ELEC4705 – Fall 2009

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LECTURE 2
Classical Physics

Contents

2.1. Operators

The most sophisticated and powerful method of expressing classical physics is through the development of differential equations. Often these are partial differential equations which are functions of time (t) and space $\vec{r} = \{x, y, z\}$. We do this through operators. We replace the simple math and laws such as $F = ma$ with more powerful expressions – very often differential operators.

We will now define some of these operators. Operators are mathematical objects that "operate" in other words perform a manipulation on a function. An example is integration or differentiation. Using operators we can reformulate the laws of physics in powerful concise expressions that allow us to solve difficult problems – sometimes with difficulty!

2.1.1. The "Del" operator – ∇

We have the operator ∇ defined as below.

$$\nabla = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z} \tag{2.1}$$

As an example we have

$$\nabla v = \frac{\partial v}{\partial x}\hat{i} + \frac{\partial v}{\partial y}\hat{j} + \frac{\partial v}{\partial z}\hat{k}.$$

A summery of some mathematical operations based on ∇ is shown in Table 1.

Table 1. Operations based on ∇

operation	name	result
∇ V	Gradient	Vector
∇ . V	Divergence	Scaler
∇^2V	Laplacian	Scaler
$\nabla\times$ V	Curl	Vector

The gradient can be thought of as essentially the slope of a function, although in 2/3D it has a direction. Divergence is the change in the flux of vector field – such as gas velocity or current density. It represents the source or destruction of what ever is flowing. The Laplacian determines the curvature of a function. The Curl obtains the rotation of a field (turbulence in water flow).

2.2. Formulation of Classical Physics

Most of classical physics (what was know before 1905) can be summarized as follows:

- Maxwell's Equations
- Conservation of charge (can be deduced from Maxwell's equations)
- Force laws
- Laws of motion
- Law of gravitation

2.2.1. Maxwell's Equations

Maxwell's equations in electromagnetism are described as follows:

- The source of an electrical field is the existence of electrical charge i.e. flux of E throguh a closed surface $\propto$ charge inside.

$$\nabla . E = \rho/\varepsilon_0 \tag{2.2}$$

- Flux of B through a closed surface $= 0$, i.e. there is no magnetic monpole.

$$\nabla . B = 0 \tag{2.3}$$

- According to the Farday's law of induction we have:

$$\nabla \times E = -\frac{\partial B}{\partial t} \tag{2.4}$$

(A changing magnetic field will induces an electric field)
- According to Ampere's law a current or a time varing electric field induces a magnetic field as:

$$c^2 \nabla \times B = \frac{\partial E}{\partial t} + \frac{j}{\varepsilon_0} \tag{2.5}$$

Maxwell's equation in free space: In free space we have $\rho = 0$ and $J = 0$ so we have:

$$\begin{aligned} \nabla . E &= 0 \\ \nabla . B &= 0 \\ \nabla \times E &= -\frac{\partial B}{\partial t} \\ \nabla \times B &= \frac{1}{c^2}\frac{\partial E}{\partial t} \end{aligned} \tag{2.6}$$

From the math we can obtain using identities and algebra:

$$\nabla \times (\nabla \times A) = \nabla(\nabla . A) - \nabla^2 A \quad \longrightarrow \tag{2.7}$$

$$\begin{aligned} \nabla \times (\nabla \times E) &= \nabla(\nabla . E) - \nabla^2 E \quad \longrightarrow \\ -\frac{\partial}{\partial t} \nabla \times B &= 0 - \nabla^2 E \quad \longrightarrow \\ \frac{\partial}{\partial t}\frac{1}{c^2}\frac{\partial E}{\partial t} &= \nabla^2 E \quad \longrightarrow \\ \nabla^2 E - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} &= 0 \end{aligned} \tag{2.8}$$

And equation 2.8 is the Maxwell's equation in free space for the electric field. There is a corresponding one for the magnetic field. It is a simple wave equation.

2.2.2. Conservation of Charge

Basically conservation of charge means that electrical charge can not be created or destroyed. In other words it says that the total amount of charge inside any region can only change by the amount that passes in or out of the region, which is expressed as the continuity equation as follows:

$$\nabla . J + \frac{\partial \rho}{\partial t} = 0 \tag{2.9}$$

which states that the divergence of charge flow equals to the change of charge in time.

