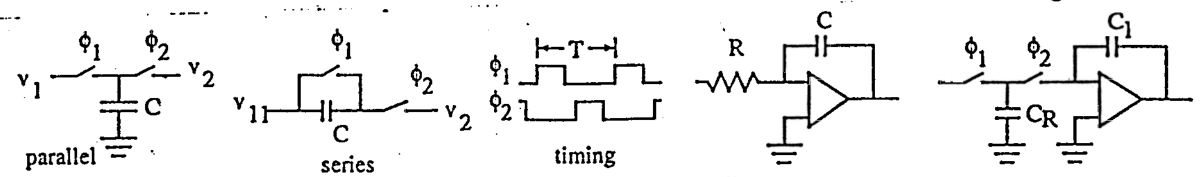
1) Switched-Capacitor Filter Summary

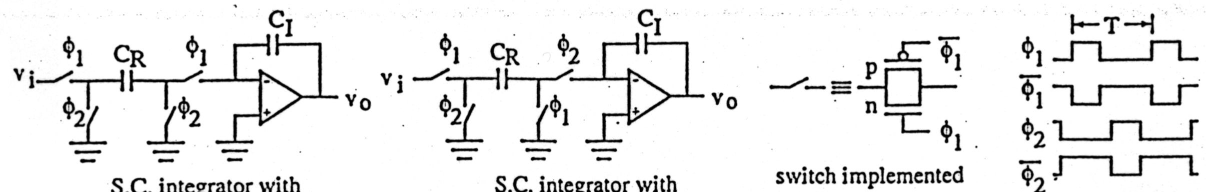


parallel — series — timing — discrete integrator — S.C. integrator

Switched-Capacitor "Resistors"

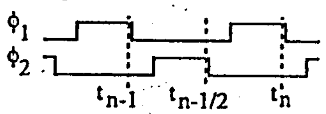
$$R_{equiv} = \frac{T}{C_R} \quad \text{thus :} \quad \frac{v_o}{v_i} = \frac{1}{sC_I R_{equiv}} = \frac{1}{sT}\frac{C_R}{C_I} = \frac{\alpha}{sT} \quad \text{where} \quad \alpha = \frac{C_R}{C_I}$$

For a general filter, the pole frequency is given by $1/(2\pi RC_I) = f_{pole} = \alpha/(2\pi T) = \alpha f_{clock}/(2\pi)$. - To avoid problems with stray capacitances, stray free resistors are used:



S.C. integrator with stray-free resistor — S.C. integrator with inverting stray-free resistor — switch implemented with pass gate — overall timing

$$\sum i = \sum \frac{\Delta Q}{T} = \sum \frac{C\Delta V}{T} = 0$$



Where ΔV represents the difference in the voltage across a capacitor before and after the transition, expressed as $v_n, v_{n-\frac{1}{2}}$ etc. Result: difference equation. For integrators shown above:

1) $v_{o,n} = v_{o,n-1} - \alpha v_{in}$ for regular integrator with t_n at the end of ϕ_1
2) $v_{o,n} = v_{o,n-1} + \alpha v_{i,n-\frac{1}{2}}$ for negative resistor with t_n at the end of ϕ_2
3) $v_{o,n} = v_{o,n-1} + \alpha v_{i,n-1}$ for negative resistor with t_n at the end of ϕ_1

Z transform - replace the subscripts $n-1$, $n-\frac{1}{2}$ and n with z^{-1}, $z^{-\frac{1}{2}}$, z^{-0} multiplier. For example, the integrators from last section become:

$$1)\ \frac{v_o}{v_i} = -\frac{\alpha}{1-z^{-1}} \qquad 2)\ \frac{v_o}{v_i} = \frac{\alpha z^{-\frac{1}{2}}}{1-z^{-1}} \qquad 3)\ \frac{v_o}{v_i} = \frac{\alpha z^{-1}}{1-z^{-1}}$$

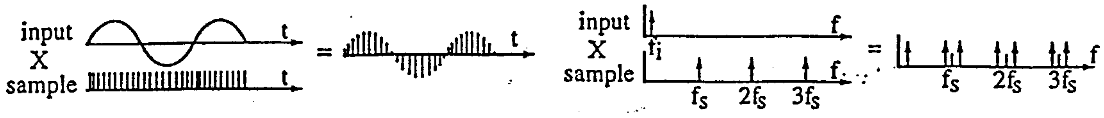
Here, z^{-1} represents a delay of 1 clock period T, where $z = e^{sT}$

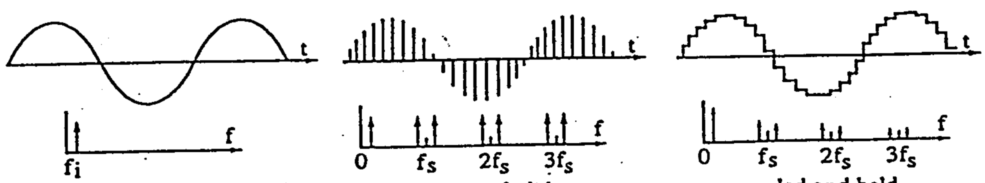
2) The z transform is just a substitution for the Laplace transform for a delay. Here, the delay $T = 1/f_s$ is one period of the sampling clock.

$$z = e^{sT} = e^{j2\pi fT} = e^{j2\pi\frac{f}{f_s}} = 1\angle 2\pi\frac{f}{f_s} = \cos\left(2\pi\frac{f}{f_s}\right) + j\sin\left(2\pi\frac{f}{f_s}\right)$$

This is a vector of magnitude 1 and angle is $\frac{f}{f_s}$ as a fraction of 360°. This results in the following table for frequency and values of z^{-1}.

$\frac{f}{f_s}$	0 →	1/8 ↘	1/4 ↓	1/2 ←	1 →
z^{-1}	$1 = 1\angle 0$	$\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}} = 1\angle -45$	$-j1 = 1\angle -90$	$-1 = 1\angle -180$	$1 = 1\angle -360 = 1\angle 0$





continuous — pure sampled data — sampled and held