ELEC4705 – Fall 2009

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LECTURE 11
Light

Contents

11.1. Introduction

We now wish to look at the interaction of light with matter. We have a description of matter (solid crystals) as a band structure created by a regular array of atoms and an electronic band structure. We now need a useful description of light.

Classically, light is a propagating electromagnetic wave (electric field + magnetic field). For simple situations we can calculate the electric field Using Coulomb's law.

The two fields primarily describe the interaction of charges (either stationary or moving).

- The electric field E describes the force on an electric charge due to the presence of a charge distribution. Charge is also the source of electric field:
  - Coulomb's law:

$$E = \frac{Q}{4\pi\varepsilon_0 |r|^2} \tag{11.1}$$

  - For an arbitrary charge distribution.

$$E = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho}{|r|^2} d^3 r \tag{11.2}$$

- The magnetic field B is a complementary field that produces a force on an electrical charge moving with velocity v. Producing a net force of:

$$Force = qE + \frac{q}{c}.v \times B \tag{11.3}$$

11.2. Maxwell's Equations

For complex situations to determine the two fields we use Maxwell's equations (which we saw briefly in the 1st lecture).

If E is the electric field and B is the magnetic field we have Maxwell's equations as below:

- The source of electrical field is the electrical charge.

$$\nabla \cdot E = \rho/\varepsilon_0 \tag{11.4}$$

  where ρ is the charge density. For a semiconductor this would be the sum of the two mobile charges plus the fixed charges $\rho = (N_D - N_A - n + p)q$

- A current or time varying field induces a magnetic field.

$$\nabla \times B = -\mu_0 \frac{\partial}{\partial t}(\varepsilon_0 E) + j \quad (11.5)$$

$$= -\mu_0 \frac{\partial}{\partial t}(\varepsilon_0 E + P) \quad (11.6)$$

where for a material composed of atoms $P = Nqr$ is the polarization density which will capture material effects. In the material charges will be present that can form currents to flow. We define r as the displacement of a charge q from its equilibrium position due to an excitation. The time derivative of the polarization represents effect of moving these charges (ie a current).

- In general when we have both the magnetic and electric field with Faraday's law of induction we have

$$\nabla \times E = -\frac{\partial B}{\partial t} \quad (11.7)$$

Which means A changing magnetic field induces an electric field.

- There is no magnetic monopole.

$$\nabla . B = 0 \quad (11.8)$$

11.2.1. Electromagnetic waves in vacuum

In free space from Maxwell's equations we will have the wave equation as:

$$\nabla^2 E = \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} \quad (11.9)$$

$$\frac{\partial^2 E}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} \quad \textit{in one dimensional systems} \quad (11.10)$$

Where $c = (\mu_0 \varepsilon_0)^{-0.5}$ is the speed of light in SI units. The solution to the wave equation is a sinusoidal wave given by equation 11.11 and shown in figure 1.

$$E(x,t) = A\cos(kx - wt + \phi)\hat{p} \quad (11.11)$$

The standard wave parameters describe the solution:

Frequency: $w = 2\pi v$ (radians per second)
Wavevector: $k = 2\pi/\lambda = w/c$ (inverse length, spatial frequency)
Wavelength: $\lambda = 2\pi/k$ (length, e.g. nm)
Polarization: p (a unit vector)
Phase: ϕ (radians)

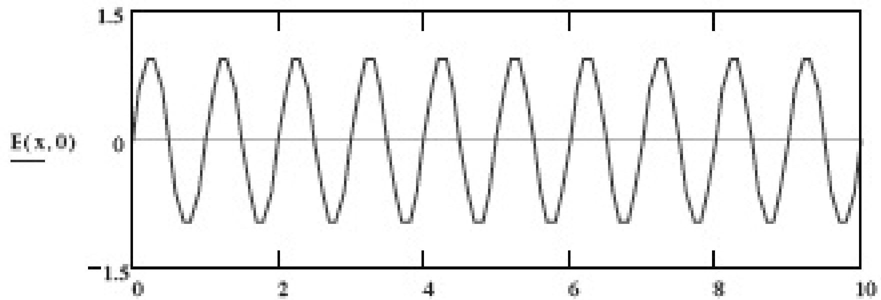

Figure 1. An electromagnetic wave

Intensity: $I = \frac{1}{2}\sqrt{\frac{\varepsilon_0}{\mu_0}}|E_0|^2$ (watts/area)