2.2.3. Force laws

An example is the force acting on a charged particle in presence of electromagnetic fields as given by Lorentz force equation:

$$F = q(E + v \times B) \tag{2.10}$$

2.2.4. Laws of Motion

According to classical physics we have the force on moving particles as follows:

$$F = \frac{dp}{dt} \tag{2.11}$$

$$F = ma \tag{2.12}$$

where

m: is the mass of the particle
a : is the acceleration
p : is the momentum

2.2.5. Laws of Gravitation

Newton's law states that the force acting on two particles due to their gravity is inversely proportional to the distance between them and is given by:

$$F = -G\frac{m_1 m_2}{r^2} \tag{2.13}$$

where G is the gravity constant.

2.2.6. Atomic Theory

Matter consists of particles (electron, protons, neutrons). These particles have qualities (mass, charge). Are distinguishable (we can label them). They have very specific positions, trajectories and predictable actions (deterministic). They are subject to forces and governed by laws. Materials were atoms held together by forces (a bit unspecified) and described by "bulk" properties (conductivity, hardness, etc).

2.2.7. Others – primarily derived from the previous and atomic theory

Wave Equation: Sound, EM, light, etc

$$\frac{\partial^2 u}{\partial t^2} = c^2\nabla^2 u$$

Diffusion Equation: Heat, gasses, electrons, holes etc.

$$\frac{\partial u}{\partial t} = D\nabla^2 u + S(\vec{r})$$

Fluid Flow: Naiver-Stokes Equation

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v}\right) = -\nabla p + \nabla\cdot\mathbb{T} + \mathbf{f}$$

Stress-Strain: Continuum mechanics

$$\dot{\rho} + \rho\nabla\cdot v = 0 \text{ (one of a number of linked equations)}$$

The following are described as forming Classical mechanics:

- Newtonian mechanics, the original theory of motion (kinematics) and forces (dynamics)
- Hamiltonian mechanics, a theoretical formalism, based on the principle of conservation of energy
- Lagrangian mechanics, another theoretical formalism, based on the principle of the least action
- Celestial mechanics, the motion of heavenly bodies: planets, comets, stars, galaxies, etc.
- Astrodynamics, spacecraft navigation, etc.
- Solid mechanics, elasticity, the properties of (semi-)rigid bodies
- Acoustics, sound (= density variation propagation) in solids, fluids and gases.
- Statics, semi-rigid bodies in mechanical equilibrium
- Fluid mechanics, the motion of fluids
- Soil mechanics, mechanical behavior of soils
- Continuum mechanics, mechanics of continua (both solid and fluid)
- Hydraulics, mechanical properties of liquids
- Fluid statics, liquids in equilibrium
- Applied mechanics, or Engineering mechanics
- Biomechanics, solids, fluids, etc. in biology
- Biophysics, physical processes in living organisms
- Statistical mechanics, assemblies of particles too large to be described in a deterministic way
- Relativistic or Einsteinian mechanics, universal gravitation

Most of engineering is (or has been) based on classical physics.

2.2.8. Classical versus Modern or Quantum Physics

The following are categorized as being part of Quantum mechanics:

- Particle physics, the motion, structure, and reactions of particles
- Nuclear physics, the motion, structure, and reactions of nuclei
- Condensed matter physics, quantum gases, solids, liquids, etc.
- Quantum statistical mechanics, large assemblies of particles

Philosophically, some very big changes with QM. Classical mechanics is deterministic. If you know the starting point and all the physics you know where everything is going. No Free Will? QM is stochastic and random. Particles are never precisely pinned down to a place/time or state. Back comes Free Will? Perhaps? Some people think that QM and consciousness are very closely linked. A lot weird results and a lot of arguments (Einstein and Bohr). Still not closed or understood.

There is natural tendency to draw a line between the two. However, it is difficult and leads to more problems. Get into SciFi stuff such as many worlds, teleportation etc.

