First and Second Order Filters

- These functions are useful for the design of simple filters or they can be cascaded to form high-order filter functions

First Order Filters

General first order bilinear transfer function is given by:

$$T(s) = \frac{a_1 s + a_o}{s + \omega_o}$$

pole at s = - ω_o and a zero at s = - a_o / a_1 and a high frequency gain that approaches a_1

- The numerator coefficients (a_o, a_1) determine the type of filter (e.g. low-pass, high-pass, etc.)

Transfer Functions

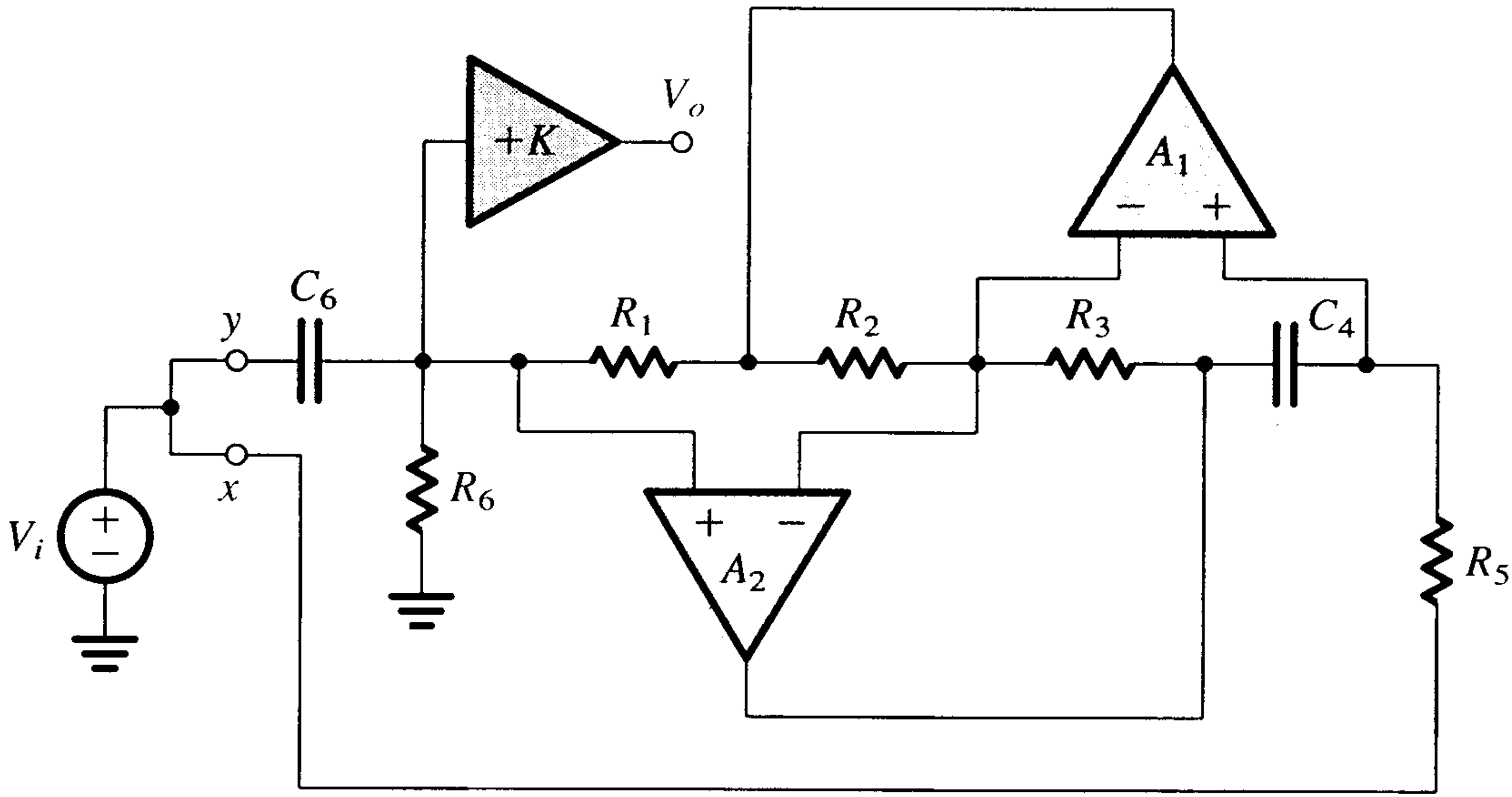
Low Pass
$$T(s) = \frac{1/LC}{s^2 + s\left(1/CR\right) + 1/LC}$$

High Pass
$$T(s) = \frac{s^2}{s^2 + s\left(w_o/Q\right) + w_o^2}$$

Bandpass
$$T(s) = \frac{s\left(1/CR\right)}{s^2 + s\left(1/CR\right) + \left(1/LC\right)}$$

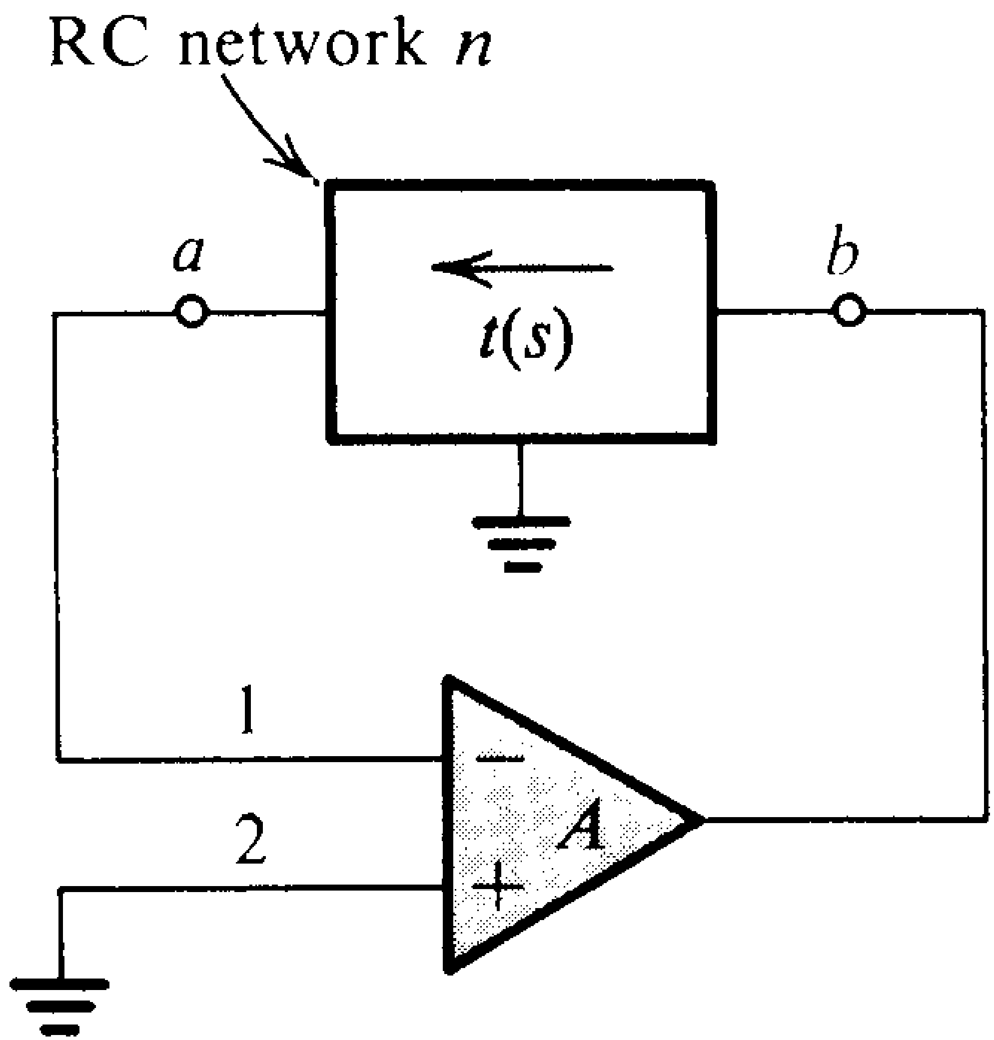
Notch
$$T(s) = \frac{s^2 + w_o^2}{s^2 + s\left(w_o/Q\right) + w_o^2}$$

All Pass
$$T(s) = 0.5 - \frac{s\left(w_o/Q\right)}{s^2 + s\left(w_o/Q\right) + w_o^2}$$

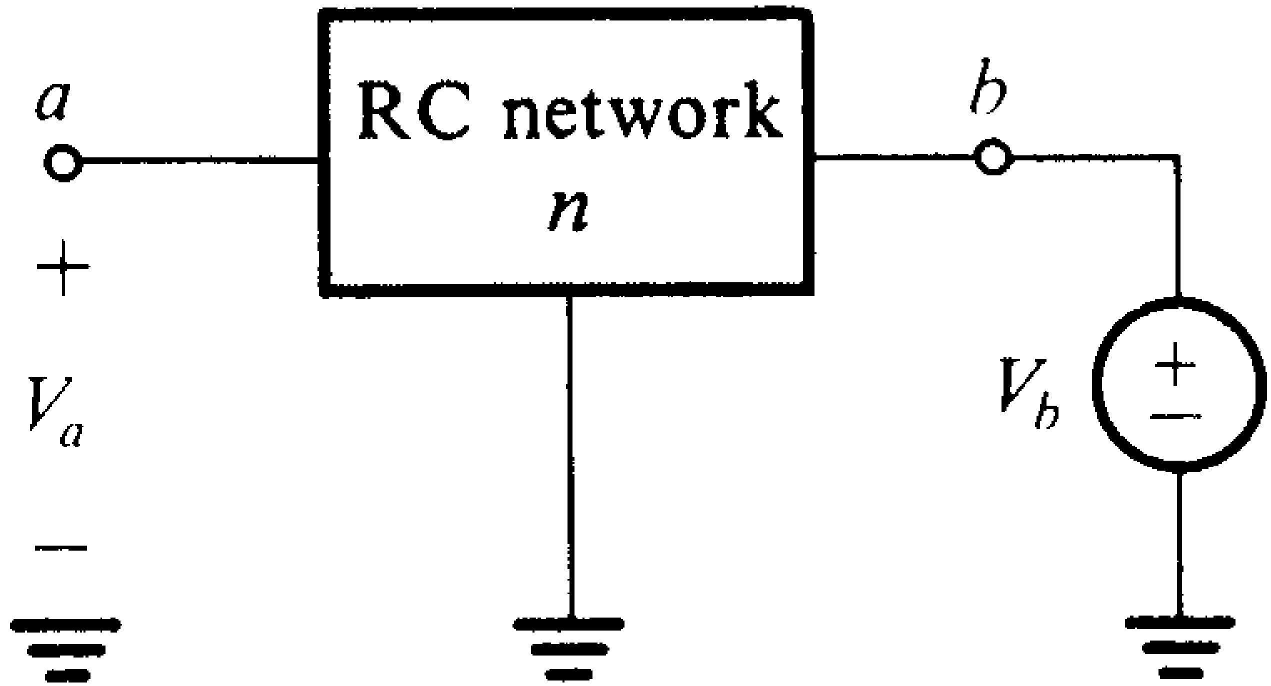


(d) Notch at ω_0

Consider the Following Circuit



(a)



$$t(s) \equiv \frac{V_a}{V_b}$$

(b)

$$t(s) = \frac{N(s)}{D(s)} \qquad \text{Loop Gain } = L(s) = A\,t(s) = \frac{A\,N(s)}{D(s)}$$

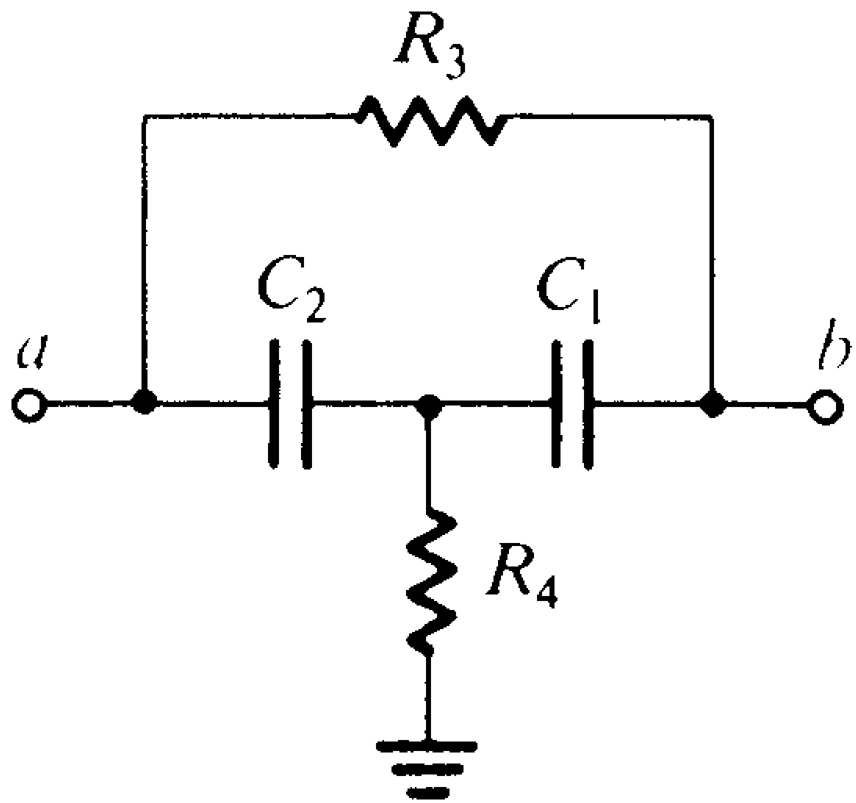
For stability. 1+ L (s) = 0 which results in the poles s_P of the closed - loop circuit as

$$t(s_P) = -\frac{1}{A}$$

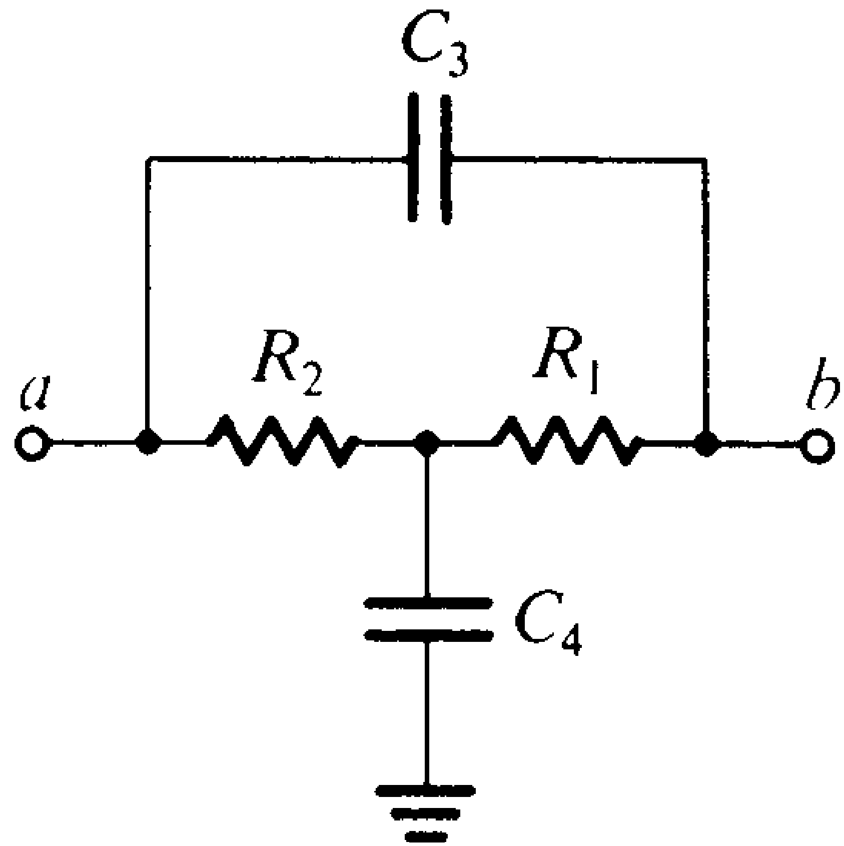
Assuming an ideal opamp with A = ∞ the poles are obtained from

$$N(s_P) = 0$$

- That is, the poles are identical to the zeros of the RC network
- Since our objective is to realize a pair of complex conjugate poles we should select an RC network that has complex conjugate zeros
- The simplest such networks are Bridged - T networks



$$t(s) = \frac{s^2 + s\left(\frac{1}{C_1} + \frac{1}{C_2}\right)\frac{1}{R_3} + \frac{1}{C_1 C_2 R_3 R_4}}{s^2 + s\left(\frac{1}{C_1 R_3} + \frac{1}{C_2 R_3} + \frac{1}{C_1 R_4}\right) + \frac{1}{C_1 C_2 R_3 R_4}}$$



$$t(s) = \frac{s^2 + s\left(\frac{1}{R_1} + \frac{1}{R_2}\right)\frac{1}{C_4} + \frac{1}{C_3 C_4 R_1 R_2}}{s^2 + s\left(\frac{1}{C_4 R_1} + \frac{1}{C_4 R_2} + \frac{1}{C_3 R_2}\right) + \frac{1}{C_3 C_4 R_1 R_2}}$$

- The pole polynomial of the active filter circuit will be equal to the numerator polynomial of the Bridged - T network

$$s^2 + s\frac{\omega_o}{Q} + \omega_o{}^2 = s^2 + \left(\frac{1}{C_1} + \frac{1}{C_2}\right)\frac{1}{R_3} + \frac{1}{C_1\, C_2\, R_3\, R_4}$$

$$\omega_o = \frac{1}{\sqrt{C_1\, C_2\, R_3\, R_4}} \qquad Q = \left[\frac{\sqrt{C_1\, C_2\, R_3\, R_4}}{R_3}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)\right]^{-1}$$

- Common implementation $C_1 = C_2 = C$, $R_3 = R$, $R_4 = R / m$

$$m = 4\,Q^2 \qquad CR = \frac{2Q}{\omega_o}$$

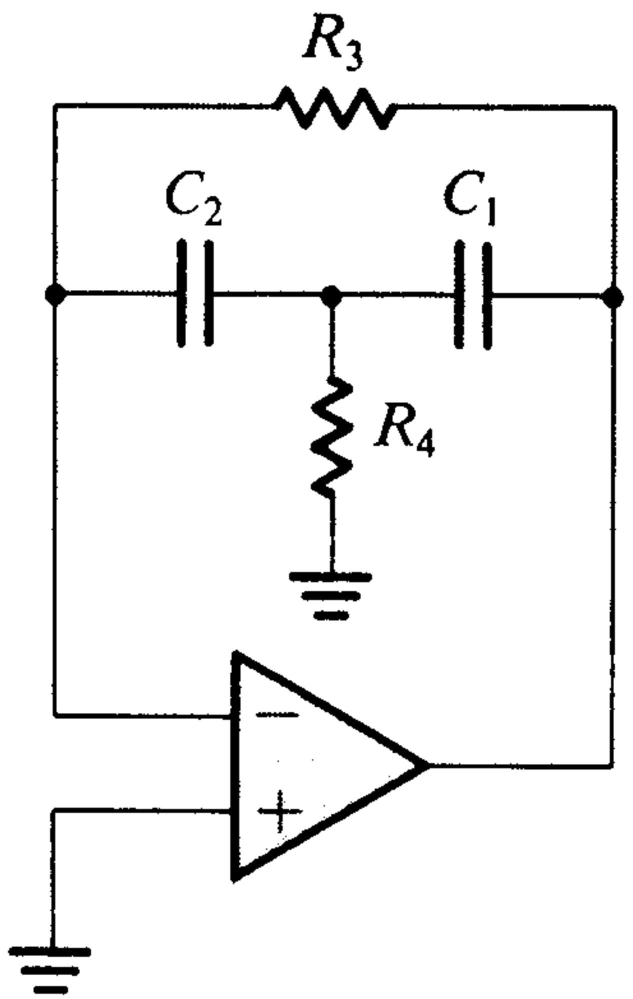
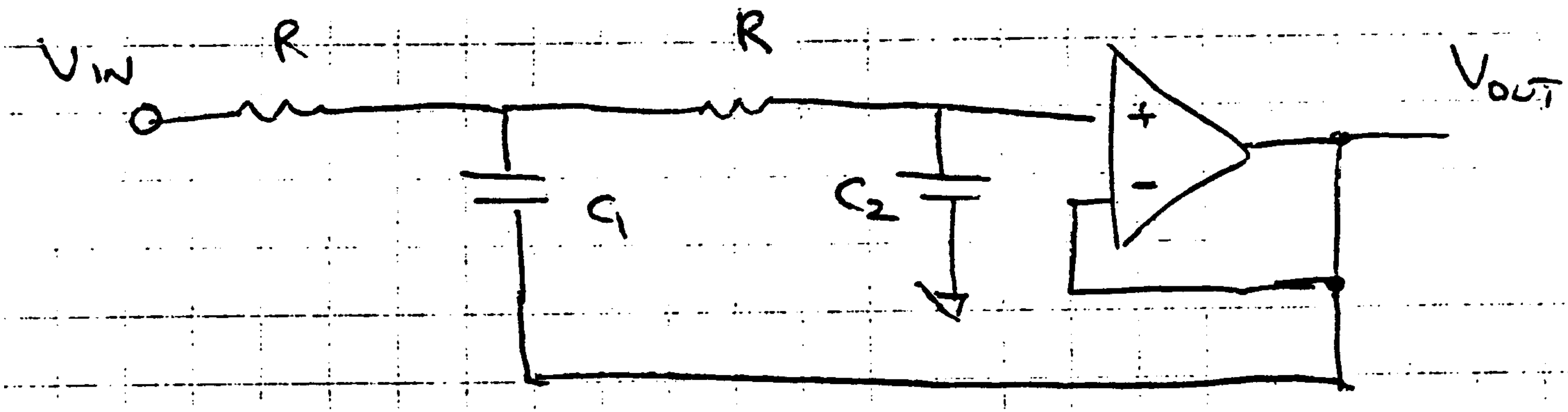


Fig. 11.29 An active-filter feedback loop generated using the bridged-T network of Fig. 11.28(a).