Phase velocity: $c = w/k$ (3×10^8 meters per second)

Notes:

- *In vacuum, and most of the rest of the time, electromagnetic waves are transverse, i.e.* $E \cdot K = 0$ *as shown in figure 2.*

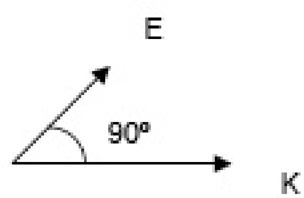

Figure 2. transverse waves

- *An electromagnetic wave has a B field as well as shown in figure 3. For a transverse wave the two fields are at right angles to each other and perpendicular to the direction of propagation.*

11.3. Photons, QM

As with electrons Quantum mechanics tells us that light has a dual nature – it propagates as a wave, but interacts like a particle. Therefore an EM wave can only absorb and emit energy in discrete packets or quanta. The quantum of EM radiation is called a photon. For photon's

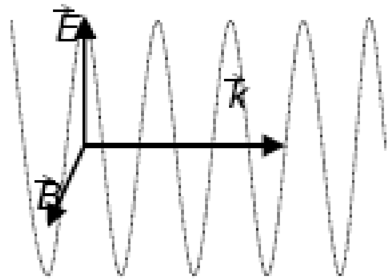

Figure 3. E-B-K graph

energy and momentum we have:

$$E = \hbar w \tag{11.12}$$
$$P = \hbar k \tag{11.13}$$

The photon energy vs. wavelength is shown in figure 4.

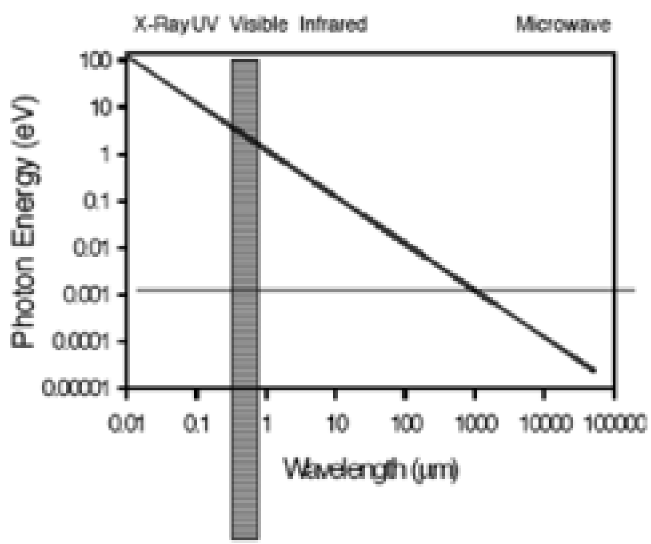

Figure 4. photon energy vs. wavelength

Photons are **not fermions**, however, they are **bosons** this means that there is no limit to how many photons can occupy a single energy state. Quite different statistics!.

11.4. Light in materials

When light propagates through a material it primarily interacts with the electrons as they are very light and easily energized by the electromagnetic energy. It is useful to use both classical and QM models of the electrons in materials. Classically, we have two types of electrons – bound and free. QM describes electrons as existing in a band structure which limits the available energy transitions. Although free electron absorption can be important most optical properties are determined by bound electron transitions.

