNR_{out}} = \left[1 + \frac{2P_{ASE}}{hf\ \Delta f_{sp}}\right]\frac{1}{G}$$