- A common implementation of the single amplifier biquad is the Salen - key filter
- A low - pass filter can be seen below
- Similar circuits are available for the other filter configurations



$$w_c = \frac{1}{\sqrt{R_2 C_1 C_2}}$$

$$Q = 0.5\sqrt{C_1 / C_2}$$

Sensitivity

- Because of telerances in component values and because of the finite opamp gain the response of the actual filter will deviate from the ideal response
- As a means of predicting such deviations, the filter designer employs the concept of sensitivity
- For second order filters one is normally interested in finding how sensitive their poles are relative to variations (Both initial tolerances and future changes) in RC component values and amplifier gain
- These sensitivities can be quantified using the classical sensitivity function

$$S_x^y = \lim_{\Delta x \to 0} \frac{\Delta y / y}{\Delta x / x} = \frac{\P\, y}{\P\, x} \frac{x}{y}$$

- Here x denotes the value of a component and y denotes a circuit parameter of interest (e.g. ω_o, Q). For small changes

$$S_x^y = \frac{\Delta y / y}{\Delta x / x}$$

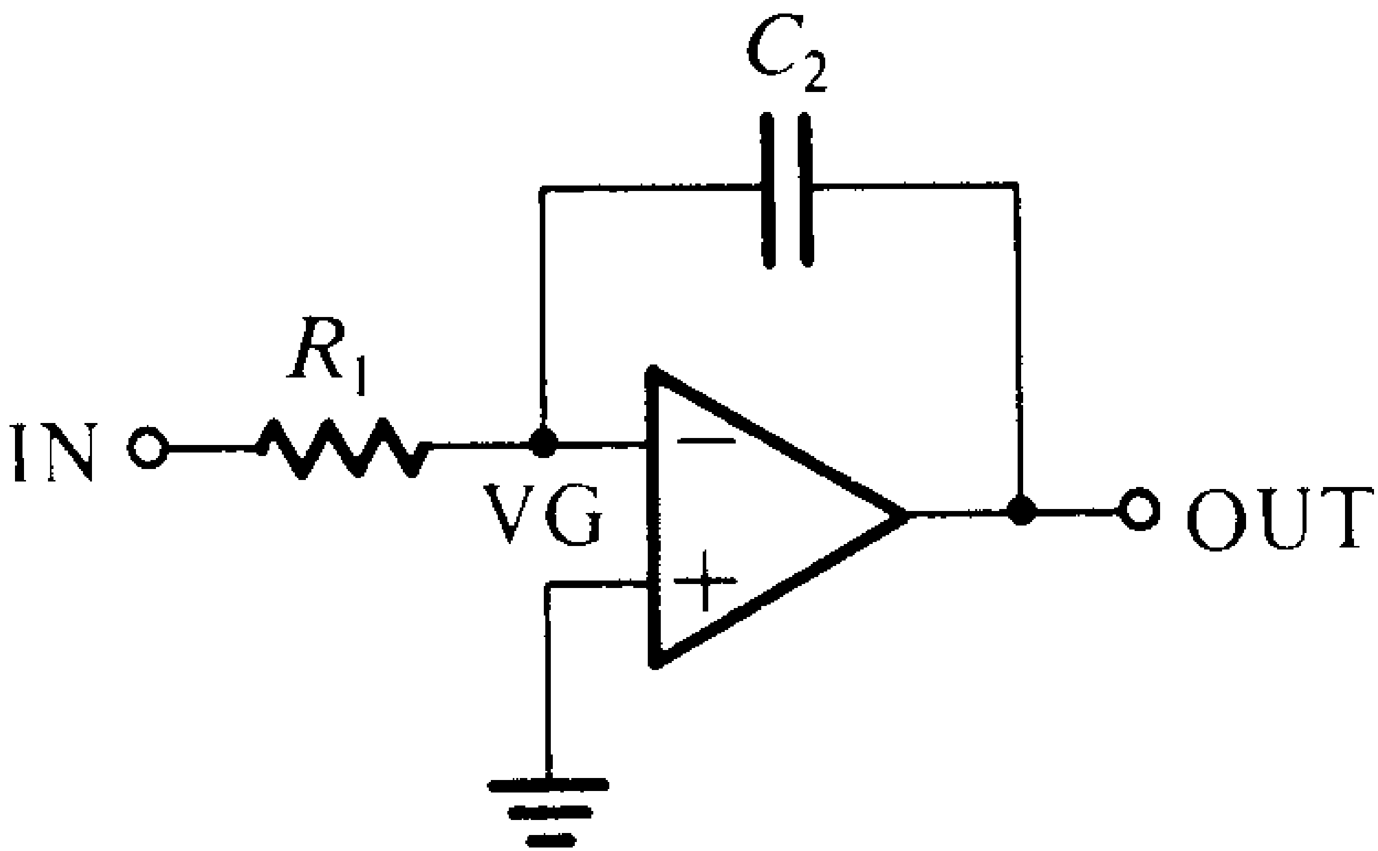
Thus we use the value of S_x^y to determine the per unit change in y due to a given per - unit change in x

- If the sensitivity of Q relative to a particular resistance R_1 is 5, then a 1% increase in R_1 results in a 5% increase in the value of Q

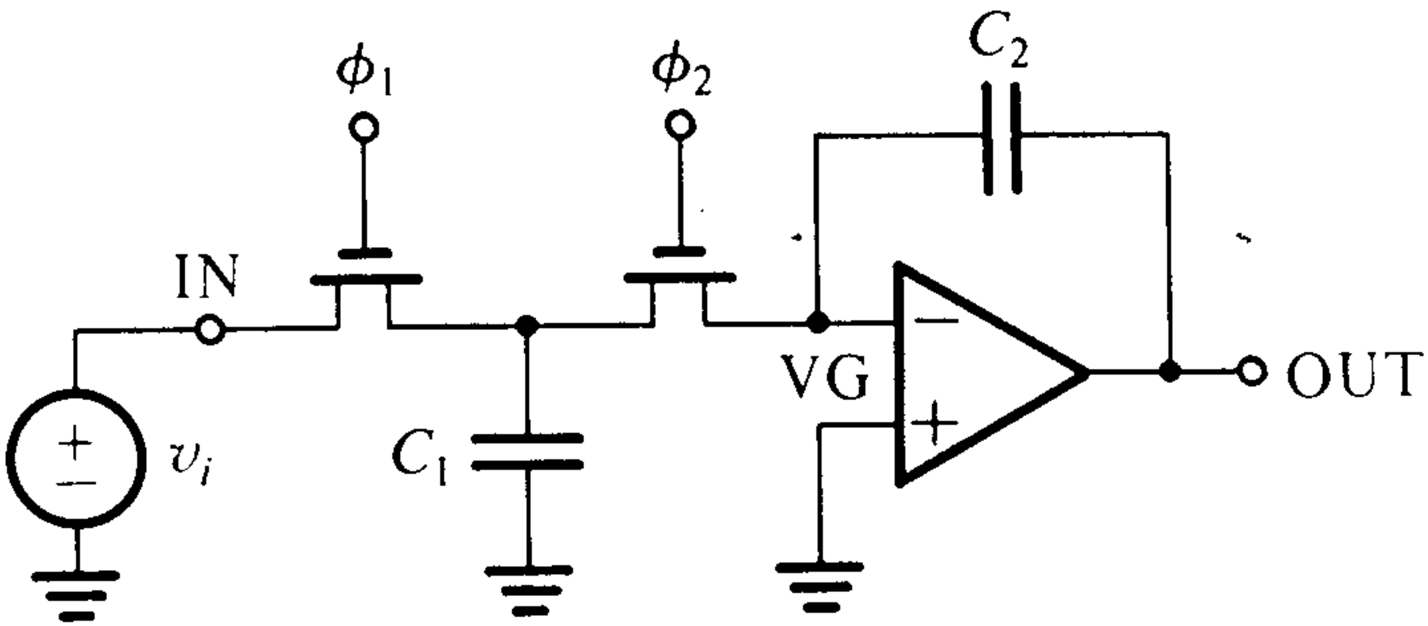
Switched - Capacitor Filters

- Active RC filters are difficult to implement totally on an IC due to the requirements of large - valued capacitors and accurate RC time constants
- The switched capacitor filter technique is based on the realization that a capacitor switched between two circuit nodes at a sufficiently high rate is equivalent to a resistor connecting these two nodes

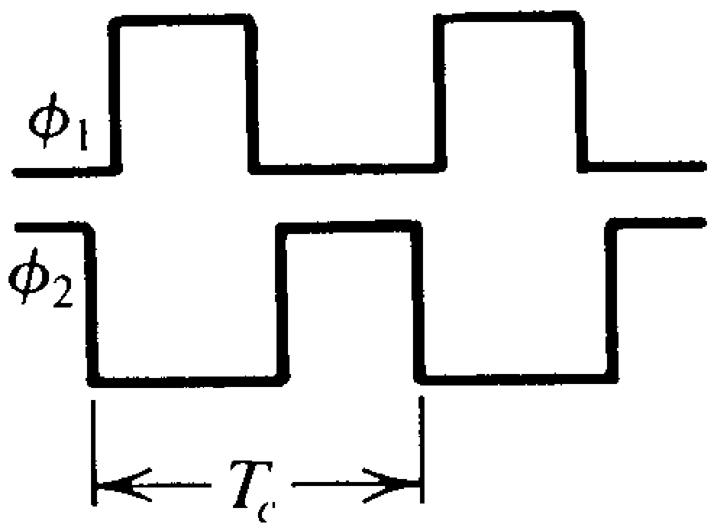
Consider the Following Circuit

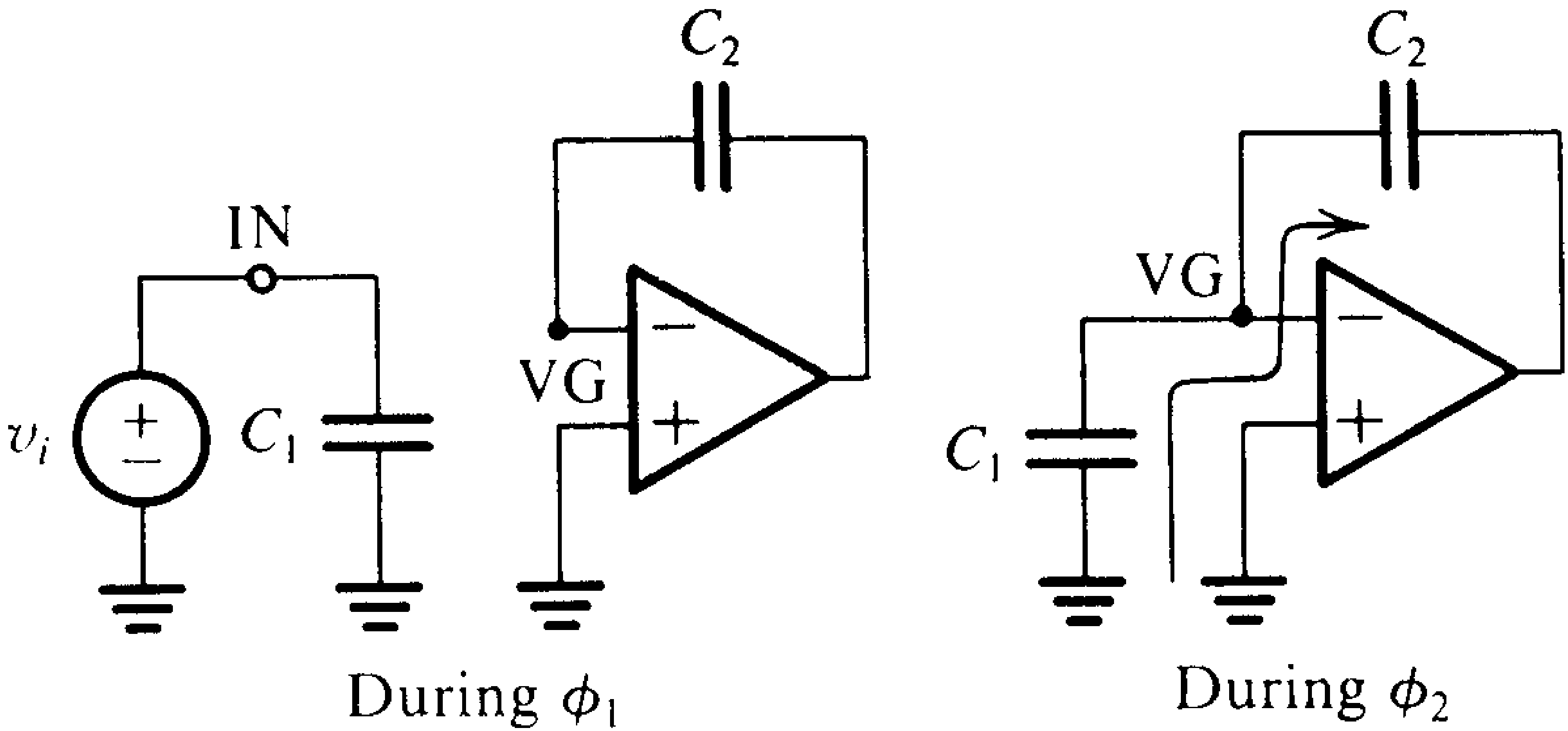


(a)



(b)





(d)

Fig. 11.35 Basic principle of the switched-capacitor filter technique. **(a)** Active-RC integrator. **(b)** Switched-capacitor integrator. **(c)** Two-phase clock (nonoverlapping). **(d)** During ϕ_1, C_1 charges up to the current value of v_i and then, during ϕ_2, discharges into C_2.

- From the above circuit we see that during each clock period T_C an amount of charge $q_{C1} = C_1 v_1$ is subtracted from the input source and supplied to the integrator capacitor C_2
- The average current flow between the input node and virtual ground (VG) is

$$i_{AV} = \frac{C_1 v_i}{T_C}$$

If T_C is sufficiently short one can think of the process as continuous and define an equivalent resistance R_{EQ} that is in effective resistance between nodes IN and VG

$$R_{EQ} = v_i / i_{AV} = T_C / C_1$$

The time constant for the integrator is:

$$\text{Time constant} = C_2 R_{EQ} = T_C \frac{C_2}{C_1}$$

- Thus the time constant that determines the frequency response of the filter is determined by the clock period T_C and the capacitor ratio C_2 / C_1. Both of these parameters can be well controlled in an IC process
- Note the dependence on capacitor ratio rather than absolute value. The accuracy of capacitor ratios in MOS technology are on the order of 0.1%
- For reasonable clock frequencies (100 kHz) and not too large capacitor ratios (10) one can obtain relatively large time constants (10^{-4} s)
- Switched capacitor filter ICs offer a low cost high order filter on a single IC
- The clock frequency must be higher than any frequency component of the signal (typically 100x)
- Can be easily programmed by changing the clock frequency
- Some of clock signal feeds through to the output, signals near the clock frequency can be aliased into the passband, overall increase in the noise floor

Oscillators and Waveform Shaping Circuits

- In the design of electronic systems the need frequently arises for signals having prescribed waveforms (e.g. sinusoidal, square, triangle, pulse, etc.)
- Commonly used in computers, control systems, communication systems and test and measurement systems
- Two common ways for generating sinusoids
  - Positive feedback loop with non-linear gain limiting mechanism
  - Appropriately shaping other waveforms such as triangle waves
- Circuits that directly generate square, triangle and pulse waveforms generally employ circuit blocks known as multivibrators. Three basic types are bistable, astable and monostable

Sinusoidal Oscillators

- Commonly referred to as linear sine-wave oscillators although some form of non-linearity has to be employed to limit the output amplitude

- Analysis of circuits is more difficult as s - plane analysis cannot be directly applied to the non-linear part of the circuit

- The basic structure of a sinusoidal oscillator consists of an amplifier and a frequency selective network connected in a positive feedback loop

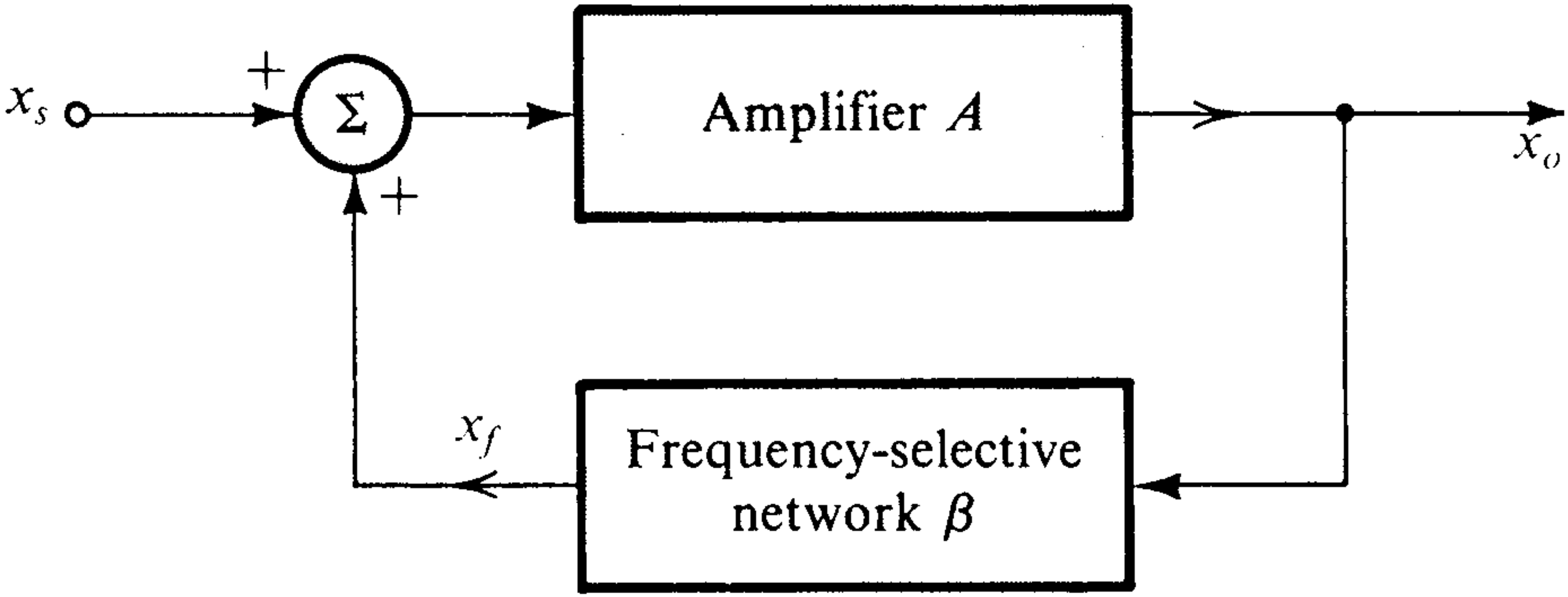


Fig. 12.1 The basic structure of a sinusoidal oscillator. A positive-feedback loop is formed by an amplifier and a frequency-selective network. In an actual oscillator circuit no input signal will be present; here an input signal x_s is employed to help explain the principle of operation.