11.4.1. Classical Models

The classical model of bound electron in a material is a spring model. Using this model we will add a polarization term to wave equation which will describe the effect of the material,

$$\frac{\partial^2}{\partial x^2}E = \frac{1}{c^2}\frac{\partial^2}{\partial t^2}(E + \frac{1}{\varepsilon_0}P) \tag{11.14}$$

Where,

$$P = -Ner \qquad \textit{Polarization density} \tag{11.15}$$

will capture all the material effects.

The basic model is shown in figure 5 with a heavy atom center (essentially immobile) and a light electron bound to the atom by a spring. The atom is at in an orbital where the effect of an oscillating E field is to cause the electron to vibrate through a small distance r. This oscillating charge will induce a electromagnetic field which modifies the original electric field.

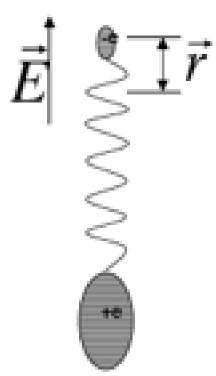

Figure 5. spring model for light

11.4.2. Susceptibility and Dielectric Constant

The modification of the electric field is captured in the P to determine an expression for P in terms of the original field **we assume a linear response**: i.e. $r \propto E$. Then we will have:

$$P = \varepsilon_0 \chi E \tag{11.16}$$

$$\varepsilon = \varepsilon_0(1+\chi) \tag{11.17}$$

where we define the optical parameters:

χ**:** linear susceptibility
ε**:** permittivity

If we know χ and ε then we know everything (almost).

To determine the effect of P on the electric field we use the wave equation including the presence of matter. Which simplifies to,

$$\frac{\partial^2 E}{\partial x^2} = \frac{1+\chi}{c^2}\frac{\partial^2 E}{\partial t^2} \tag{11.18}$$

$$n \equiv \sqrt{1+\chi} = \sqrt{\frac{\varepsilon}{\varepsilon_0}} \tag{11.19}$$

where n is called the **index of refraction**.

The solution of the wave solution will be as equation 11.20.

$$E(x,t) = E_0 \cos(nk_0 x - wt + \phi)\hat{p} \tag{11.20}$$

which can be written as

$$E(x,t) = E_0\, \hat{p}\, Re[e^{i(nk_0 x - wt + \phi)}] \tag{11.21}$$

Note:

$e^{i\theta} = \cos\theta + i\sin\theta$ where $i = \sqrt{-1}$.

$Re[e^{i\theta}] = \cos\theta$,

$Im[e^{i\theta}] = \sin\theta$

This result can be generalized by including losses in the electron vibration. This models the absorption of light and its conversion to heat. We do this by defining a **Complex Refractive index** as $N = n + ik$. This gives the following wave equation,

$$E(x,t) \sim e^{i(Nk_0x - wt)} \tag{11.22}$$

$$E(x,t) = E_0\,\hat{p}\,e^{-\alpha x/2}\cos(nk_0x - wt) \tag{11.23}$$

where

- $e^{-\alpha x/2}$ is the damping term
- $\cos(nk_0x - wt)$ is the wave term
- Absorption coefficient: $\alpha = 2\kappa k_0 = 2\,Im(N)k_0$
- Refractive index: $n = Re(N)$
- Phase velocity $= c/n$

The wave solution in matter (equation 11.20) is shown in figure 6.

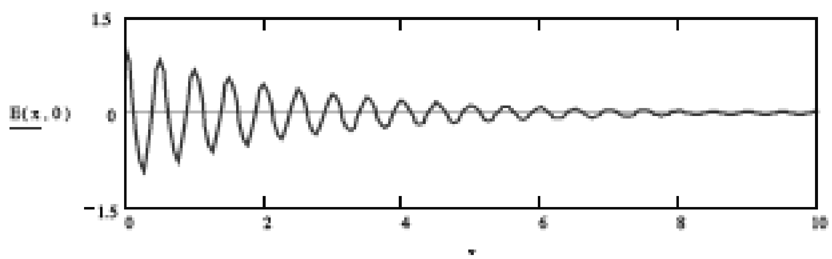

Figure 6. wave solution in matter $n = 2$, $\alpha = 0.8$, $N = 2 + i0.063$

Some notes about the wave solution in matte:

- Phase velocity $= c/n$ i.e. light slows down.
- $\lambda = \lambda_0/n$ i.e. Wavelength is shorter.
- $k = nk_0$ i.e. Wavevector is larger.
- $\alpha = 2\kappa k_0$ Optical absorption or gain.