We have a very good intuition for CM. It is, basically, the physics of our time and length scale. We understand Newton's laws because we evolved to understand them! If we did not we would die! QM is the physics of the small and the energetic. This is not our natural domain. We have a very poor feel for the physics. Everything is vague often counter intuitive and sometimes down right bizarre.

However, QM is correct – never been shown to be wrong, CM is an approximation of QM.

Science proceeds by a series of approximations to the truth. We have an agreed set of facts, which grows over time as our experimental techniques improve. As these facts invalidate a theory another theory replaces the last one. Paradigm shifts are used to describe this phenomena. Usually, the last theory is found to be a limited approximation of the latest one. A theory makes predictions based on premises and facts and the application of logic and math, if the predictions are found to be in error then the theory is lacking or incorrect. No predictions, then no theory, just speculation.

* Material from:http://en.wikipedia.org/wiki/Mechanics#Classical_versus_quantum

2.3. Problems with classical physics

These simple laws with extensions and modifications were found to correctly predict a vast array of effects and properties. However, a number of effects became experimentally confirmed that could not be explained by these classical theories. Three of the most prominent were:

(a) The photoelectric effect
(b) Black body radiation
(c) Heat capacity of solids

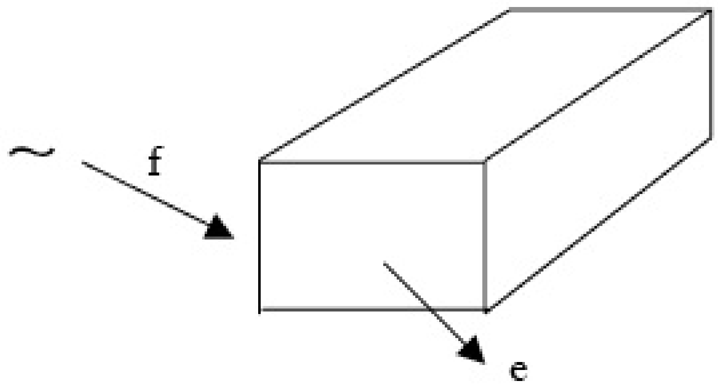

Figure 1. Electron emision

2.3.1. The photoelectric effect

Consider Figure 1. Suppose a light with frequency f is incident on a solid, then experimentally electrons are emitted from the surface if we have,

$$\hbar\omega > \phi \tag{2.14}$$

where

- $\omega = 2\phi\, f_0$ and f_0 is the threshold light frequency.
- ϕ is a function of the material called the work function.
- $\hbar = h\,/\,2\pi$ and h is a fundamental constant in physics called Planks constant, $h = 6.626068 \times 10^{-34}\, j.s$

The photoelectric effect is independent of the intensity of the light striking the solid sample for frequencies below f_0. The existence of the threshold frequency cannot be explained on the basis of classical physics.

According to the classical analysis, light is like a classical EM wave, therefore the higher intensity we have, the more energy is delivered to the solid surface and so the more electron emission we should have, **which seemed to be wrong!**.

In 1900 Max Planck announced a new formula that fitted the experimental evidences well, but at that time he didn't have a theoretical explanation for his formula. Later he came up with a theory that was more satisfactory. Quantum theory as it was later known stated that light consisted of a set of discrete energies known as photon. Planck derived his formula based on this theory, which is now the fundamental idea of quantum mechanics. Planck's formula is given by

$$E = hf = \hbar w \tag{2.15}$$

where h is Planck's constant and f is the frequency of the phonon. Later on Einstein proposed the photon theory of light in photoelectric effect. Using Planck's idea of discrete energy, Einstein showed that the discrete energy E released from a matter is proportional to the frequency f of the incident light. His work confirmed that energy released or absorbed by an electron is in discrete form therefore under certain conditions light behaved like individual particles. Einstein won the Noble prize for his work on 1921. Thus only when a photon with energy greater than the work function of the metal (the amount of energy needed to remove an electron from a solid) collided with an electron there would be enough energy to emit an electron. This explained the behavior of the solid when illuminated, showed in Figure 2.