- In actual oscillator no input will be present, included to help explain operation
- Note the feedback signal X_F is summed with a positive sign

$$A_f(s) = \frac{A(s)}{1 - A(s)\beta(s)}$$

The loop gain is

$$L(s) = A(s)\beta(s)$$

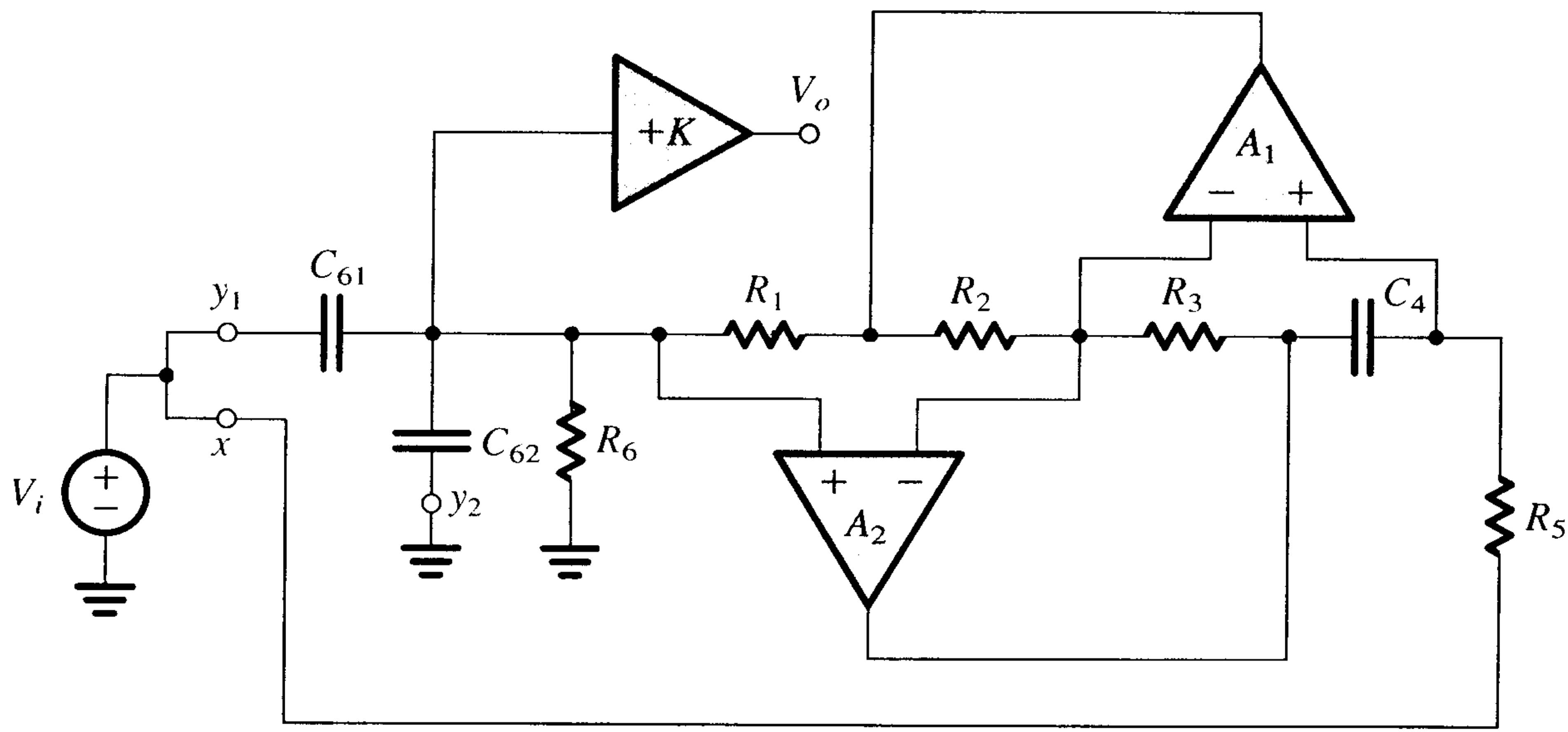
and the characteristic equation is

$$1 - L(s) = 0$$

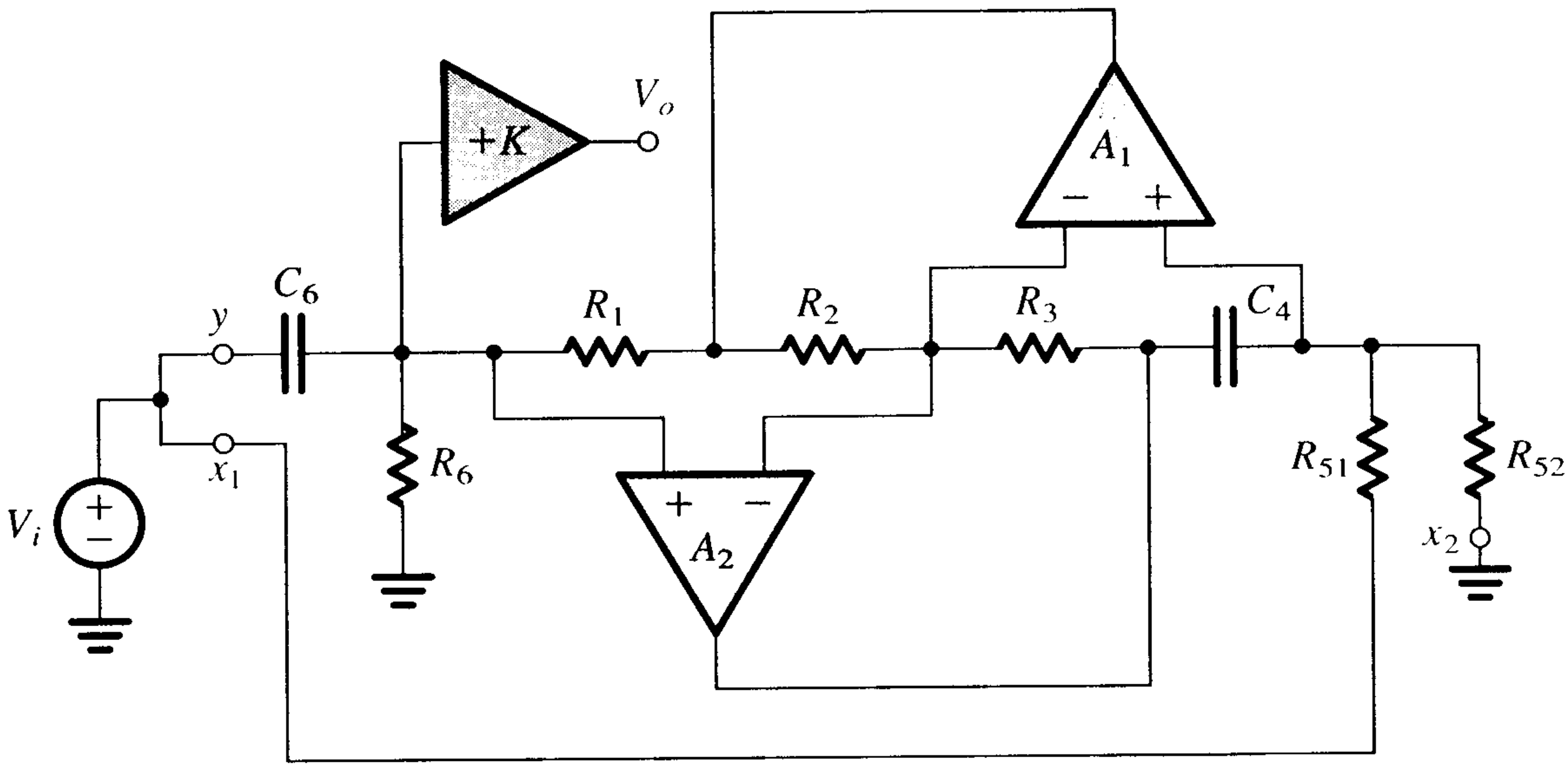
- If at a specific frequency f_o the loop gain $A\beta$ is equal to unity it follows that A_f will be infinite. Such a circuit is by definition an oscillator
- Thus for sinusoidal oscillation at ω_o

$$L(j\omega_o) = A(j\omega_o)\beta(j\omega_o) = 1 \qquad \text{Barkhausen Criteria}$$

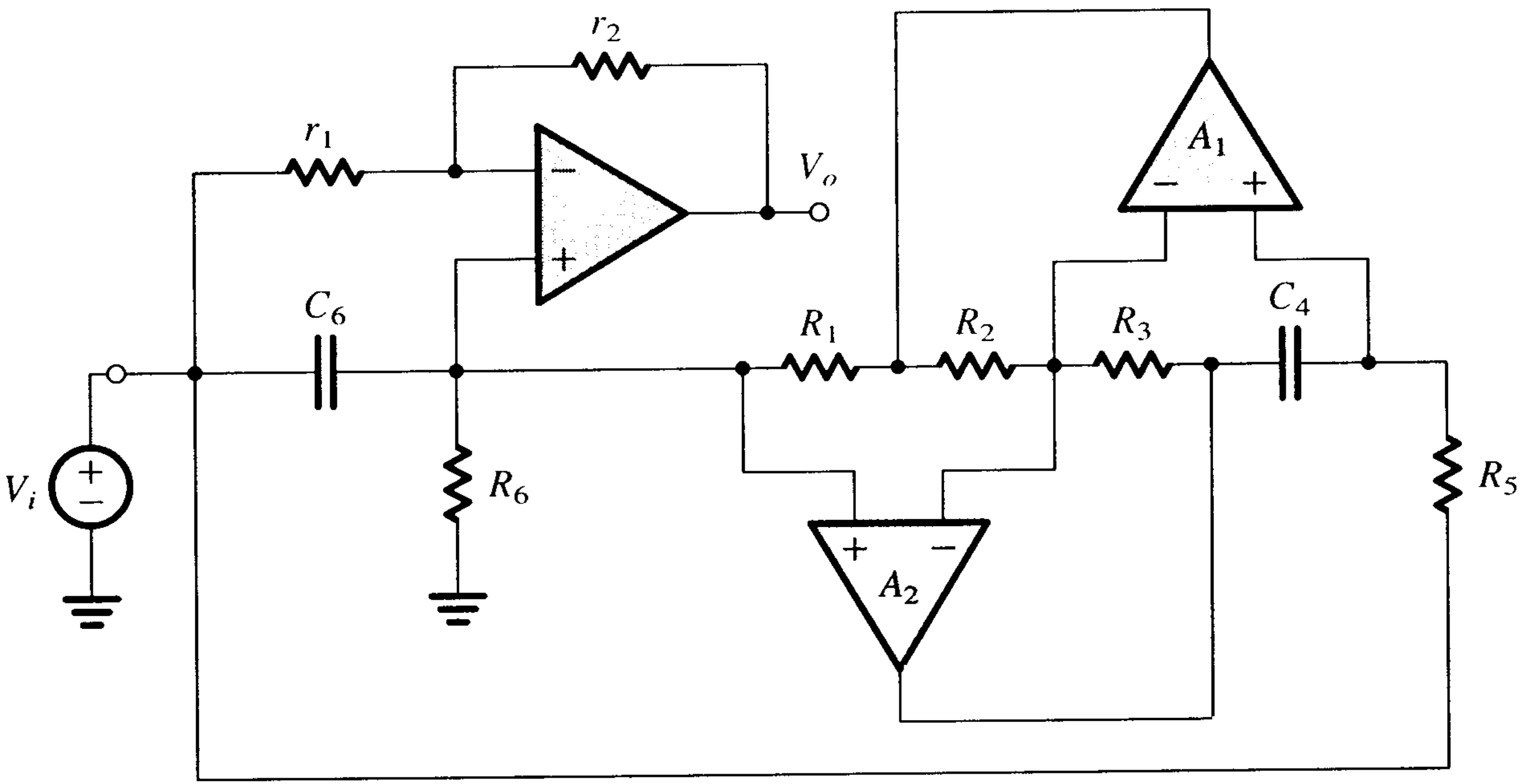
- Unity gain, zero phase shift



(e) LPN, $\omega_n \geq \omega_0$



(f) HPN, $\omega_n \leq \omega_0$



(g) All-pass

Table 11.1 DESIGN DATA FOR THE CIRCUITS OF FIG. 11.22

CIRCUIT	TRANSFER FUNCTION AND OTHER PARAMETERS	DESIGN EQUATIONS
Resonator Fig. 11.21(b)	$\omega_0 = 1/\sqrt{C_4 C_6 R_1 R_3 R_5 / R_2}$ $Q = R_6 \sqrt{\frac{C_6}{C_4} \frac{R_2}{R_1 R_3 R_5}}$	$C_4 = C_6 = C$ (practical value) $R_1 = R_2 = R_3 = R_5 = 1/\omega_0 C$ $R_6 = Q/\omega_0 C$
Low-pass (LP) Fig. 11.22(a)	$T(s) = \dfrac{K R_2 / C_4 C_6 R_1 R_3 R_5}{s^2 + s\dfrac{1}{C_6 R_6} + \dfrac{R_2}{C_4 C_6 R_1 R_3 R_5}}$	$K =$ dc gain
High-pass (HP) Fig. 11.22(b)	$T(s) = \dfrac{K s^2}{s^2 + s\dfrac{1}{C_6 R_6} + \dfrac{R_2}{C_4 C_6 R_1 R_3 R_5}}$	$K =$ High-frequency gain
Bandpass (BP) Fig. 11.22(c)	$T(s) = \dfrac{K s / C_6 R_6}{s^2 + s\dfrac{1}{C_6 R_6} + \dfrac{R_2}{C_4 C_6 R_1 R_3 R_5}}$	$K =$ Center-frequency gain
Regular notch (N) Fig. 11.22(d)	$T(s) = \dfrac{K[s^2 + (R_2 / C_4 C_6 R_1 R_3 R_5)]}{s^2 + s\dfrac{1}{C_6 R_6} + \dfrac{R_2}{C_4 C_6 R_1 R_3 R_5}}$	$K =$ Low- and high-frequency gain

Low-pass notch (LPN) Fig. 11.22(e)	$T(s) = K\dfrac{C_{61}}{C_{61}+C_{62}} \times \dfrac{s^2 + (R_2/C_4C_{61}R_1R_3R_5)}{s^2 + s\dfrac{1}{C_6R_6} + \dfrac{R_2}{C_4(C_{61}+C_{62})R_1R_3R_5}}$ $\omega_n = 1/\sqrt{C_4C_{61}R_1R_3R_5/R_2}$ $\omega_0 = 1/\sqrt{C_4(C_{61}+C_{62})R_1R_3R_5/R_2}$ $Q = R_6\sqrt{\dfrac{C_{61}+C_{62}}{C_4}\dfrac{R_2}{R_1R_3R_5}}$	K = dc gain $C_{61} + C_{62} = C_6 = C$ $C_{61} = C(\omega_0/\omega_n)^2$ $C_{62} = C - C_{61}$
High-pass notch (HPN) Fig. 11.22(f)	$T(s) = K\dfrac{s^2 + (R_2/C_4C_6R_1R_3R_{51})}{s^2 + s\dfrac{1}{C_6R_6} + \dfrac{R_2}{C_4C_6R_1R_3}\left(\dfrac{1}{R_{51}} + \dfrac{1}{R_{52}}\right)}$ $\omega_n = 1/\sqrt{C_4C_6R_1R_3R_{51}/R_2}$ $\omega_0 = \sqrt{\dfrac{R_2}{C_4C_6R_1R_3}\left(\dfrac{1}{R_{51}} + \dfrac{1}{R_{52}}\right)}$ $Q = R_6\sqrt{\dfrac{C_6}{C_4}\dfrac{R_2}{R_1R_3}\left(\dfrac{1}{R_{51}} + \dfrac{1}{R_{52}}\right)}$	K = High frequency gain $\dfrac{1}{R_{51}} + \dfrac{1}{R_{52}} = \dfrac{1}{R_5} = \omega_0 C$ $R_{51} = R_5(\omega_0/\omega_n)^2$ $R_{52} = R_5/[1 - (\omega_n/\omega_o)^2]$
All-pass (AP) Fig. 11.22(g)	$T(s) = \dfrac{s^2 - s\dfrac{1}{C_6R_6}\dfrac{r_2}{r_1} + \dfrac{R_2}{C_4C_6R_1R_3R_5}}{s^2 + s\dfrac{1}{C_6R_6} + \dfrac{R_2}{C_4C_6R_1R_3R_5}}$ $\omega_z = \omega_0$ $Q_z = Q(r_1/r_2)$ Flat gain = 1	$r_1 = r_2 = r$ (arbitrary) Adjust r_2 to make $Q_z = Q$.