11.4.3. Quantum Mechanical Model – Optical constants of real materials

The QM model uses a band structure to model the electron behaviour (replacing the spring model). In this model the allowed electron transitions determine how the light can interact with the material.

For a single isolated atom the allowed energy levels are discrete and the allowed transitions are sharply defined. For example $\Delta E = E_2 - E_0$. Light will only be absorbed or emitted at these transitional energies where $\hbar\omega = \Delta E$. At these transitions the light will be strongly absorbed and remitted causing a rapid change in both the real and complex index of refraction. The springs in the classical model are approximate models of these transitions. Spring resonance occurs when the photon energy equals the energy required for an electronic transition energy $\hbar w = E_i - E_j$.

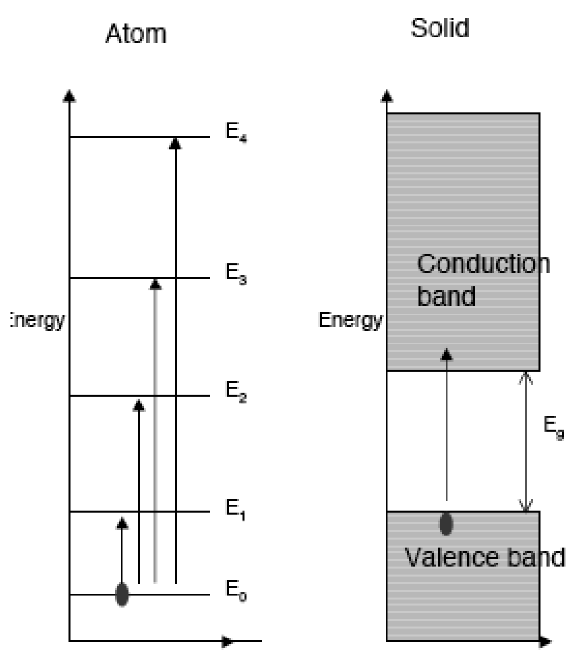

Figure 7. Band structure and energy levels

Note: It should be remembered that a transition can only occur from and occupied electron state to a empty state.

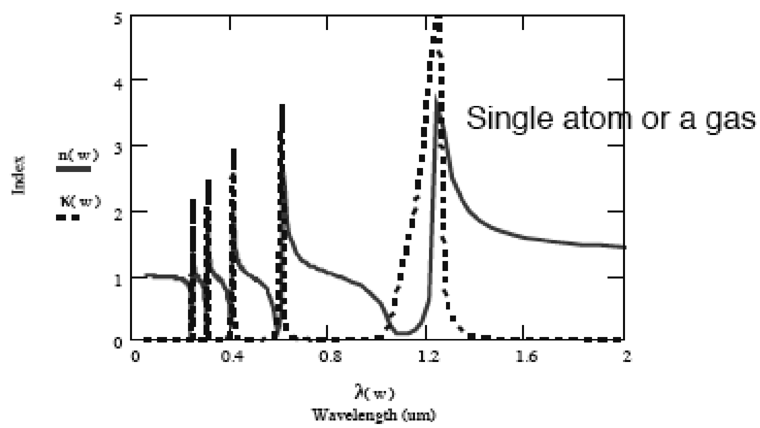

Figure 8

In a crystal the allowed transitions are from band to band - essentially bound electron transitions and with in a band (free electron transitions). For a semiconductor there is one primary transition between the valance band and the conduction band. This means that there is sudden transition from transparent to absorbing as the light energy increases beyond the value of the band gap.