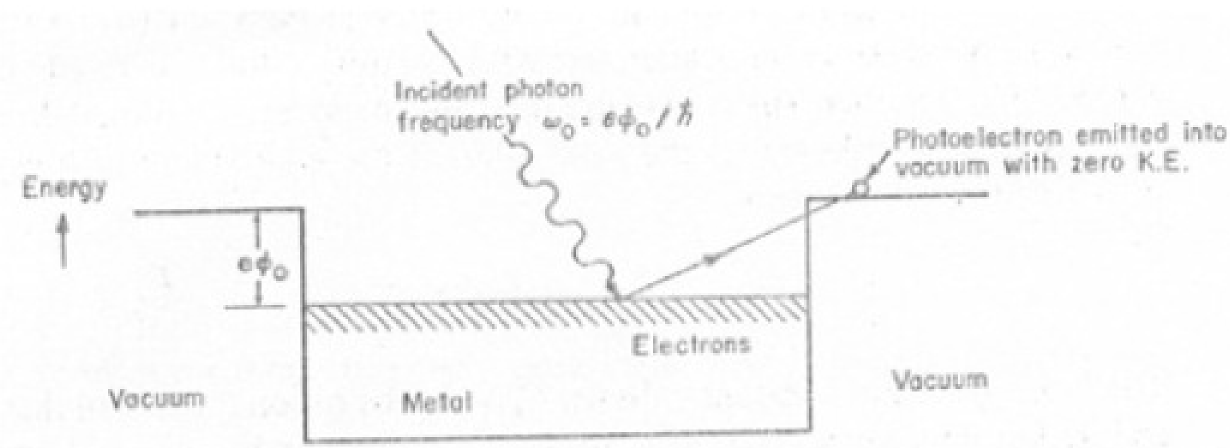

Figure 2. Einstein's model for photoelectric effect

If the energy of a photon were less than the work function of the metal then no electrons would be emitted, otherwise an electron can be emitted and the difference in energy will be equal to the kinetic energy of the emitted electron.

It was also proposed that if an electromagnetic field can behave like a particle with an energy of E, then the light particle should also have a momentum p as

$$p = \hbar k \quad , \quad k = w/c \quad (only\,for\,photons) \tag{2.16}$$

This can be derived from relativistic equations and is determined assuming that the photon has zero mass and travels at the speed of light.

This was the start of Quantum Mechanics!

Plasma Glow is another photo effect which classical physics can not explain. place a gas in a glass tube, use a large electric potential to cause break down (separation of ions and electrons). The light emitted is only at certain wavelengths figure 3. Mysteries:

- How are spectral lines created?
- Why do electrons NOT collapse into a nucleus?