Active Filter Based on Two Loop Integrator (Biquad)

- Opamp - RC circuit that realizes second order filter functions based on the use of two integrators connected in cascade in an overall feedback loop
- Consider second order high pass filter

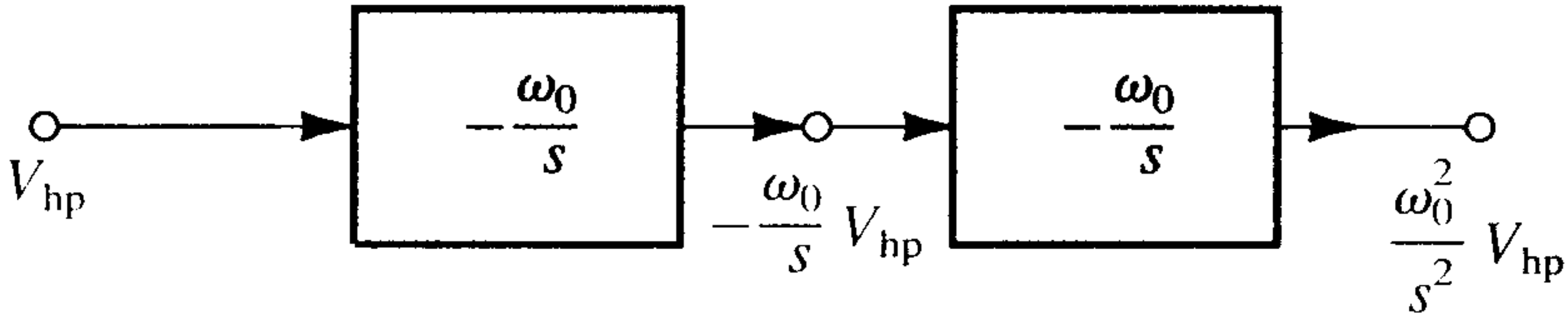
$$T(s) = \frac{V_{hp}}{V_i} = \frac{k\,s^2}{s^2 + s\left(\omega_o / Q\right) + \omega_o^2}$$

where k is the high frequency gain. Rearranging the equation gives

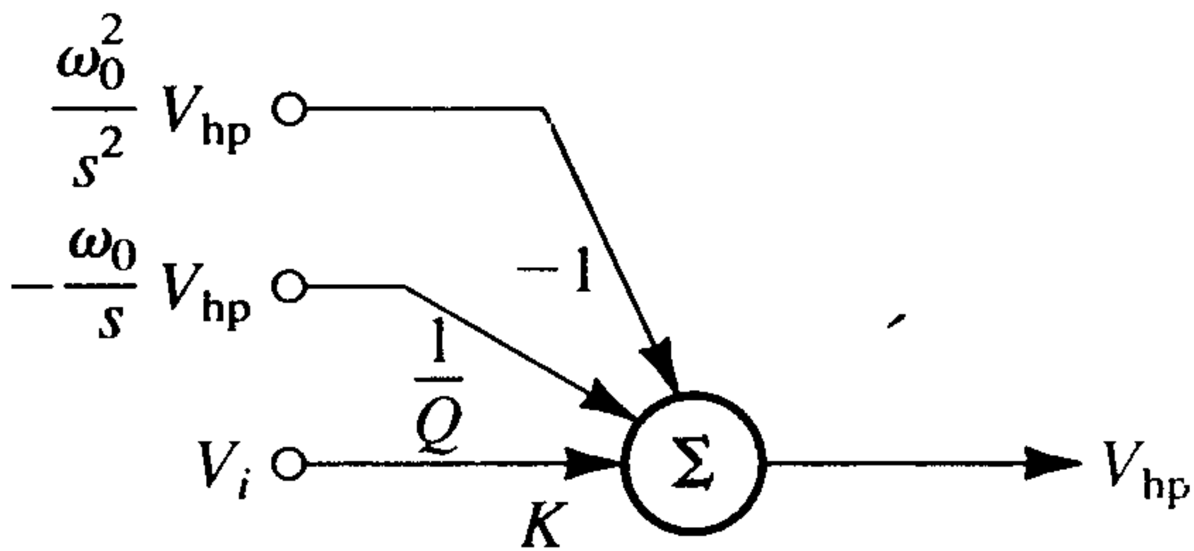
$$V_{hp} + \frac{1}{Q}\left(\frac{\omega_o}{s}V_{hp}\right) + \left(\frac{\omega_o^{\ 2}}{s^{\ 2}}V_{hp}\right) = kV_i$$

The signal $\frac{\omega_o}{s}V_{hp}$ can be obtained by passing V_{hp} through an integrator with a time constant equal to $1/\omega_o$. Passing the resulting signal through another identical integrator generates $\left(\frac{\omega_o^{\ 2}}{s^{\ 2}}\right)V_{hp}$

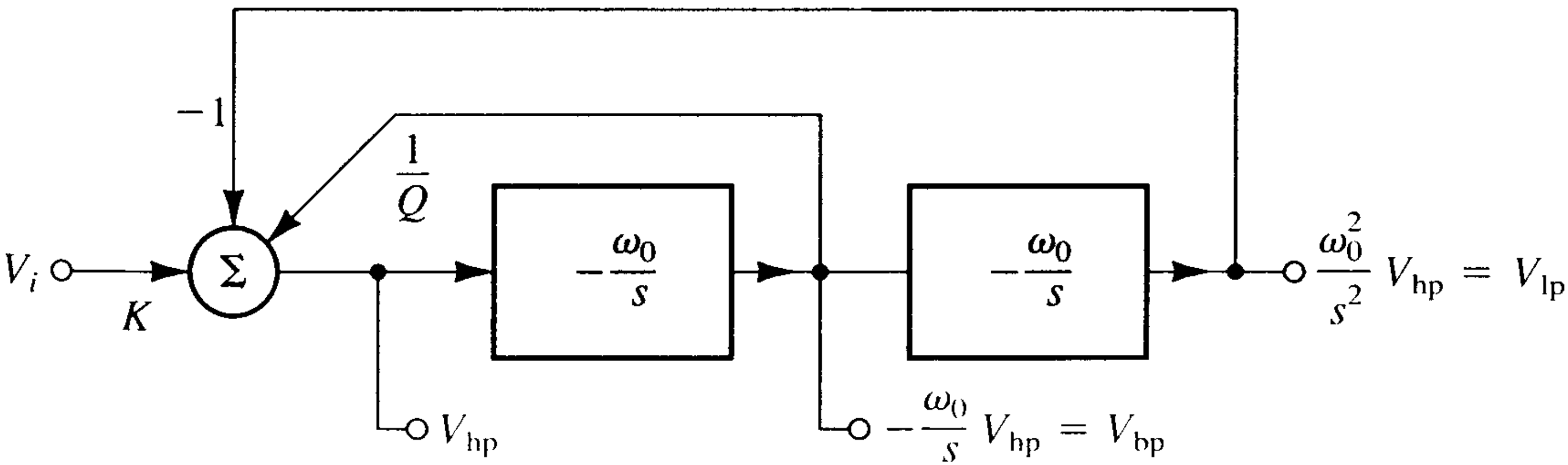
$$V_{hp} = kV_i - \frac{1}{Q}\frac{\omega_o}{s}V_{hp} - \frac{\omega_o^{\ 2}}{s^2}V_{hp}$$


$-\frac{\omega_0}{s}$
$-\frac{\omega_0}{s}$
V_{hp}
$-\frac{\omega_0}{s} V_{hp}$
$\frac{\omega_0^2}{s^2} V_{hp}$

(a)


$\frac{\omega_0^2}{s^2} V_{hp}$
$-\frac{\omega_0}{s} V_{hp}$
V_i
-1
$\frac{1}{Q}$
K
Σ
V_{hp}

(b)



(c)

Fig. 11.23 Derivation of a block-diagram realization of the two-integrator-loop biquad.

- The signal at the output of the first integrator is

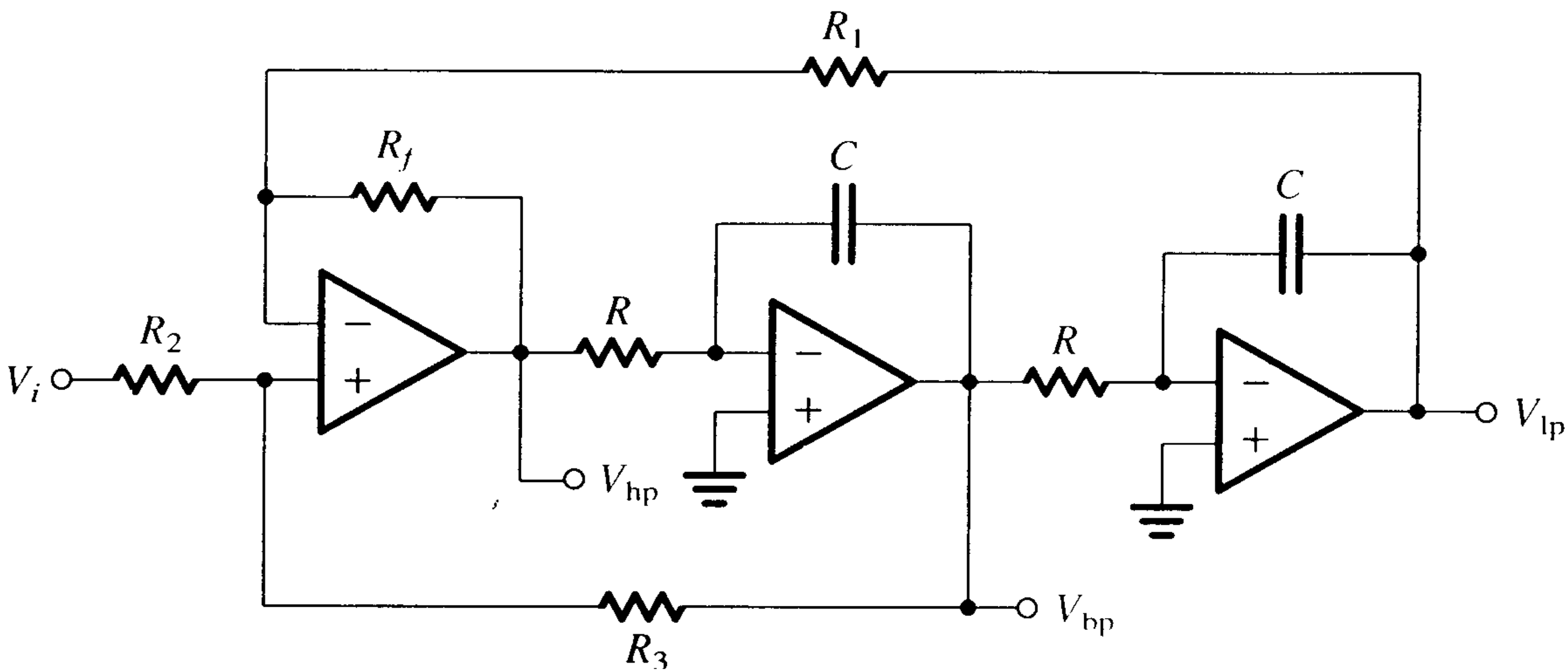
$$\frac{\left(\frac{-w_o}{s}\right)V_{hp}}{V_i}$$ which is a bandpass function

$$\frac{\left(\frac{-w_o}{s}\right)V_{hp}}{V_i} = -\frac{k w_o s}{s^2 + s\left(\frac{w_o}{Q}\right) + w_o^2} = T_{bp}(s)$$

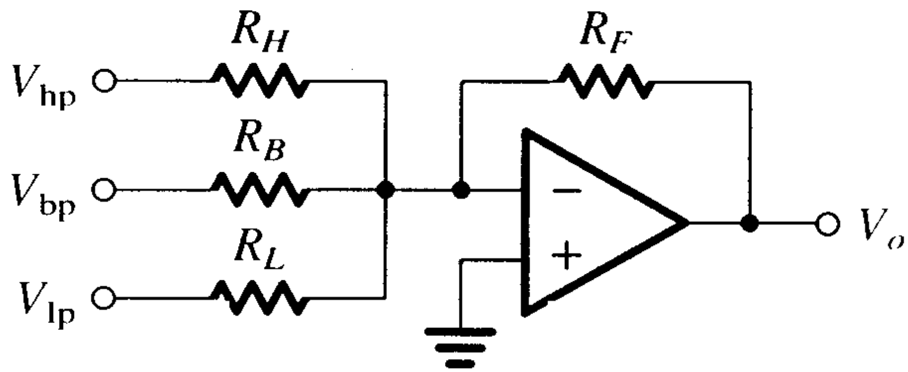
- Similarly the output of the second integrator is a low pass function

$$\frac{\left(\frac{w_o^2}{s^2}\right)V_{hp}}{V_i} = \frac{k w_o^2}{s^2 + s\left(\frac{w_o}{Q}\right) w_o^2} = T_{lp}(s)$$

- The two - integrator - loop biquad realizes three basic second order filter functions LP, BP and HP simultaneously. This circuit is commonly called the universal active filter



(a)



(b)

Fig. 11.24 (a) The KHN biquad circuit, obtained as a direct implementation of the block diagram of Fig. 11.23(c). The three basic filtering functions, HP, BP, and LP, are simultaneously realized. **(b)** To obtain notch and all-pass functions, the three outputs are summed with appropriate weights using this op amp summer.

- If $R_f / R = 1$

$$CR = 1/\omega_o \qquad R_3/R_2 = 2Q - 1 \qquad k = 2 - \left(1/Q\right)$$

By summing the LP, BP, and HP outputs the overall transfer function of the KHN biquad and summer is

$$\frac{V_o}{V_i} = -\frac{k\left(R_F/R_H\right)s^2 - s\left(R_F/R_B\right)\omega_o + \left(R_F/R_L\right)\omega_o^{\ 2}}{s^2 + s\left(\omega_o/Q\right) + \omega_o^{\ 2}}$$

- An alternative two - loop integrator which uses three opamps in single ended mode is the Tow - Thomas biquad. Details can be found on pages 804 & 805 of the text
- Although Two - Integrator loop biquads are versatile and easy to design, their performance is adversely affected by the finite bandwidth of the opamps

Single Amplifier Biquad Filters

- Second order filter functions can also be implemented with a single amplifier. These minimal realizations are low power and low cost, however, they suffer from greater dependence on opamp gain and bandwidth and are generally more sensitive to tolerances in the resistors and capacitors

- The single amplifier biquads (SABs) are therefore generally limited to less stringent filter specifications ($Q < 10$)

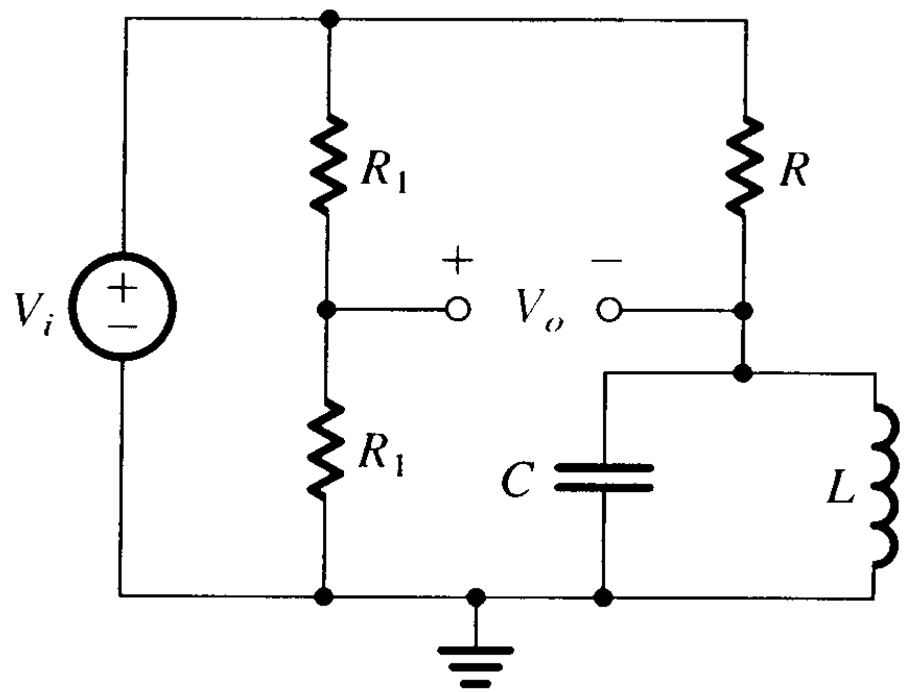


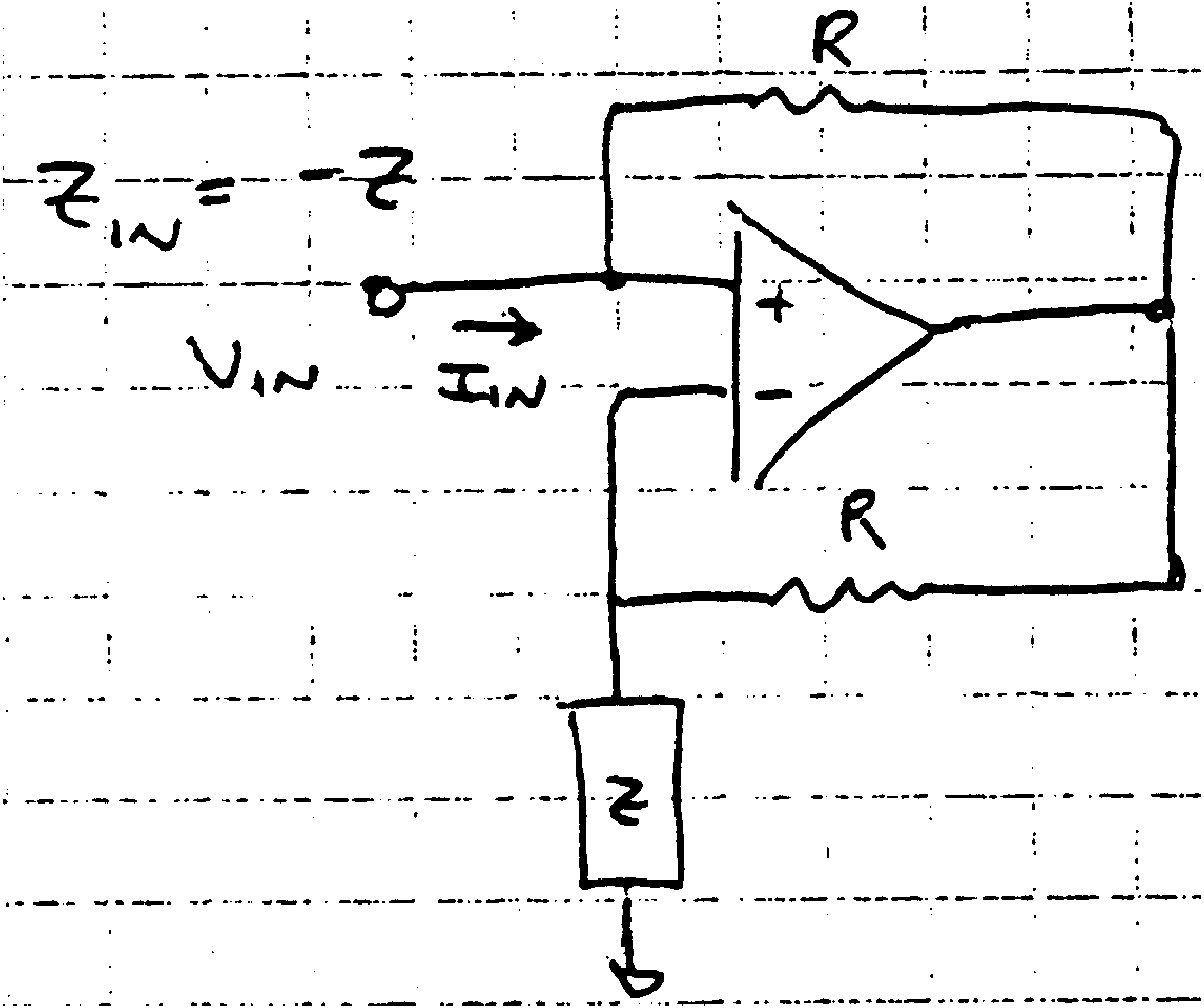
Fig. 11.19 Realization of the second-order all-pass transfer function using a voltage divider and an LCR resonator.

