Name:

Student Number:

Quiz 1 ELEC 4705
Thursday Oct. 17 2019

1. (10 marks) Crystals and Electrons

   (a) When we bring three finite electron wells close together what happens to the energy levels for an electron in any of the wells? Why does this occur?

   (b) How do we represent the potential energy of an electron in a solid material? What impact does this have on the solution for the Schroedinger equation?

   (c) As we form a solid piece of material from a large number of atoms what impact does this have on the Energy versus momentum ($E(k)$) relationship of the electron.

   (a) Each energy state associated with a well will split into three slightly different energy states. This occurs because as the wells come close enough they couple (interact) and the wave function is split over the the three wells (similar to tunneling behaviour).

   (b) The potential energy must be represented by a periodic function (i.e. a Fourier Series) as the crystal is periodic. The solution of the SCE under this case is the Bloch solution $\Psi(x) = U(x)e^{ikx}$.

   (c) As a solid is formed the energy levels associated with an individual well form into bands or allowable energies separated by band gaps of disallowed energies.

2. (10 marks) Band structures and conduction

   (a) What is our basic model for potential energy of an electron in a finite piece of material? How does the finite size of the material impact the $E(k)$ relationship?

   (b) Is the electron a Fermion or Boson? Why is this important for how electrons "fill" band structures.

   (c) In order to calculate the number of electrons at a particular energy in a band structure ($n(E)$) we need to define two functions. Name them and briefly describe what they represent.

   (d) An electron in an electric field (E) will be subject to a force $F = qE$ if we assume that $F = ma$ holds for the electron how does the electron behave? Does this make sense? If not what is the additional physical mechanism we use to "fix" the behavior of the electron so that it does make sense. Specifically what does this mechanism do?

(a) Our model is of a large number of atomic sized wells inside a large well representing the finite crystal (Wells with in a well). The finite size of the material limits the number of atoms to N and each atom will contribute one state to a band. Therefore each band can accept 2N electrons.

(b) The electron is a Fermion. This is important as due to the Pauli exclusion principle each electron state in the band can only be filled with two electrons (spin up/spin down) and the bands will fill up.

(c) The two functions are:

- $g(E)$ **the density of states function which describes the number of states available at the energy E.**
- $F(E)$ **the Fermi function which describes the probability of a particular state at the energy E being filled.**

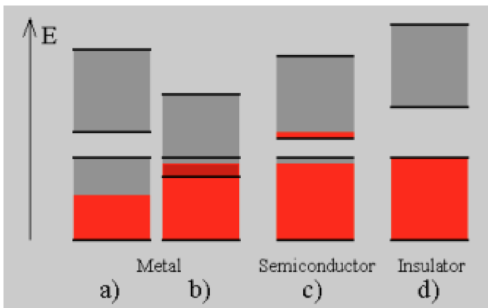
(d) As the force is constant the acceleration will be constant and velocity will increase linearly to infinity. This does not make sense as for a given field we observe a constant current (i.e. electron velocity). The mechanism that limits the velocity to a constant velocity is scattering. Scattering causes the electron to change momentum and lose energy during its acceleration limiting the velocity on average to the drift velocity.

3. (10 Marks) Semiconductors

   (a) Draw the band structure of a metal, semiconductor and insulator. What distinguishes the semiconductor from the insulator?

   (b) What are conduction electrons and holes. Why can they carry current?

   (c) How do we engineer a semiconductor such as Silicon to create regions of material that differ in the number of electrons and holes?



(a) **Insulator has a larger band gap.**

(b) **When an electron is excited from the valance band to the conduction band it creates a "conduction electron" in the conduction band and leaves behind an empty state in the valance band which we call a "hole". They can both carry current as there are states nearby to occupy. The conduction band is very empty and the conduction electron can easily move to a another state nearby in k space. The valance band is full but the hole can easily migrate in the band by displacing an electron and moving to a nearby state.**

(c) **We engineer the silicon by adding dopants (impurities) to it.**