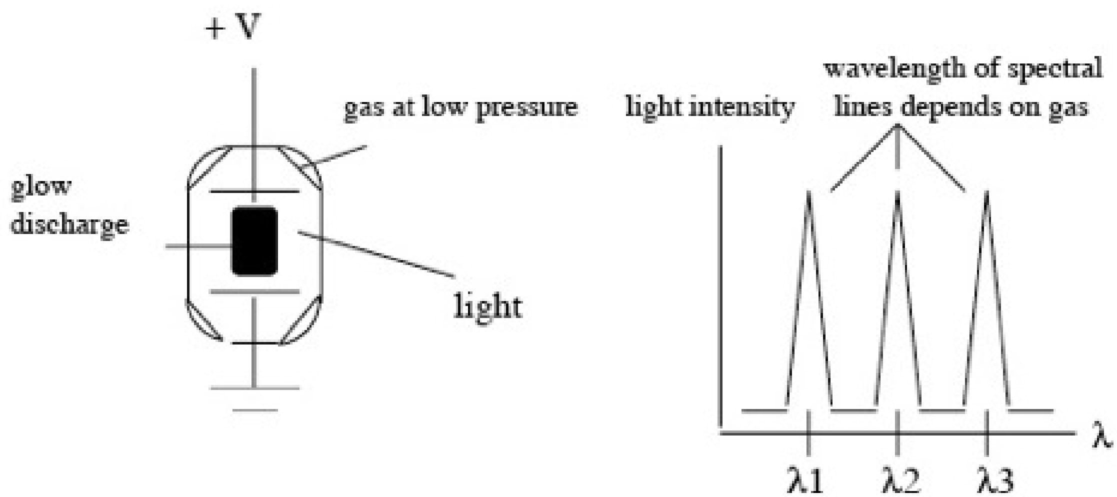

Figure 3. Plasma Glow

2.3.2. Black Body Radiation

A black body is a theoretical object that absorbs 100% of the radiation that hits it. Black body radiation is the Radiation emitted by an opaque or hot body. The black body radiation curve in figure 4 shows that the black body does radiate energy at every wavelength. It also shows that the black body emits at a peak wavelength, at which most of the radiant energy is emitted. This is represented by the area under the curve. Figure 4 also shows that as the temperature increases, the peak wavelength emitted by the black body decreases. Classical physics failed in explaining the form of these intensity curves shown in figure 4. The best approximation (Rayleigh-Jeans law) matched the experimental curves only in the very long wavelength region and predicted infinite power in the short wavelength region (absurd!) see figure 5.

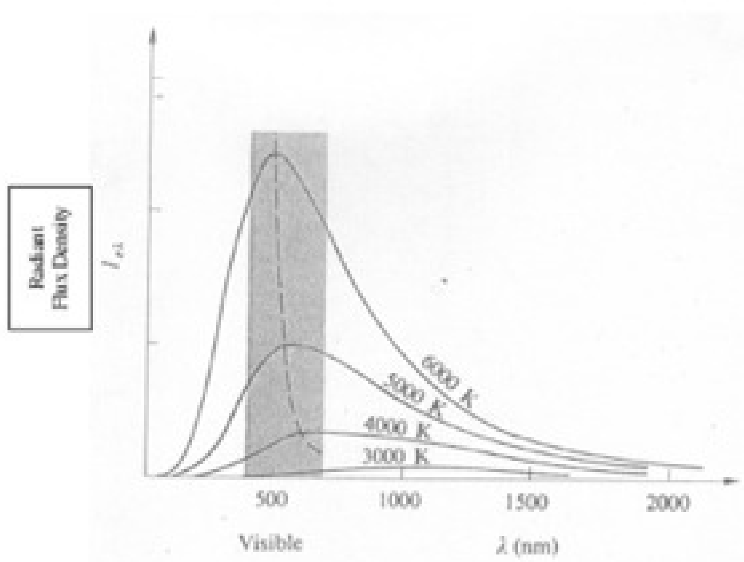

Figure 4. Black Body Radiation

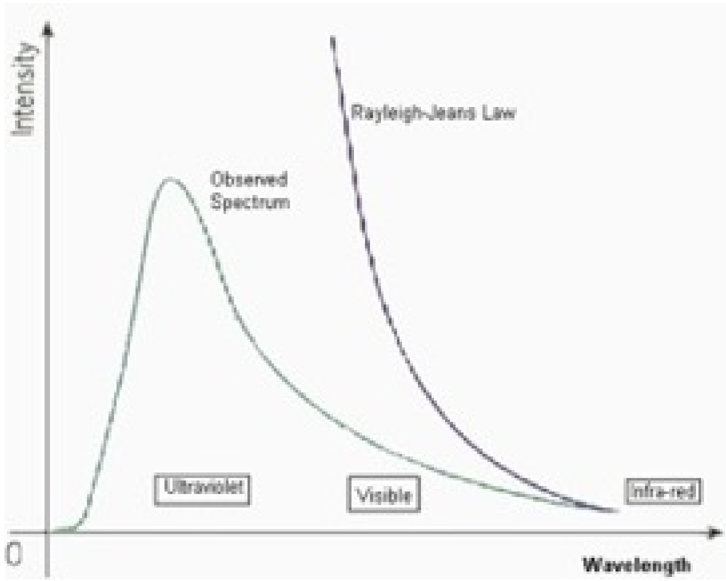

Figure 5. Rayleigh approximation for black body radiation

2.3.3. Heat capacity of solids

Heat capacity of a solid is the rate of the increase of internal energy per unit temperature rise see figure 6. Dulong and Petit law stated that the heat capacity at constant volume of all elementary solids is approximately $2.49 \times 10^4 J.kilomole^{-1}K^{-1}$ i.e. $3R$, as is shown in figure 6.