Active Filters

- In this section we study a family of opamp - RC circuits that realize the various second order filter functions by replacing the inductor L in the LCR resonator with an opamp - RC circuit that has an inductive input impedance

- Many opamp - RC circuits have been proposed for simulating the operation of an inductor

- One of the simplest is the negative impedance converter (NIC)



R
ZIN = -Z
+
VIN
IIN
-
R
Z

- Can be used to convert a capacitor to a "backward" inductor

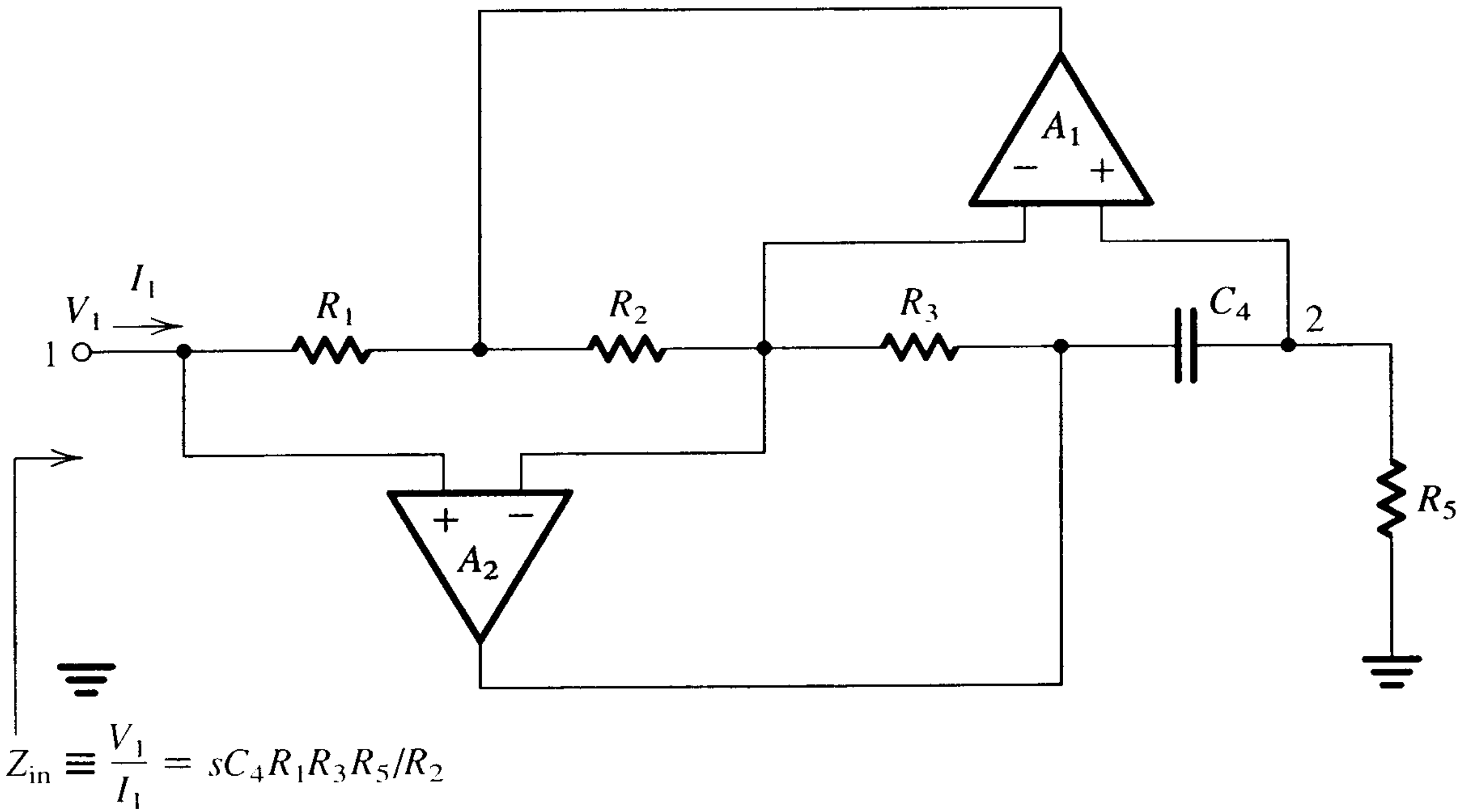
$$Z_C = \frac{1}{j\omega C} \qquad I_{IN} = -v_{IN}\, j\omega C$$

$$Z_{IN} = \frac{v_{IN}}{I_{IN}} = \frac{v_{IN}}{-v_{IN}\, j\omega C} = -\frac{1}{j\omega C} = \frac{j}{\omega C}$$

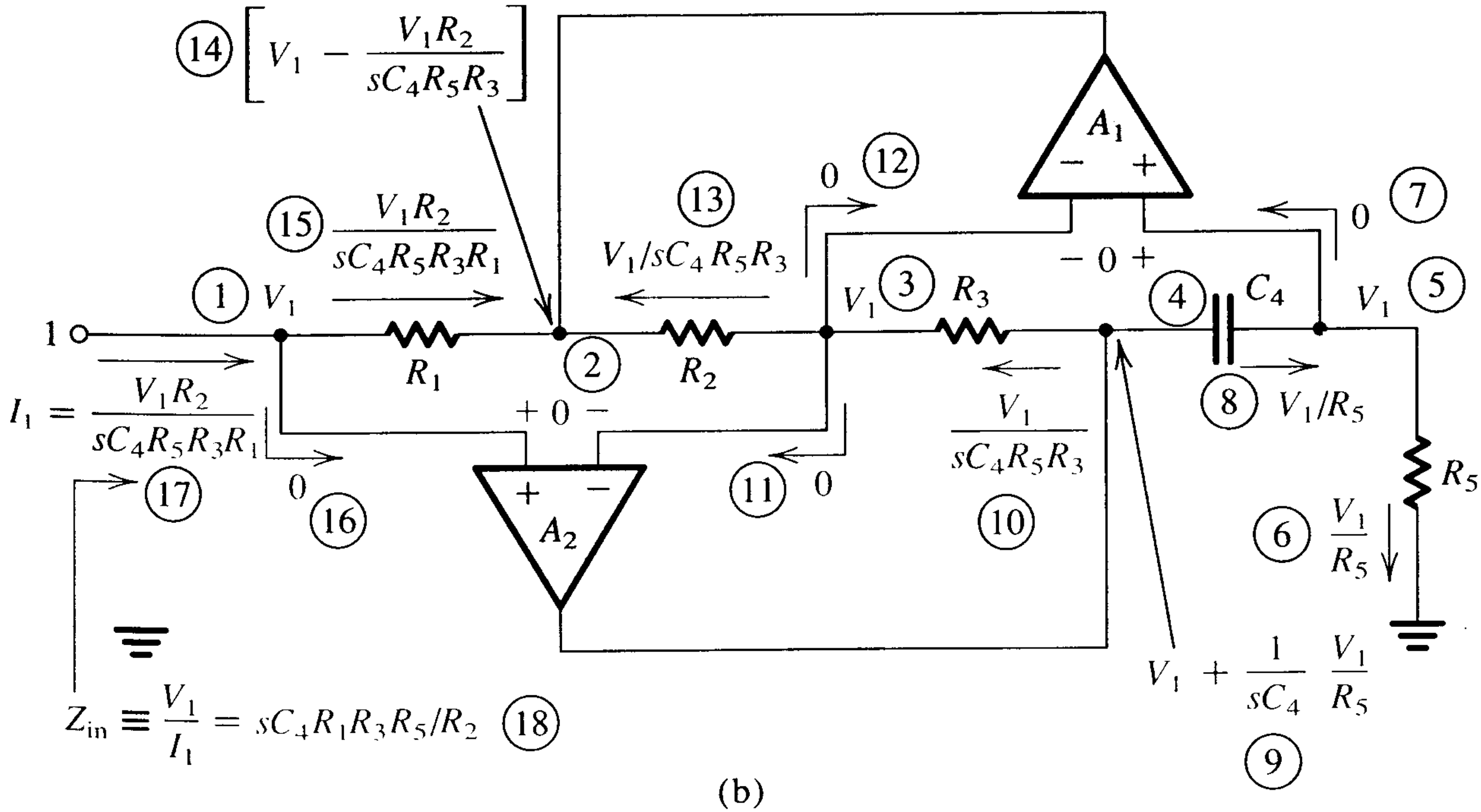
Equivalent to an inductor of value $\frac{1}{\omega C}$

- One of the best circuits for simulating an inductor is the Antoniou inductance simulation circuit. This circuit is very tolerant to the non-ideal properties of opamps

- The circuit is shown below



(a)



(b)

Fig. 11.20 **(a)** The Antoniou inductance-simulation circuit. **(b)** Analysis of the circuit assuming ideal op amps. The order of the analysis steps is indicated by the circled numbers.

The effective inductance of the circuit is

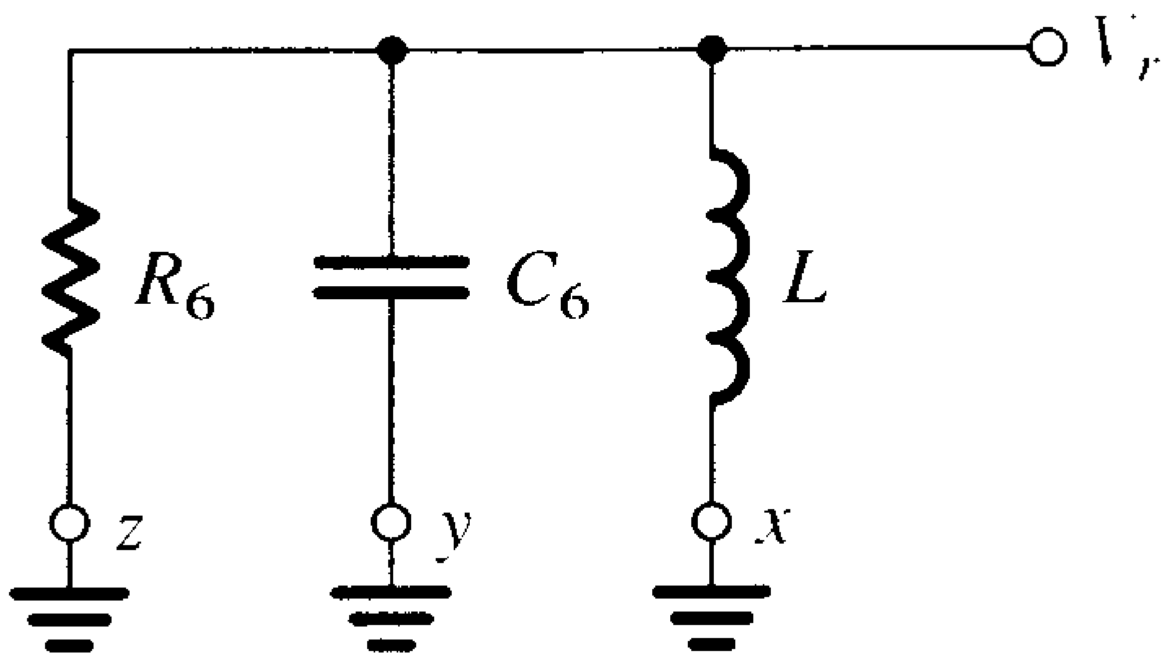
$$L = \frac{C_4 R_1 R_3 R_5}{R_2}$$

Typically $R_1 = R_2 = R_3 = R_5 = R$

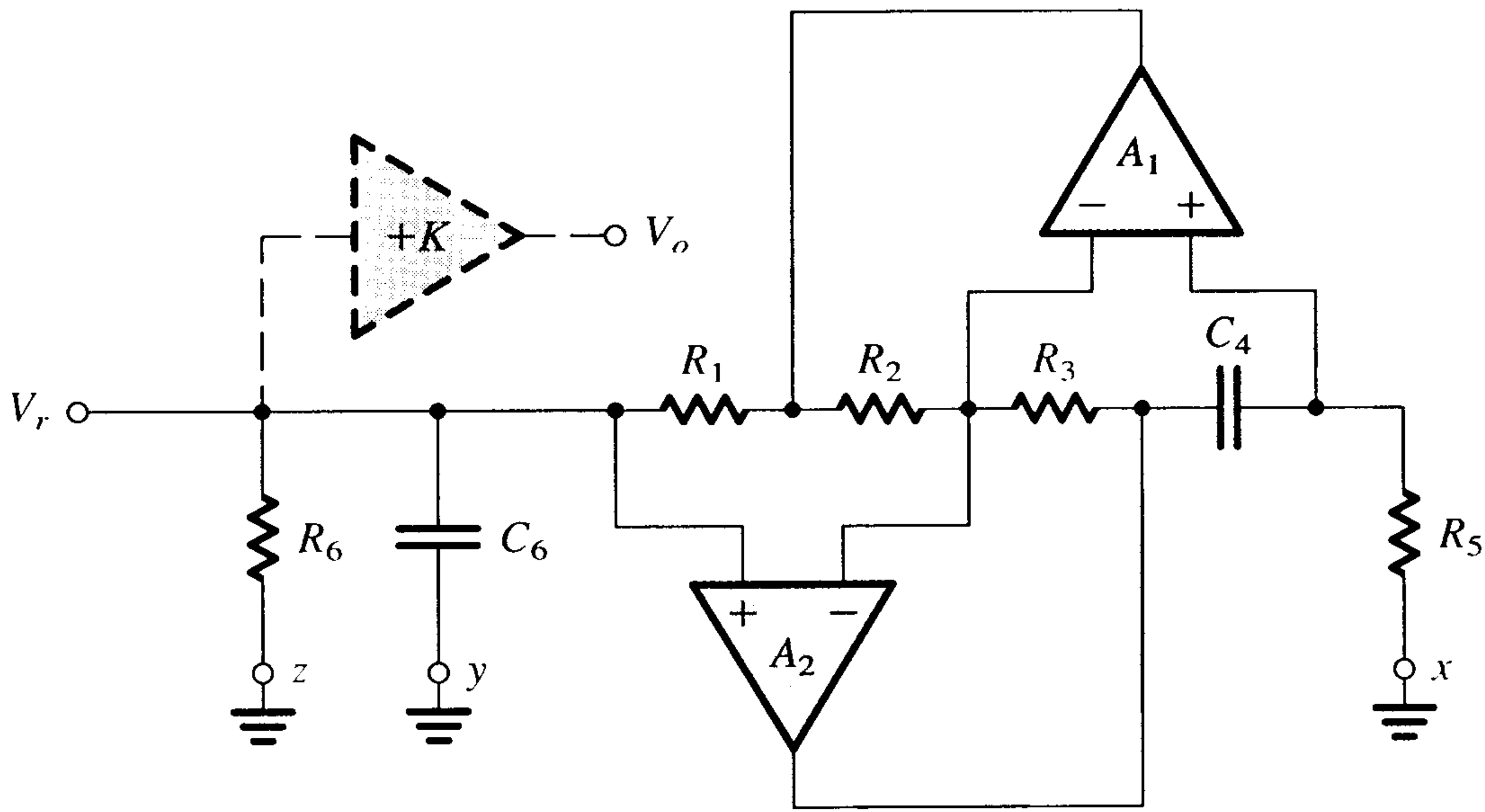
$C_4 = C$

$$L = C R^2$$

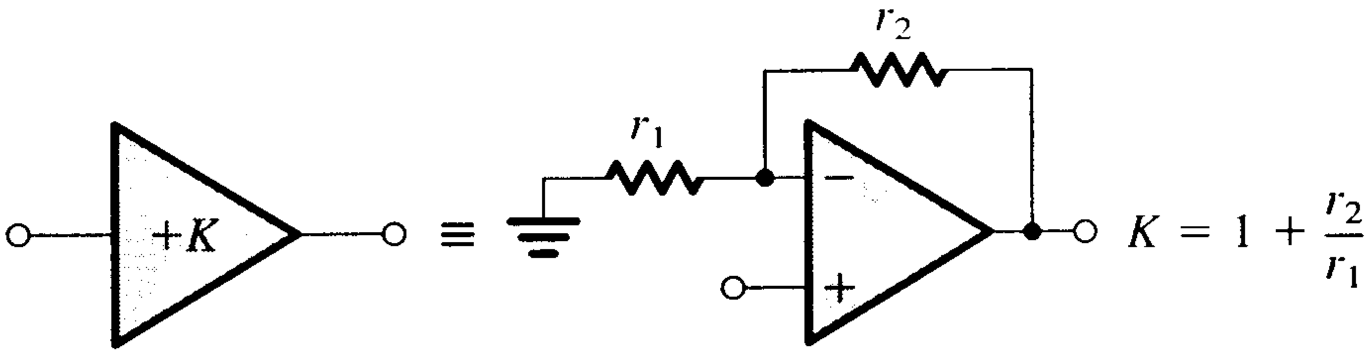
Opamp - RC Resonator



(a)



(b)



(c)

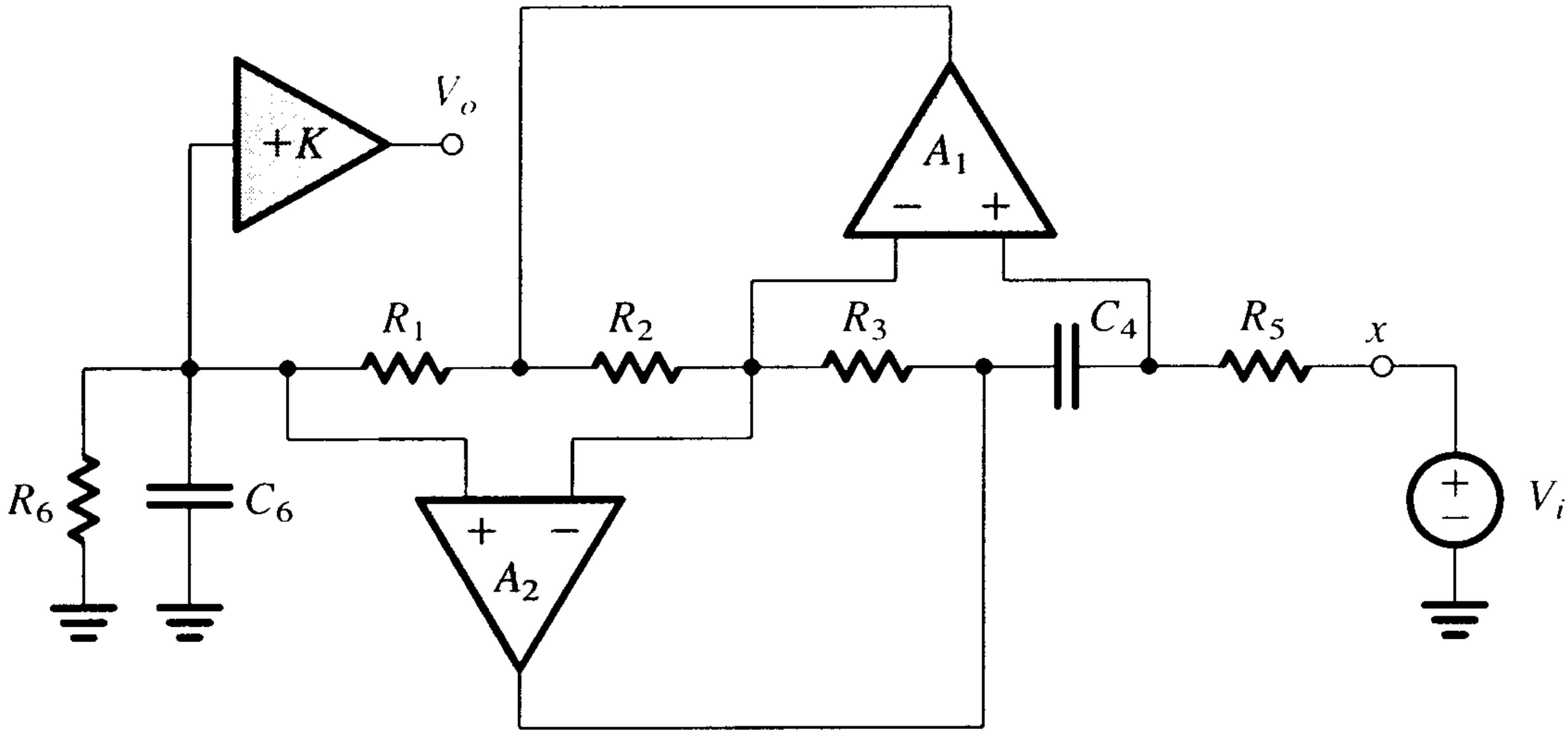
Fig. 11.21 (a) An LCR resonator. **(b)** An op amp–RC resonator obtained by replacing the inductor *L* in the LCR resonator of (a) with a simulated inductance realized by the Antoniou circuit of Fig. 11.20(a). **(c)** Implementation of the buffer amplifier *K*.