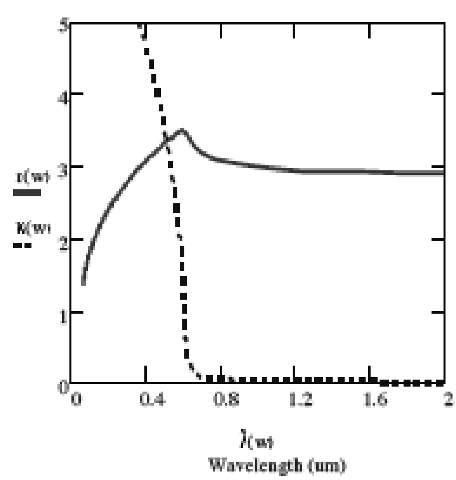

Figure 9

11.5. Optics at interfaces

Other important optical effects happen at interfaces.

Snell's law describes the specular reflection of light at a interface of two materials of different indexes n_1 and n_2.

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_1}{n_2} \tag{11.24}$$

Reflection coefficients for two different polarizations (E – parallel to the surface R_p and normal R_s),

$$R_s = |\frac{n_1\cos\theta_1 - n_2\cos\theta_2}{n_1\cos\theta_1 + n_2\cos\theta_2}|^2 \quad , \quad R_p = |\frac{-n_2\cos\theta_1 + n_1\cos\theta_2}{n_2\cos\theta_1 + n_1\cos\theta_2}|^2 \tag{11.25}$$