Later on it was predicted by the Debye model and the Einsteins earlier model that heat capacity is a function of temperatue see figure 6. Note that, as expected, the heat capacity is zero at absolute zero, and rises to a value of three as the temperature becomes much larger.

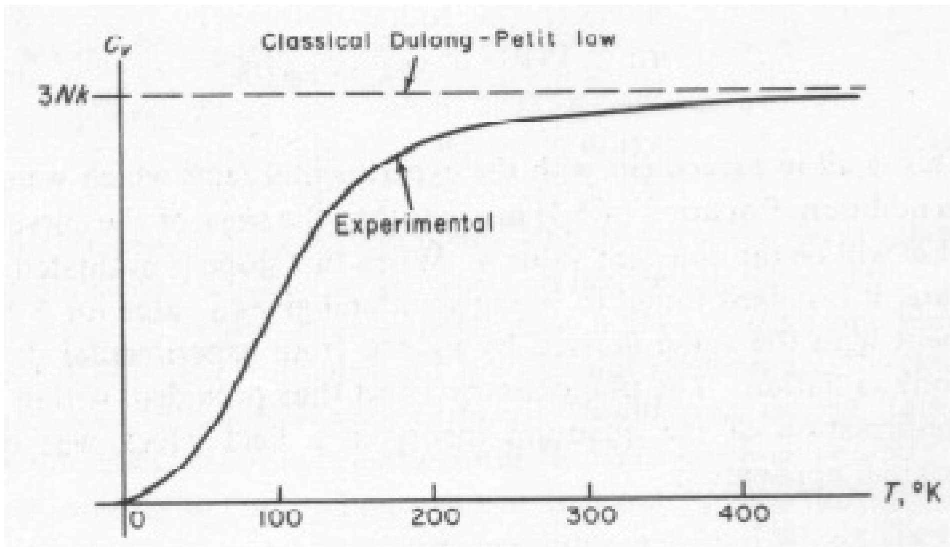

Figure 6. Heat Capacity

2.4. Complementary Notes – on operators

The divergence of a vector flux density (B, D) is the outflow of flux from a small closed surface per unit volume. It tells us how much flux is leaving a small volume on a per unit-volume basis.
Charges are the sources (+) and sinks (-) of electric field. If in the figure 7 there is a difference between the amount of flux flowing into and out of the closed surface A, it will be due to the presence of sources or sinks within A.

There is no magnetic counterpart to the electric charge, i.e., no isolated magnetic poles have ever been found. Unlike E, B does not diverge from or converge toward some kind of magnetic charge (a monopole source or sink). Lines of B are themselves continuous and closed, thus, there would be an equal number of lines of B entering and emerging from A.

The curl can be described as the circulation of a vector field per unit area.

- A time varying B generates a circulating E, see figure 8.
- A current flow (i) or time varying E induces a magnetic field, see figure 9.