$$\omega_o = 1/\sqrt{LC_6} = 1/\sqrt{C_4 C_6 R_1 R_3 R_5 / R_2}$$

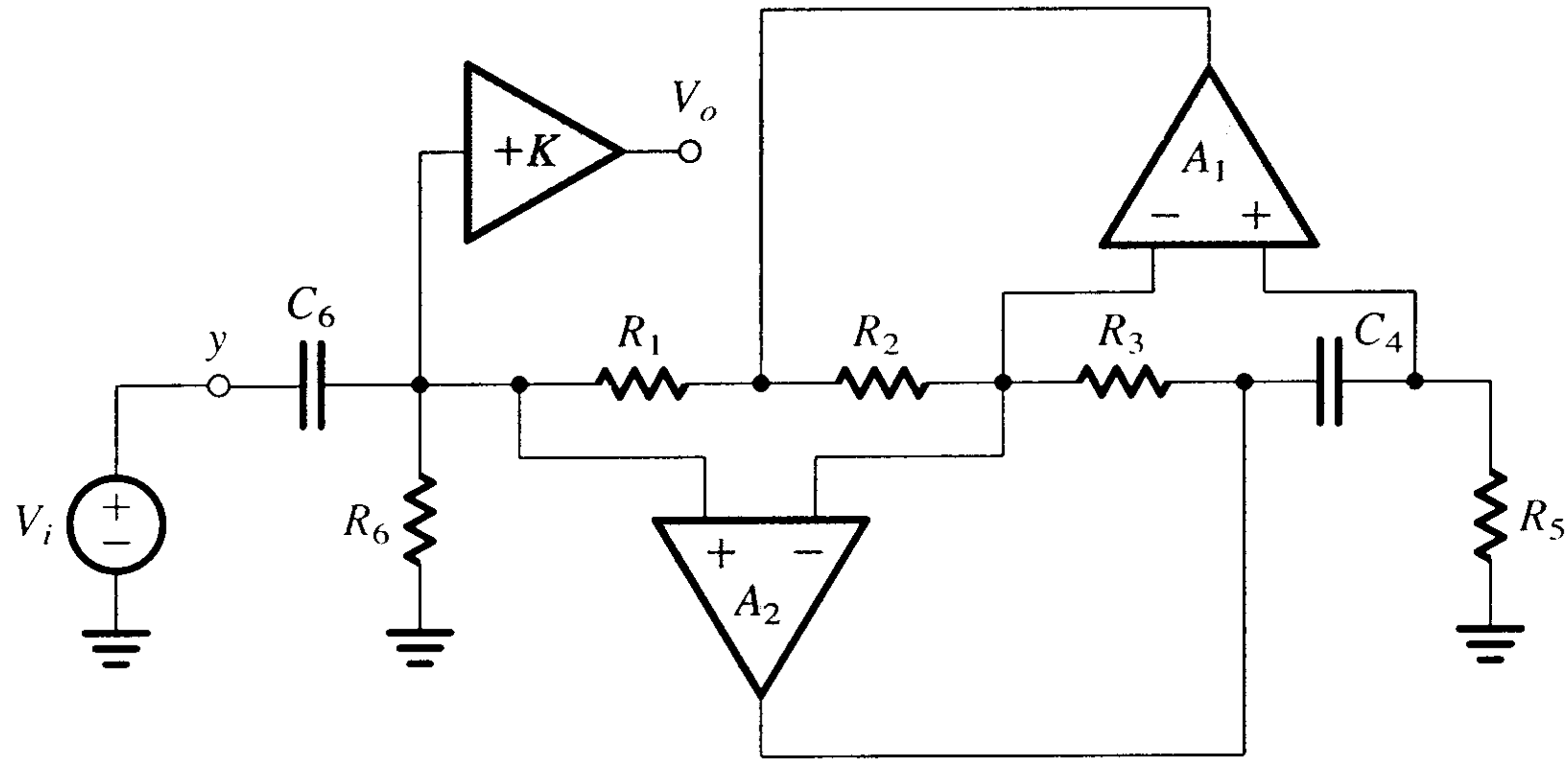
$$Q = \omega_o C_6 R_6 = R_6 \sqrt{\frac{C_6}{C_4} \frac{R_2}{R_1 R_3 R_5}}$$

If $C_4 = C_6 = C$ $\quad\quad$ $R_1 = R_2 = R_3 = R_5 = R$

$$\omega_o = 1/CR \qquad Q = R_6/R$$

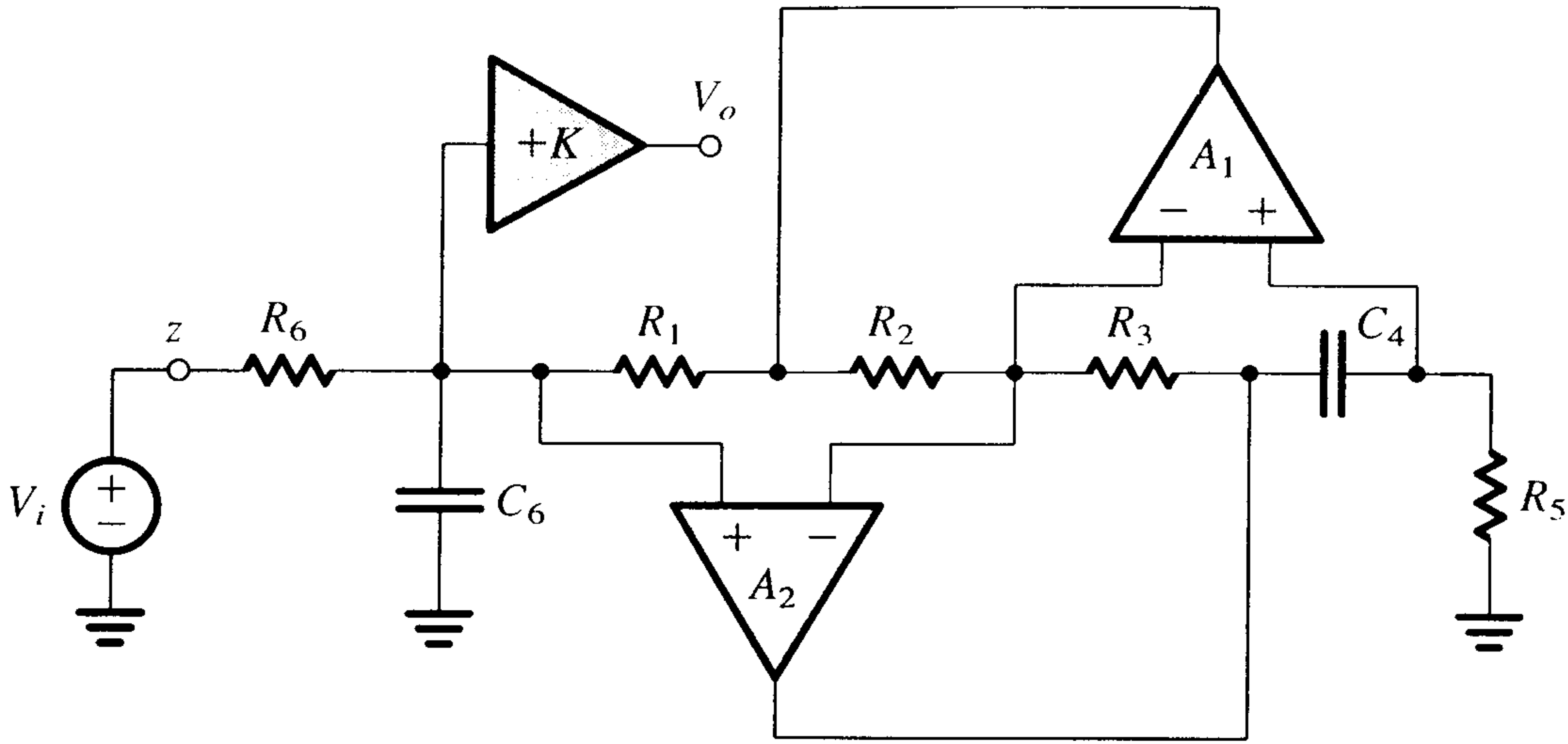


(a) LP

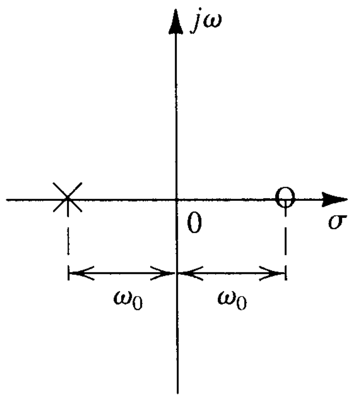


(b) HP

Fig. 11.22 Realizations for the various second-order filter functions using the op amp–RC resonator of Fig. 11.21(b). **(a)** LP; **(b)** HP; **(c)** BP, **(d)** notch at ω_0; **(e)** LPN, $\omega_n \geq \omega_0$; **(f)** HPN, $\omega_n \leq \omega_0$; **(g)** all-pass. The circuits are based on the LCR circuits in Fig. 11.18. Design equations are given in Table 11.1.



(c) BP

<table>
<tr><th>$T(s)$</th><th>Singularities</th><th>$|T|$ and ϕ</th><th>Passive Realization</th><th>Op Amp-RC Realization</th></tr>
<tr>
<td>$T(s) = -a_1 \dfrac{s - \omega_0}{s + \omega_0}$
$a_1 > 0$</td>
<td>
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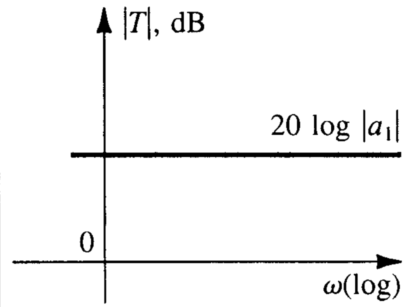
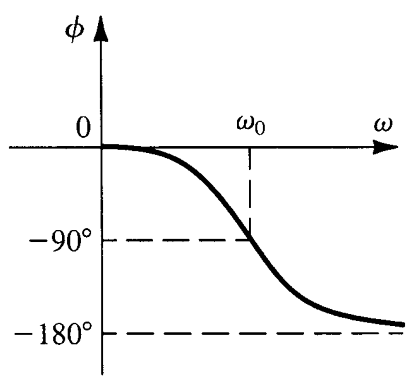
<td>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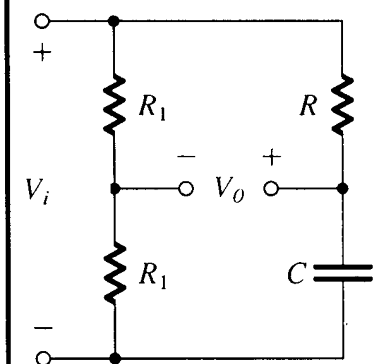
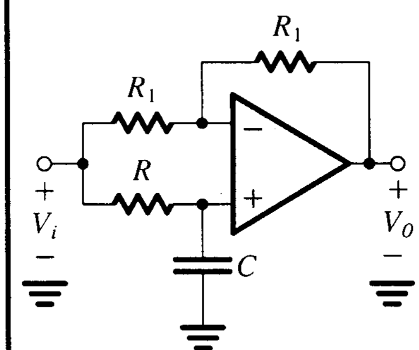
<td>

$CR = 1/\omega_0$
Flat gain $(a_1) = 0.5$</td>
<td>

$CR = 1/\omega_0$
Flat gain $(a_1) = 1$</td>
</tr>
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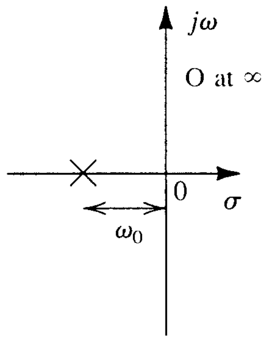
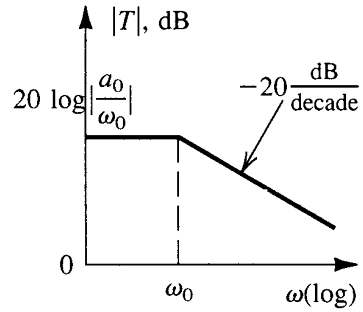
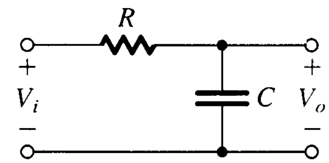
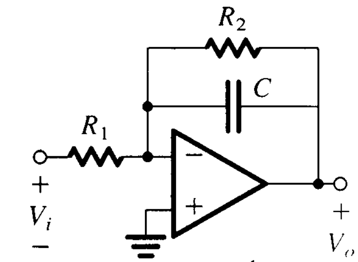
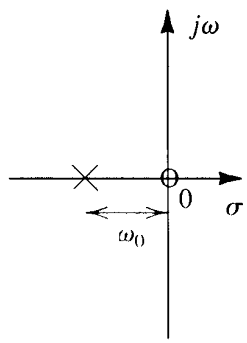
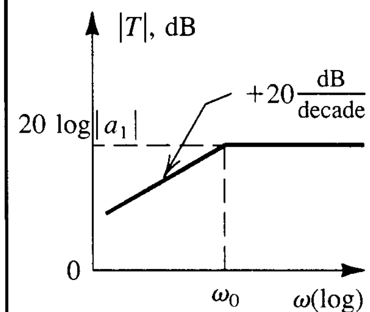
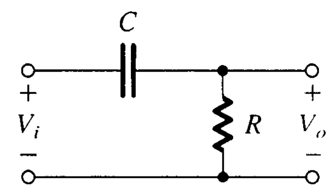
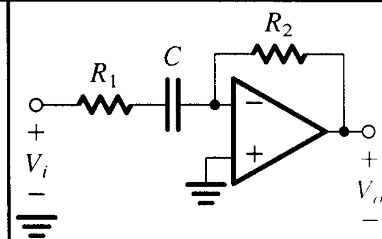
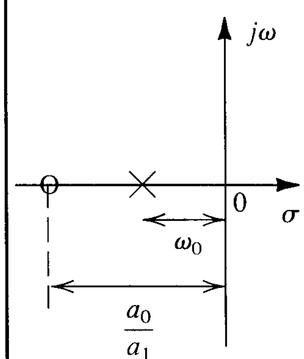
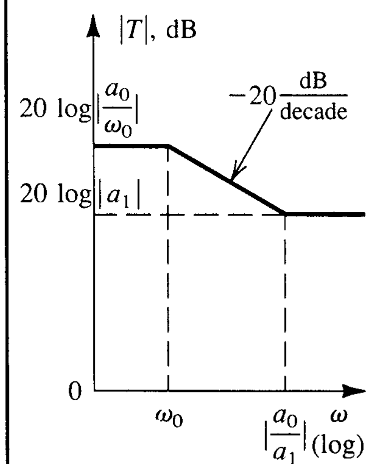
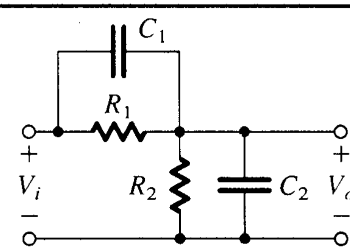
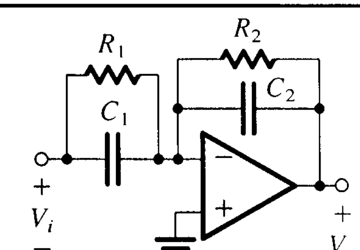
Filter Type and $T(s)$	s-Plane Singularities	Bode Plot for $\lvert T\rvert$	Passive Realization	Op Amp-RC Realization
(a) Low-Pass (LP) $T(s) = \dfrac{a_0}{s + \omega_0}$			 $CR = \dfrac{1}{\omega_0}$ dc gain = 1	 $CR_2 = \dfrac{1}{\omega_0}$ dc gain $= -\dfrac{R_2}{R_1}$
(b) High-Pass (HP) $T(s) = \dfrac{a_1 s}{s + \omega_0}$			 $CR = \dfrac{1}{\omega_0}$ High-frequency gain = 1	 $CR_1 = \dfrac{1}{\omega_0}$ High-frequency gain $= -\dfrac{R_2}{R_1}$
(c) General $T(s) = \dfrac{a_1 s + a_0}{s + \omega_0}$			 $(C_1 + C_2)(R_1 // R_2) = \dfrac{1}{\omega_0}$ $C_1R_1 = \dfrac{a_0}{a_1}$ dc gain $= \dfrac{R_2}{R_1 + R_2}$ HF gain $= \dfrac{C_1}{C_1 + C_2}$	 $C_2R_2 = \dfrac{1}{\omega_0}$ $C_1R_1 = \dfrac{a_1}{a_0}$ dc gain $= -\dfrac{R_2}{R_1}$ HF gain $= -\dfrac{C_1}{C_2}$

Second Order Functions

The general second order (biquadratic) filter transfer function is given by

$$T(s) = \frac{a_2 s^2 + a_1 s + a_o}{s^2 + \left(\frac{\omega_o}{Q}\right) s + \omega_o^2}$$

where ω_o and Q determine the poles according to

$$P_1, P_2 = \frac{-\omega_o}{Q} \pm j\omega_o \sqrt{1 - \left(1/4Q^2 \right)}$$

we are usually interested in the case of complex conjugate poles obtained for $Q > 0.5$