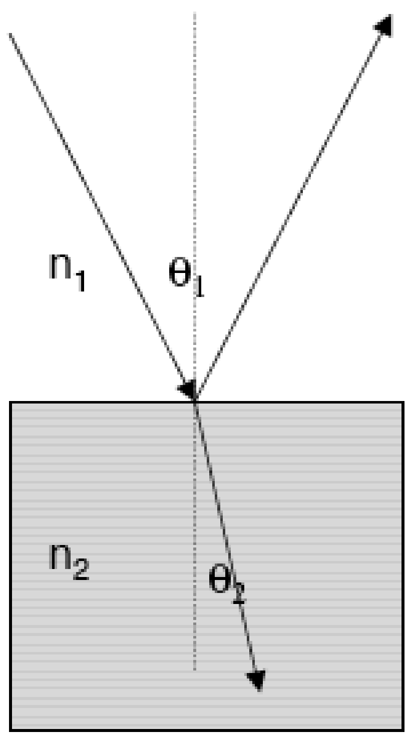

Figure 10

11.5.1. Reflectivity

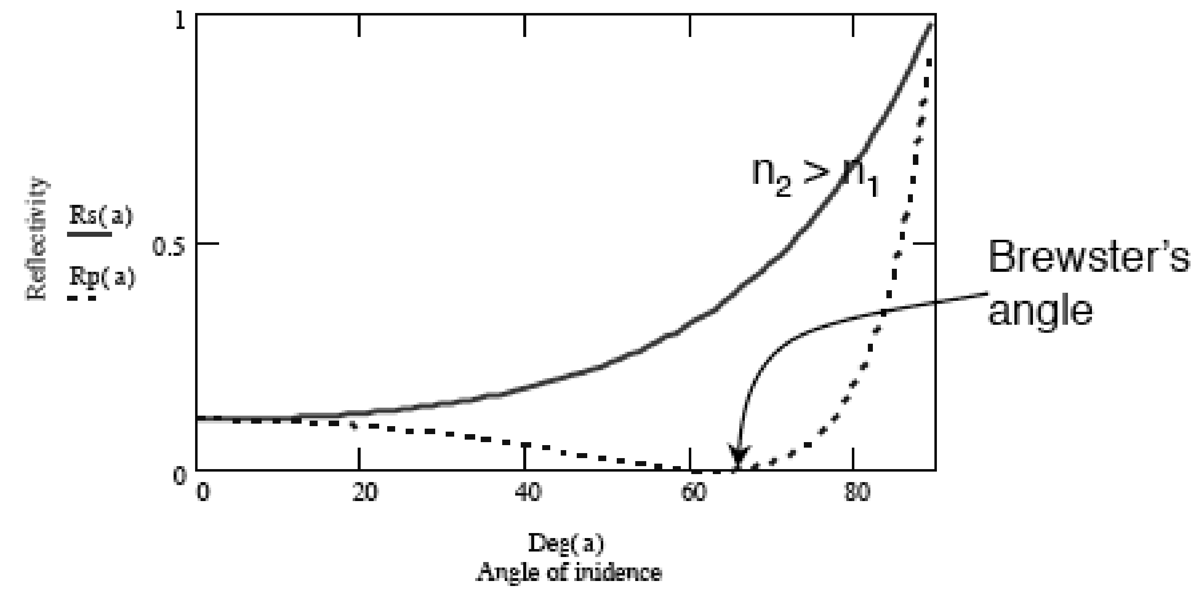

Figure 11

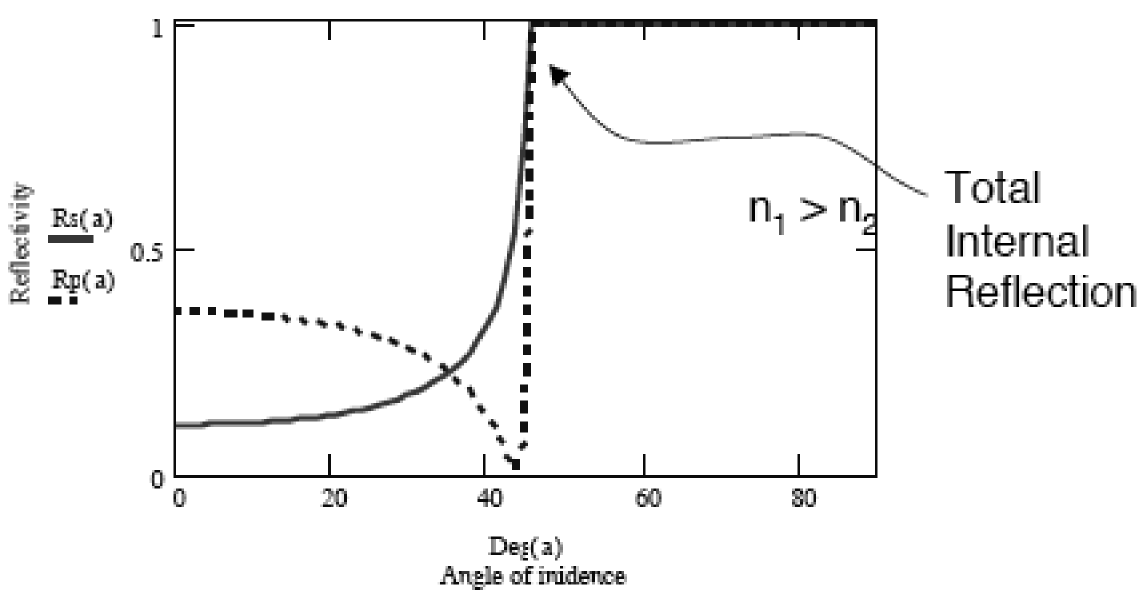

Figure 12

11.6. Summary

- Fundamental considerations lead to the conclusion that electromagnetic fields can support waves - light.
- Light is characterized by its frequency w, the wavelength λ or wavevector k, and the amplitude and polarization of the oscillating electric field.
- Light gains and loses energy only in discrete quanta of $\hbar w$ - photons.
- The optical properties of a material - the index of refraction-can be modeled well by a linear response of the electrons to the oscillating electric field.
- The band structure determines the optical properties of a material.
- The index of refraction gives us the reflectivity, transmission, bulk absorption, and speed of light in a material.