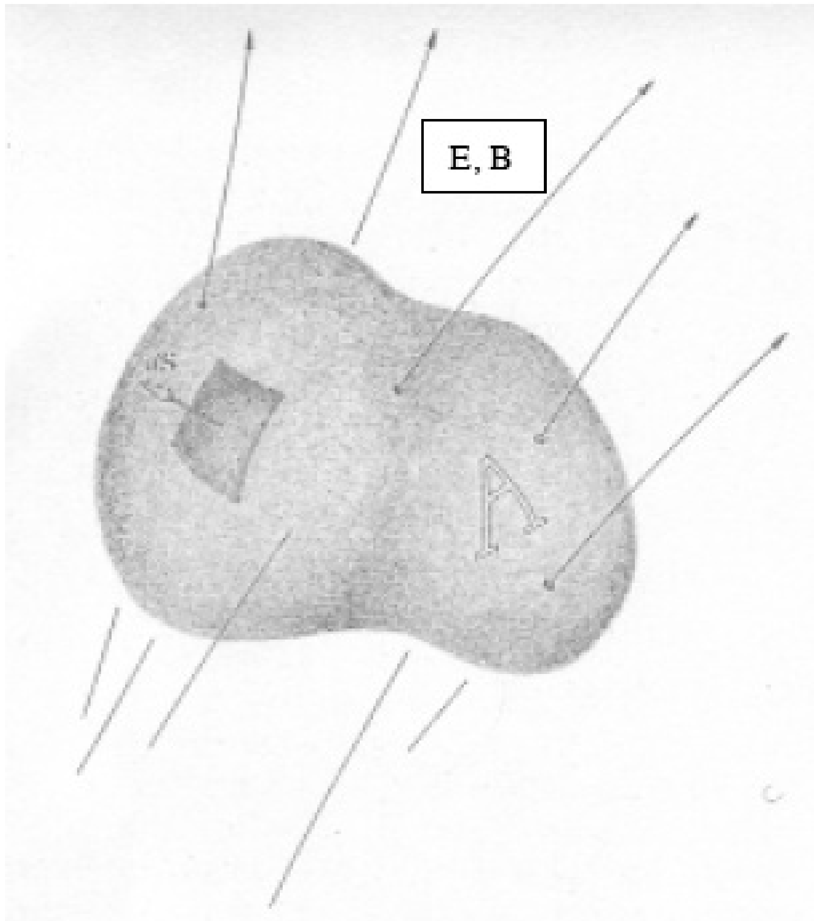

Figure 7. Divergence of flux density

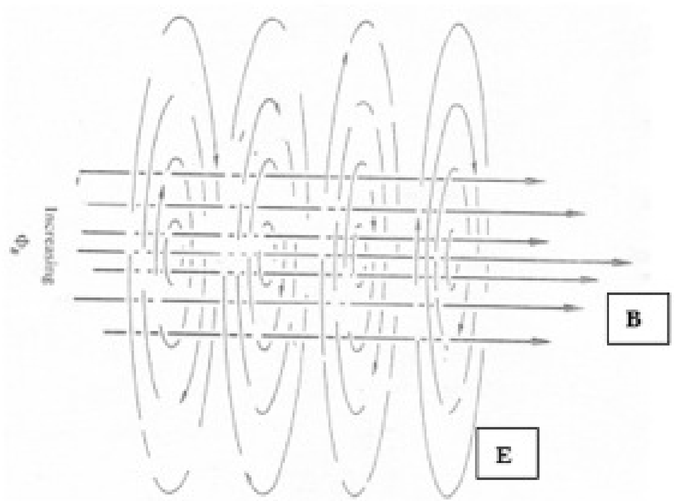

Figure 8. Time varying B generates E

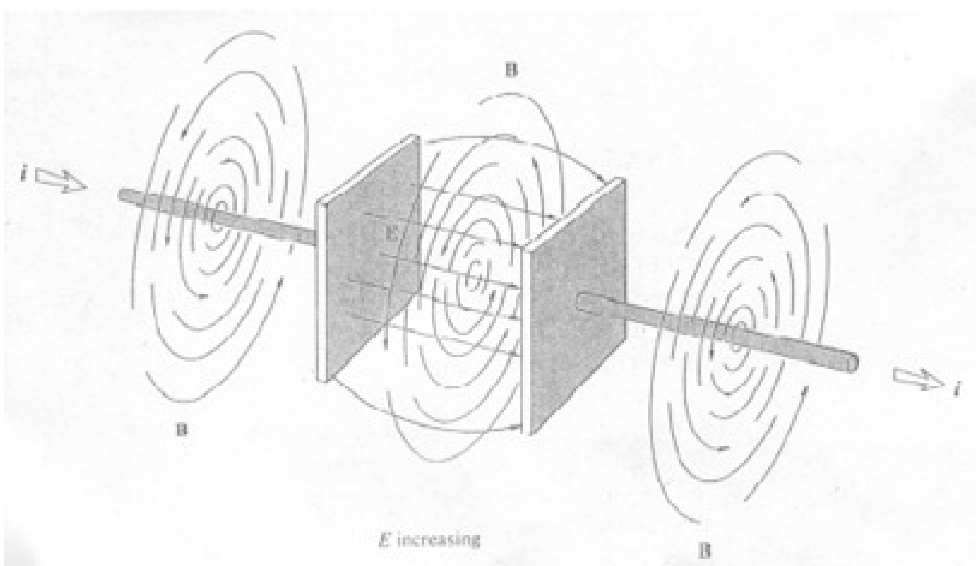

Figure 9. current induces B