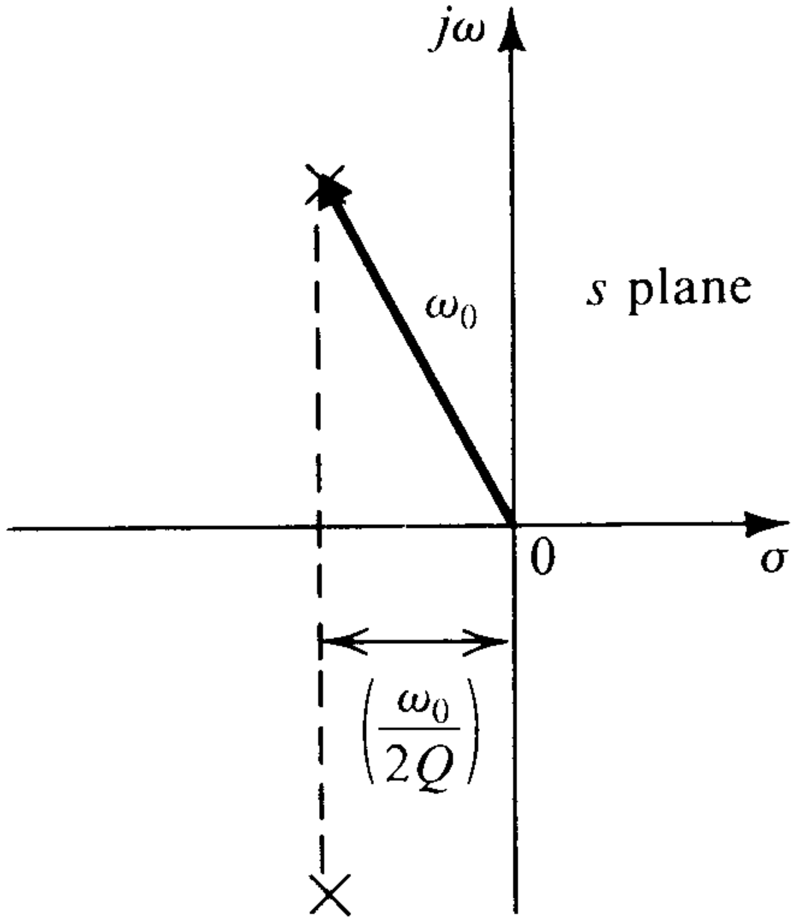
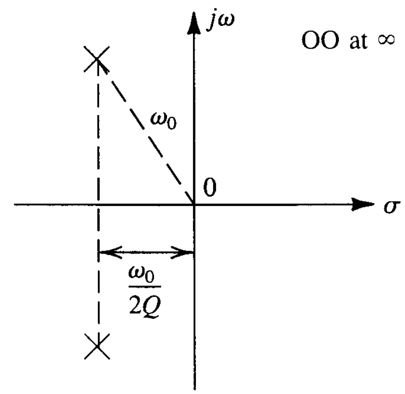
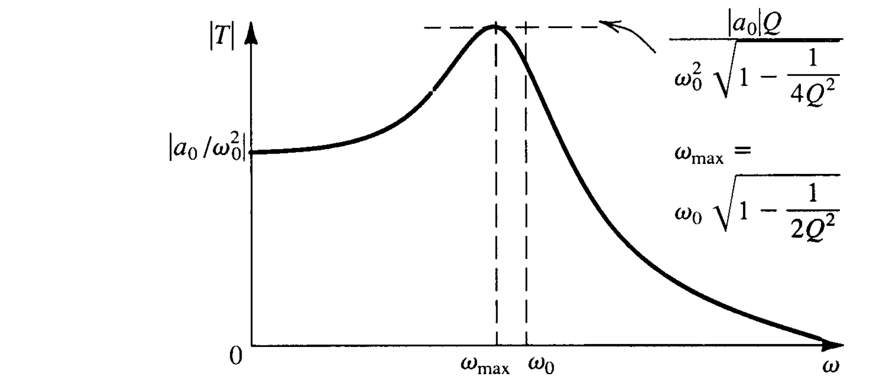
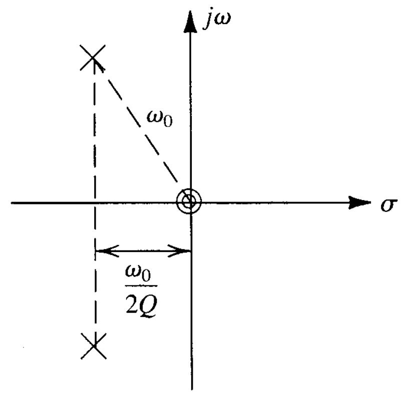
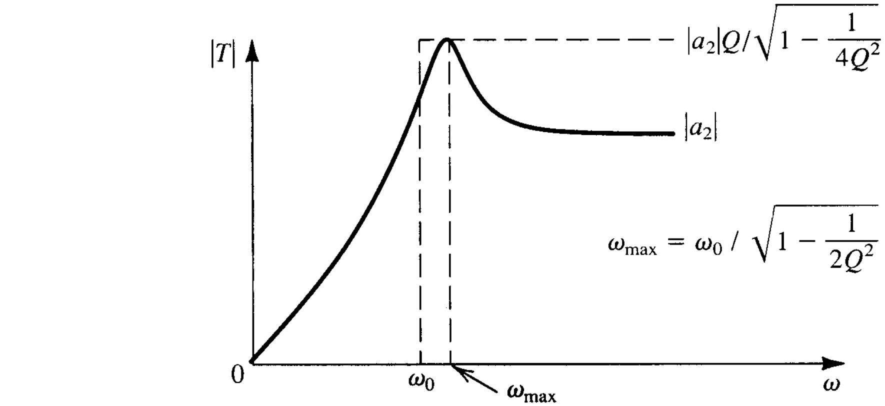
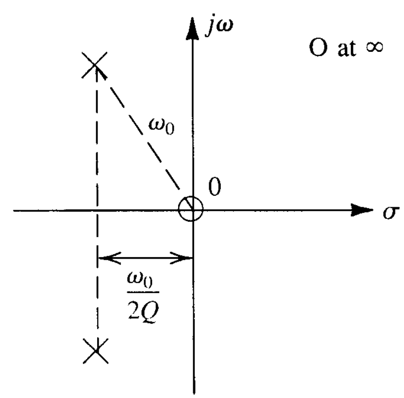
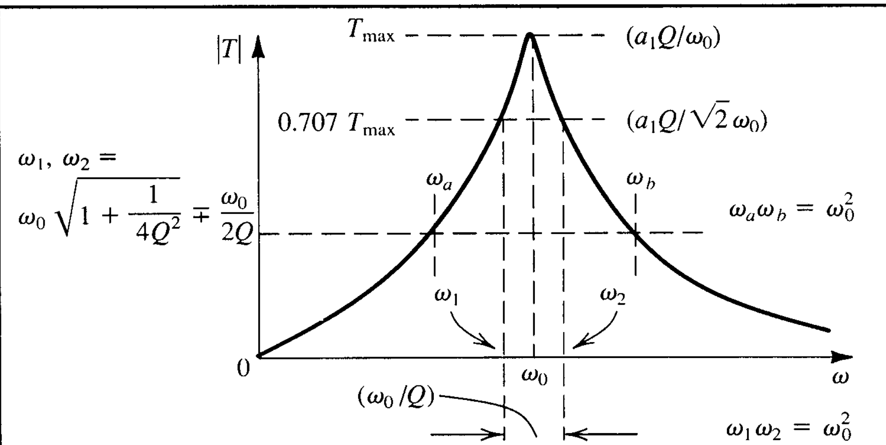
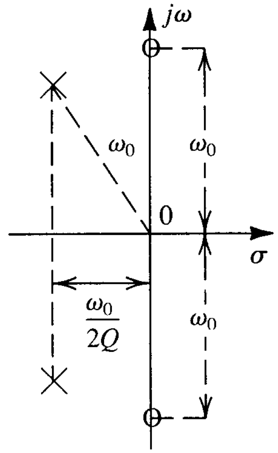
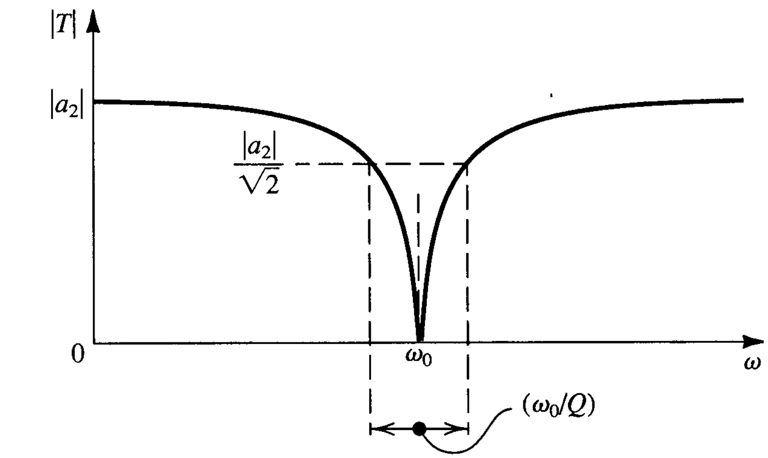
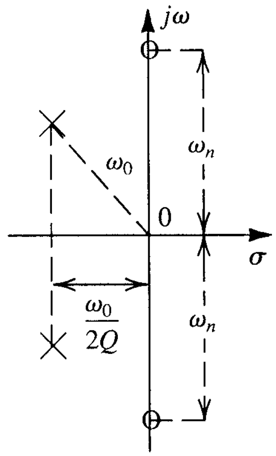
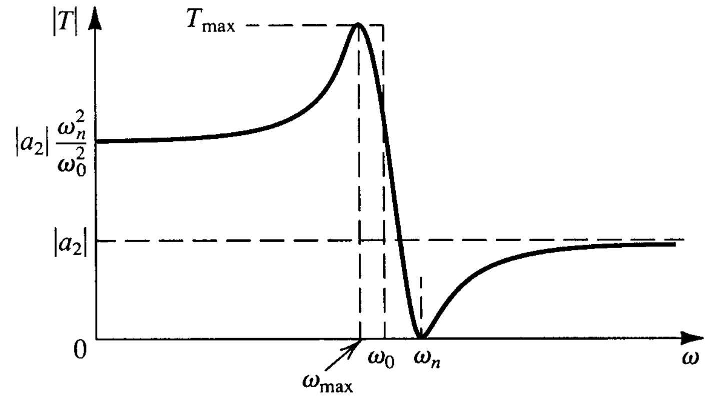
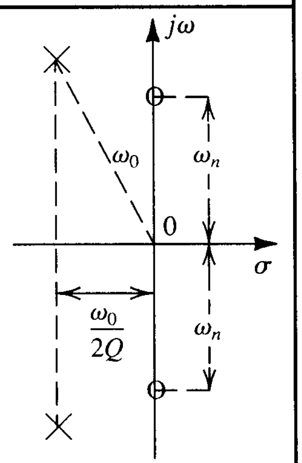
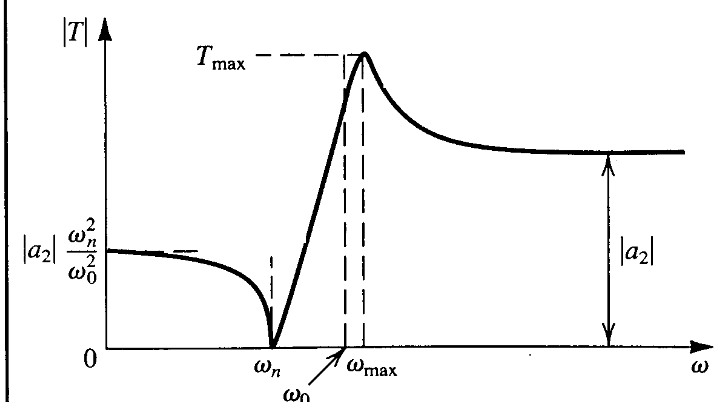


Fig. 11.15 Definition of the parameters ω_0 and Q of a pair of complex conjugate poles.

- The radial distance of the poles from the origin is the pole frequency
- The higher the value of Q the closer the poles are to the $j\omega$ axis and the more selective (higher peak and initial rolloff) the filter response becomes
- An infinite value of Q locates the poles on the $j\omega$ axis and can yield sustained oscillations
- Q is called the pole quality factor

Filter Type and $T(s)$	s-Plane Singularities	$\lvert T\rvert$
(a) Low-Pass (LP) $T(s) = \dfrac{a_0}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$ dc gain $= \dfrac{a_0}{\omega_0^2}$		
(b) High-Pass (HP) $T(s) = \dfrac{a_2 s^2}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$ High-frequency gain $= a_2$		
(c) Bandpass (BP) $T(s) = \dfrac{a_1 s}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$ Center-frequency gain $= \dfrac{a_1 Q}{\omega_0}$		

(d) Notch $T(s) = a_2 \dfrac{s^2 + \omega_0^2}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$ dc gain = high-frequency gain = a_2		
(e) Low-Pass Notch (LPN) $T(s) = a_2 \dfrac{s^2 + \omega_n^2}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$ $\omega_n \geqslant \omega_0$ dc gain = $a_2 \dfrac{\omega_n^2}{\omega_0^2}$ high-frequency gain = a_2		
(f) High-Pass Notch (HPN) $T(s) = a_2 \dfrac{s^2 + \omega_n^2}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$ $\omega_n \leqslant \omega_0$ dc gain = $a_2 \dfrac{\omega_n^2}{\omega_0^2}$ high-frequency gain = a_2		

For (e) and (f):

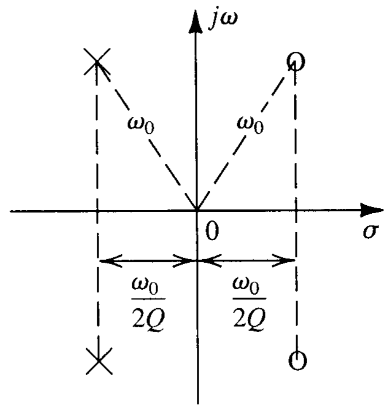
$$\omega_{max} = \omega_0 \sqrt{\frac{\left(\dfrac{\omega_n^2}{\omega_0^2}\right)\left(1 - \dfrac{1}{2Q^2}\right) - 1}{\left(\dfrac{\omega_n^2}{\omega_0^2}\right) + \dfrac{1}{2Q^2} - 1}}$$

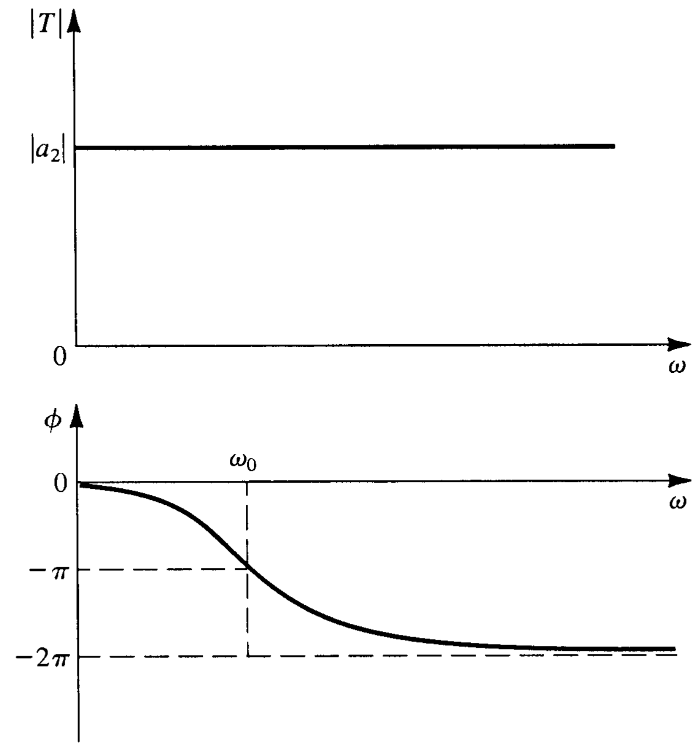
$$T_{max} = \frac{|\alpha_2| \; |\omega_n^2 - \omega_{max}^2|}{\sqrt{(\omega_0^2 - \omega_{max}^2)^2 + \left(\dfrac{\omega_0}{Q}\right)^2 \omega_{max}^2}}$$

(g) All-Pass (AP)

$$T(s) = a_2 \frac{s^2 - s\dfrac{\omega_0}{Q} + \omega_0^2}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$$

Flat gain $= a_2$





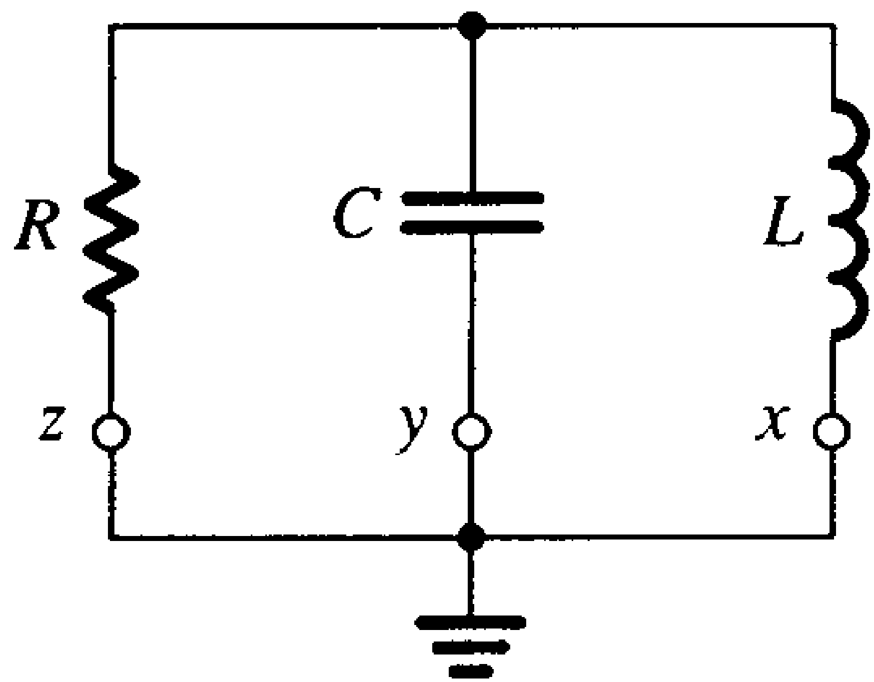
Second Order LCR Resonator

$$\frac{V_o}{I} = \frac{1}{Y} = \frac{1}{\left(1/sL \right) + sC + 1/R} = \frac{s/C}{s^2 + s\left(1/CR \right) + \left(1/LC \right)}$$

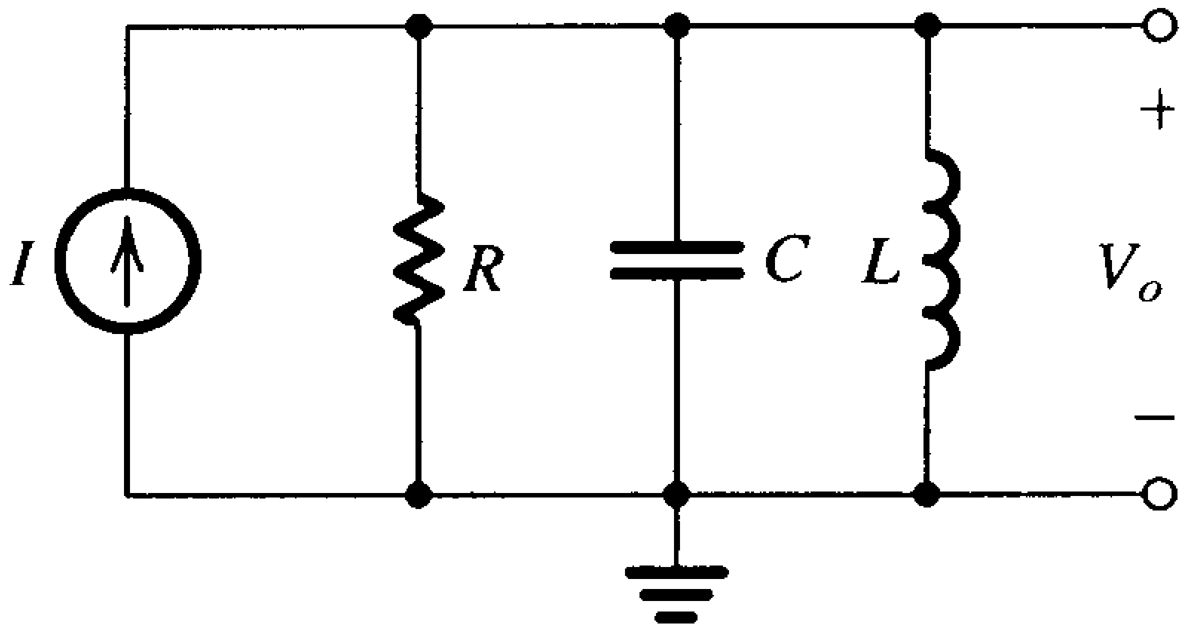
Equating the denominator to standard form gives

$$s^2 + s\left(\omega_o/Q \right) + \omega_o^2$$

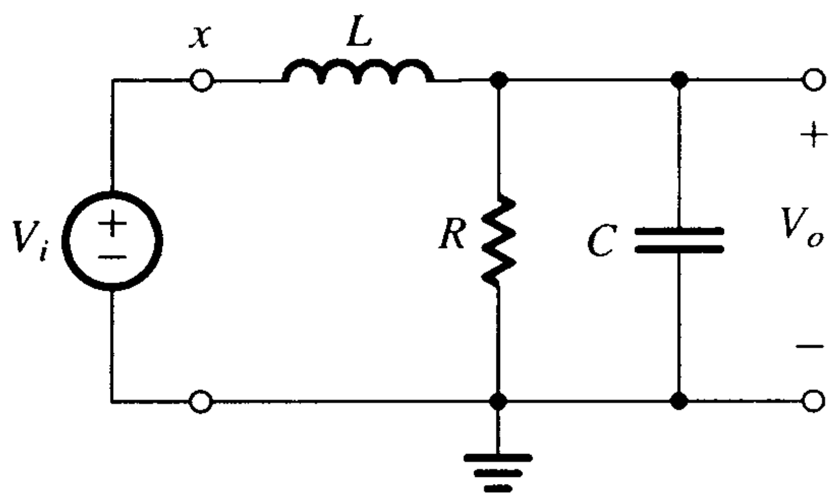
$$\omega_o^2 = 1/LC \qquad \frac{\omega_o}{Q} = 1/CR \qquad \omega_o = \frac{1}{\sqrt{LC}} \qquad Q = \omega_o C R$$



(a)

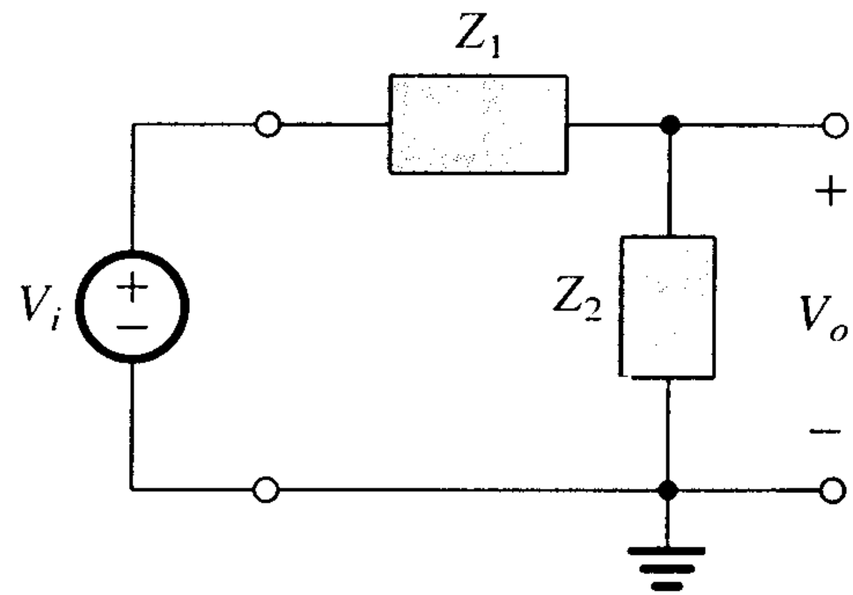


(b)

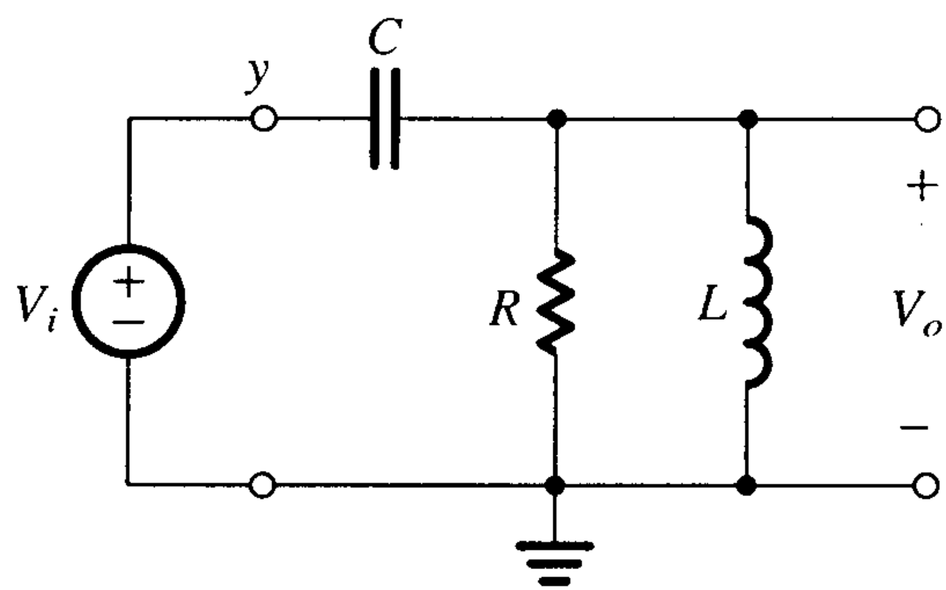


(c)

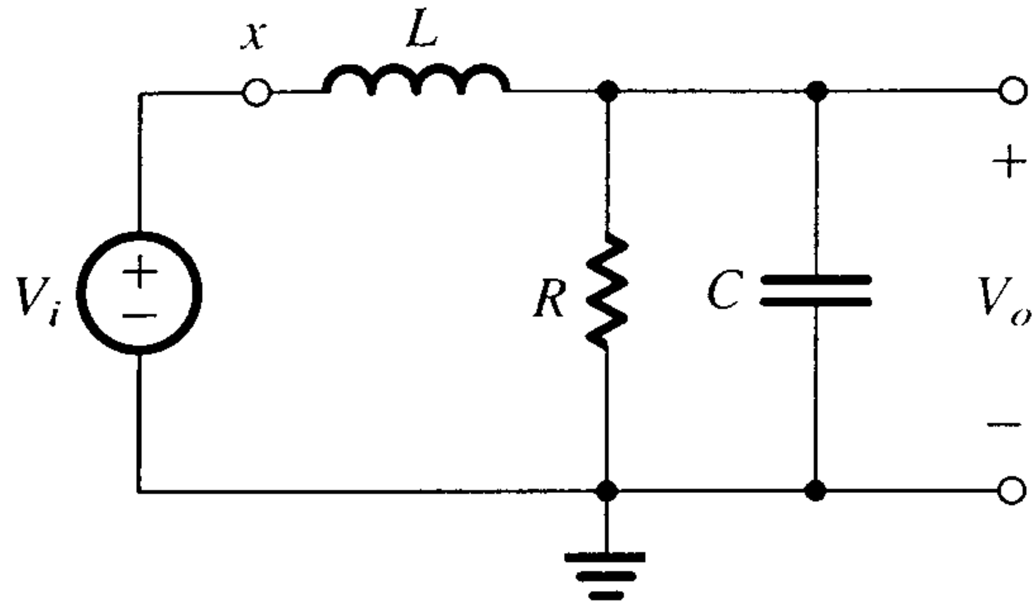
Fig. 11.17 (a) The second-order parallel LCR resonator. **(b)** and **(c)** Two ways for exciting the resonator of (a) without changing its *natural structure.* The resonator poles are the poles of V_o/I and V_o/V_i.



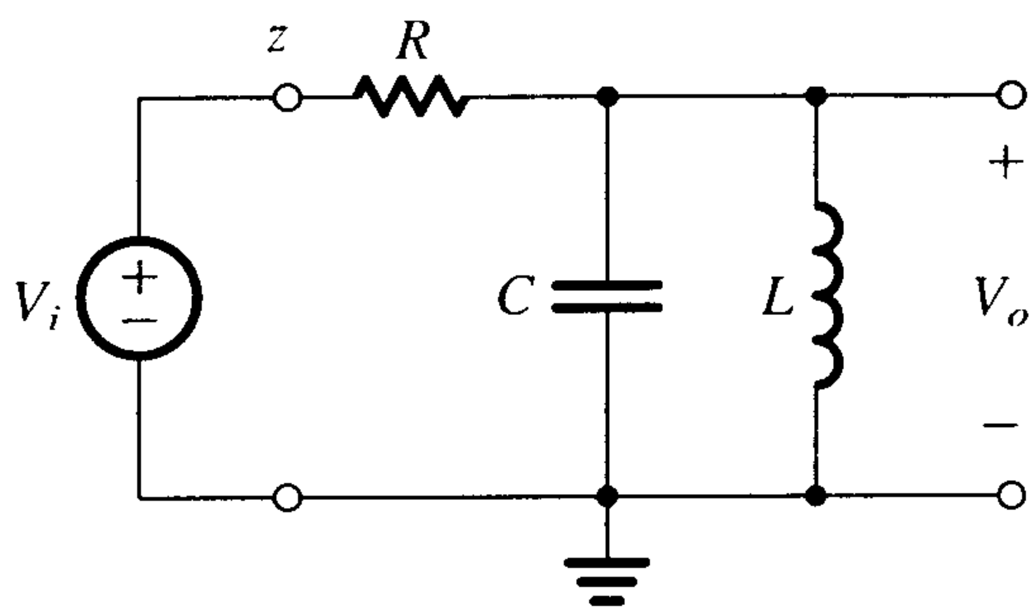
(a) General structure



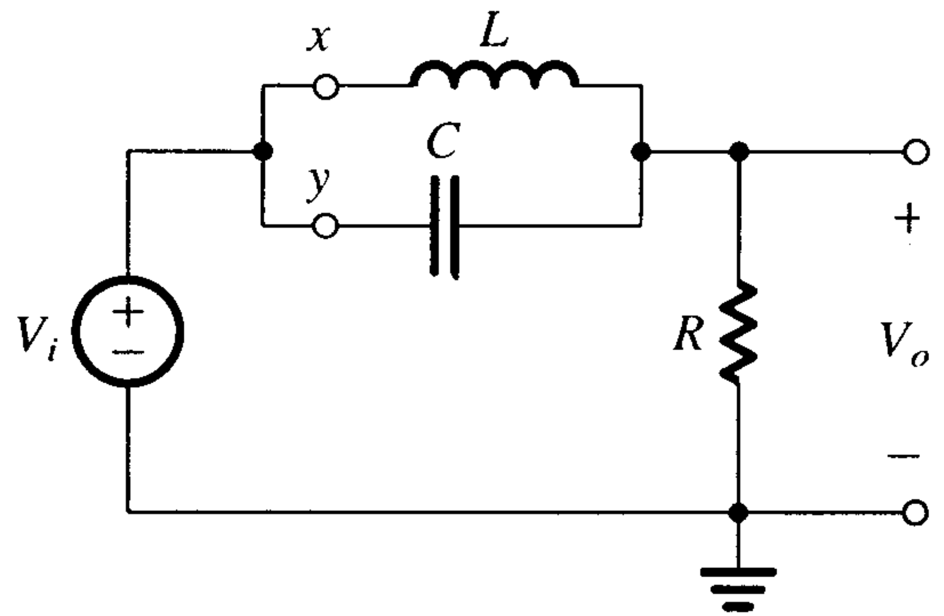
(c) HP



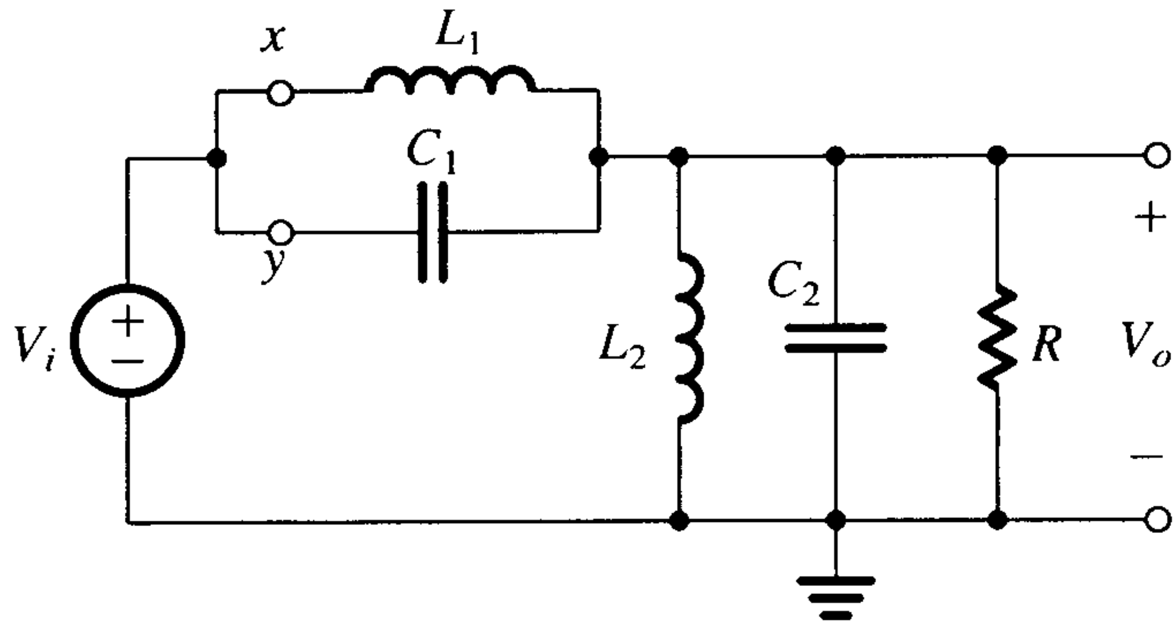
(b) LP



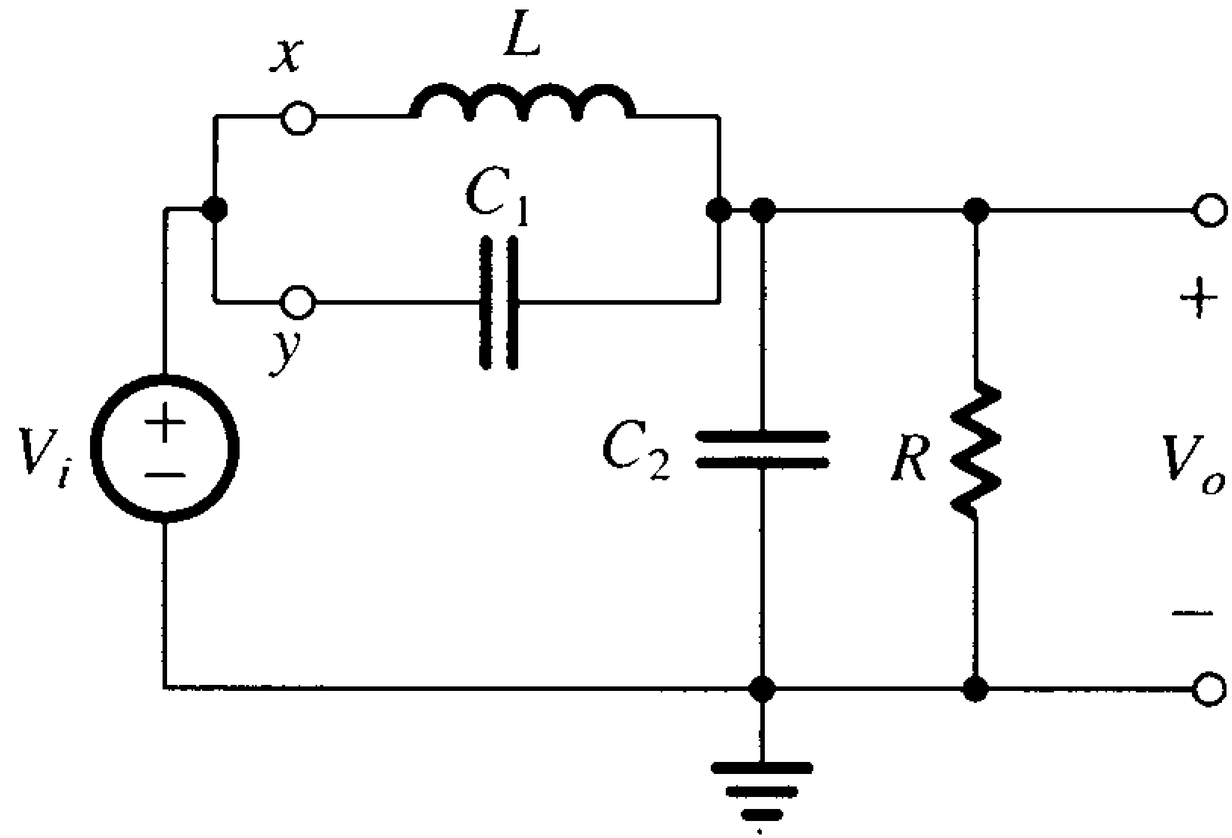
(d) BP



(e) Notch at ω_0

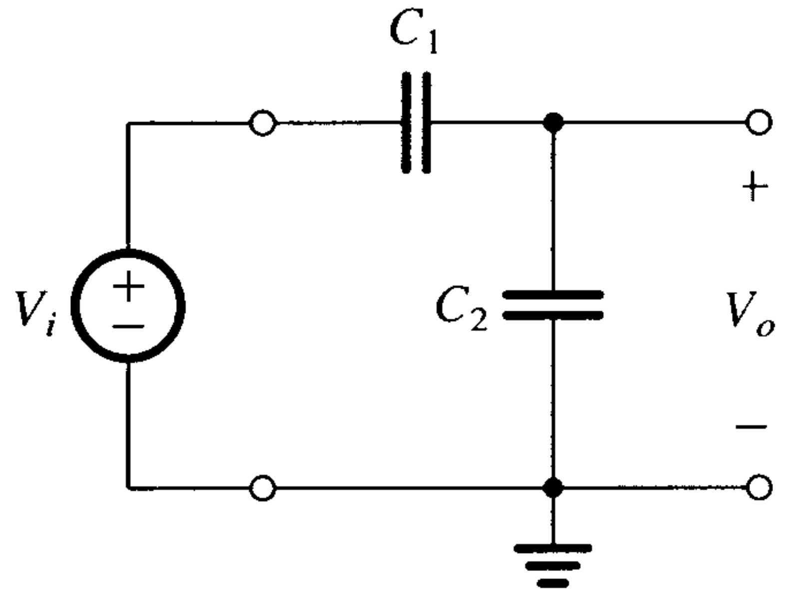


(f) General notch

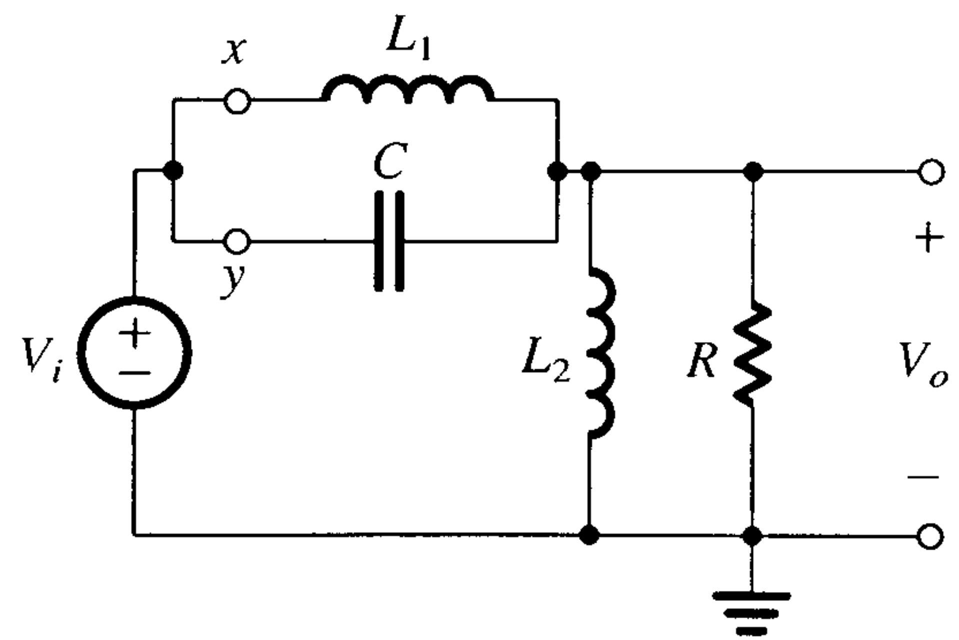


(g) LPN ($\omega_n > \omega_0$)

Fig. 11.18 Realization of various second-order filter functions using the LCR resonator of Fig. 11.17(b): **(a)** general structure, **(b)** LP, **(c)** HP, **(d)** BP, **(e)** notch at ω_0, **(f)** general notch, **(g)** LPN ($\omega_n \geq \omega_0$), **(h)** LPN as $s \to \infty$, **(i)** HPN ($\omega_n < \omega_0$).



(h) LPN as $s \rightarrow \infty$



(i) HPN ($\omega_n < \omega_0$)